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Research Article

The θ-Centralizers of Semiprime Gamma Rings

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Abstract: Let M be a 2-torsion free semiprime Γ-ring satisfying a certain assumption and θ be an endomorphism of M. Let $T: M \to M$ be an additive mapping such that $T(x\alpha y\beta x) = \theta(x)\alpha T(y)\beta\theta(x)$ holds for all $x, y \in M$ and $\alpha, \beta \in \Gamma$. Then we prove that T is a θ-centralizer. We also show that T is a θ-centralizer if M contains a multiplicative identity 1. 2010 Mathematics Subject Classification, Primary 16N60, Secondary 16W25, 16U80.

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INTRODUCTION

The notion of a Γ -ring was first introduced as an extensive generalization of the concept of a classical ring. From its first appearance, the extensions and generalizations of various important results in the theory of classical rings to the theory of Γ-rings have been attracted a wider attentions as an emerging field of research to the modern algebraists to enrich the world of algebra. Many prominent mathematicians have worked out on this interesting area of research to determine many basic properties of Γ-rings and have extended numerous remarkable results in this context in the last few decades. All over the world, many researchers are recently engaged to execute more productive and creative results of Γ-rings. We begin with the definition.

Let M and Γ be additive abelian groups. If there exists a mapping $(x, \alpha, y) \to x\alpha y$ of $M \times \Gamma \times M \to M$, which satisfies the conditions:

- $(x\alpha y) \in M$
- $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)z = x\alpha z + x\beta z$, $x\alpha(y + z) = x\alpha y + x\alpha z$
- $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ -ring.

Every ring M is a Γ-ring with M = Γ. However a Γ -ring need not be a ring. Gamma rings, more general than rings, were introduced by Nobusawa (1964). Bernes (1966) weakened slightly the conditions in the definition of Γ-ring in the sense of Nobusawa (1964).

Let M be a Γ -ring. Then an additive subgroup U of M is called a left (right) ideal of M if $M\Gamma U \subset U (U\Gamma M \subset U)$. If U is a left and a right ideal, then we say U is an ideal of M. Suppose again that M is a Γ.-ring. Then M is said to be a 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in M$. An ideal P_1 of a Γ -ring M is said to be a prime if for any ideals A and B of M, $A\Gamma B \subseteq P_1$ implies $A \subseteq P_1$ or $B \subseteq P_1$. An ideal P_2 of a Γ-ring M is said to be semiprime if for any ideal U of M, $U\Gamma U \subset P_2$ implies $U \subseteq P_2$. A Γ-ring M is said to be prime if $a\Gamma M\Gamma b = (0)$ with, $a, b \in M$ implies $a = 0$ or $b = 0$ and semiprime if $a\Gamma M\Gamma a = (0)$ with $a \in M$ implies $a = 0$. Furthermore, M is said to be commutative Γ-ring if $x\alpha y = y\alpha x$ for all $x, y \in M$ and $\alpha \in \Gamma$. Moreover, the set $Z(M) = \{x \in M : x\alpha y = y\alpha x$ for all $\alpha \in \Gamma$ and $y \in M\}$ is called the centre of the Γ-ring M.

If M is a Γ -ring, then $[x, y]_\alpha = x\alpha y - y\alpha x$ is known as the commutator of x and y with respect to α, where $x, y \in M$ and $\alpha \in \Gamma$. We make the basic commutator identities:

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$[x\alpha y, z]_\beta = [x,z]_\beta \alpha y + x[\alpha,\beta]_z y + x\alpha[y,z]_\beta$ and $[x, y\alpha z]_\beta = [x,y]_\beta \alpha z + y[\alpha,\beta]_x z + y\alpha[x,z]_\beta$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. We consider the following assumption:

- $x\alpha y\beta z = x\beta y\alpha z$ ………….. (A) for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

According to the assumption (A), the above two identities reduce to $[x\alpha y, z]_\beta = [x,z]_\beta \alpha y + x\alpha[y,z]_\beta$ and $[x, y\alpha z]_\beta = [x,y]_\beta \alpha z + y\alpha[x,z]_\beta$, which we extensively used.

An additive mapping $T: M \to M$ is a left (right) centralizer if $T(x\alpha y) = T(x)\alpha y[T(x\alpha y) = x\alpha T(y)]$ holds for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. A centralizer is an additive mapping which is both a left and a right centralizer. For any fixed $a \epsilon M$ and $\alpha \epsilon \Gamma$,, the mapping $T(x) = a\alpha x$ is a left centralizer and $T(x) = x a\alpha$ is a right centralizer. We shall restrict our attention on left centralizer, since all results of right centralizers are the same as left centralizers. An additive mapping $D : M \longrightarrow M$ is called a derivation if $D(x\alpha y) = D(x)\alpha y + x\alpha D(y)$ holds for all $x, y \epsilon M$ *and* $\alpha \epsilon \Gamma$ and is called a Jordan derivation if $D(x\alpha x) = D(x)\alpha x + x\alpha D(x)$ for all $x \epsilon M$ and $\alpha \epsilon \Gamma$. An additive mapping $T : M \longrightarrow M$ is Jordan left (right) centralizer if $T(x\alpha x) = T(x)\alpha x(T(x\alpha x) = x\alpha T(x))$ for all $x \epsilon M$ and $\alpha \epsilon \Gamma$.

Every left centralizer is a Jordan left centralizer but the converse is not in general true.

An additive mappings $T : M \longrightarrow M$ is called a Jordan centralizer if $(x\alpha y + y\alpha x) = T(x)\alpha y + y\alpha T(x)$, for all $x, y \epsilon M$ and $\alpha \epsilon \Gamma$. Every centralizer is a Jordan centralizer but Jordan centralizer is not in general a centralizer.

Bernes (1966), Luh (1969) and Kyuno (1978) studied the structure of Γ -rings and obtained various generalizations of corresponding parts in ring theory.

Zalar (1991) worked on centralizers of semiprime rings and proved that Jordan centralizers and centralizers of this rings coincide. Vukman (1997, 1999, 2001) developed some remarkable results using centralizers on prime and semiprime rings.

Ceven (2002) worked on Jordan left derivations on completely prime Γ -rings. He investigated the existence of a nonzero Jordan left derivation on a completely prime Γ-ring that makes the Γ-ring commutative with an assumption. With the same assumption, he showed that every Jordan left derivation on a completely prime Γ-ring is a left derivation on it. The commutativity condition of a prime ring has been studied by Mayne (1984) by means of a Lie ideal of R and with a nontrivial automorphism or derivation. Ullah and Chaudhary (2010) proved that if T is an additive mapping of a 2-torsion free semiprime ring with involution * satisfying $T(xx^*) = T(x)\theta(x^*) = \theta(x^*)T(x)$, then t is a θ-centralizer.

Hoque and Paul (2011) have proved that every Jordan centralizer of a 2-torsion free semiprime Γ -ring M satisfying a centain assumption is a centralizer. Hoque and Paul (2012) have proved that if T is an additive mapping on a 2-torsion free semiprimeΓ -ring M with a certain assumption such that $T(x\alpha y\beta x) = x\alpha T(y)\beta x$, for all $x, y \epsilon M$ and $\alpha,\ \beta \epsilon \Gamma$, then T is a centralizer.

Ullah and Chaudhary (2012), have proved that every Jordan θ-centralizer of a 2-torsion free semiprime Γ -ring is a θ-centralizer. Hoque *et al*. (2012) proved that T is a θ-centralizer by using a relation $2T(a\alpha b\beta a) = T(a)\alpha\theta(b)\beta\theta(a) + \theta(a)\alpha\theta(b)\beta T(a)$, where T is an additive mapping on a 2-torsion free semiprimeΓ -ringM.

Our research works inspired by the works of Hoque *et al*. (2012) in Γ -rings with θ-centralizers. Here we prove that if M is a 2-torsion free semiprime Γ-ring satisfying the assumption (A) and if $T : M \longrightarrow M$ is an additive mapping such that:

$$T(x\alpha y\beta x) = \theta(x)\alpha T(y)\beta\theta(x) \tag{1}$$

For all $x, y \epsilon M, \alpha, \beta \epsilon \Gamma$ and θ an endomorphism on M, then T is θ -centralizer. Also we show that T is a centralizer if M contains a multiplicative identity 1.

THE θ-CENTRALIZER OF SEMIPRIME GAMMA RINGS

In this section, we have given the following definitions:

Let M be a 2-torsion free semiprime Γ -ring and let θ be an endomorphism of M. An additive mapping $T : M \longrightarrow M$ is a left (right) θ-centralizer if $T(x\alpha y) = T(x)\alpha\theta(y)(T(x\alpha y) = \theta(x)\alpha T(y))$ holds for all $x, y \epsilon M$and $\alpha \epsilon \Gamma$. If t is a left and a right θ-centralizer, then it is natural to call T a θ-centralizer.

Let M be a Γ-ring and let $\alpha \epsilon M$and $\alpha \epsilon \Gamma$ be fixed element. Let $\theta : M \longrightarrow M$ be an endomorphism. Define a mapping $T : M \longrightarrow M$ by$T(x) a\alpha\ \theta(x)$. Then it is clear that is a left θ-centralizer. If $T(x) = \theta(x)\alpha a$ is defined, then T is a right θ-centralizer.

An additive mapping $T : M \longrightarrow M$ is a Jordan left (right) θ-centralizer if $T(x\alpha x) = T(x)\ \alpha\theta(x)(T(x\alpha x) = \theta(x)\alpha T(x))$holds for all $x \epsilon M$and$\alpha \epsilon \Gamma$.

It is obvious that every left θ-centralizer is a Jordan left θ-centralizer but in general Jordan left θ-centralizer is not a left θ-centralizer.

Example 2.1 Let M be a Γ-ring and let $d : M \longrightarrow M$be a left θ-centralizer, where $\theta : M \longrightarrow M$is an endomorphism. Define $M_1 = \{(x,x) : x \epsilon M\}$and $\Gamma_1 = \{(\alpha,\alpha) : \alpha \epsilon \Gamma\}$. The addition and multiplication on M_1 are defined as follows: $(x,x)+(y,y)=(x+y,x+y)$ and $(x,x)(\alpha,\alpha)(y,y)=(x\alpha y,x\alpha y)$. Under these addition and multiplication, it is clear that M_1 is a Γ_1-ring.

Define $T\big((x,x)\big)=(d(x),d(x))$ and$\theta_1\big((x,x)\big)=(\theta(x),\theta(x))$. Then T is an additive mapping on $M_1$and θ_1 is an endomorphism on M_1 . Now, let $a=(x,x)$, $\gamma=(\alpha,\alpha)$,then we have:

$$\begin{aligned}T(a\gamma a) &= T\big((x,x)(\alpha,\alpha)(x,x)\big)\\ &= T((x\alpha x,x\alpha x))\\ &= (d(x\alpha x),d(x\alpha x))\\ &= (d(x)\alpha\theta(x),d(x)\alpha\theta(x))\\ &= (d(x),d(x))(\alpha,\alpha)(\theta(x),\theta(x))\\ &= T((x,x))(\alpha,\alpha)\theta_1((x,x))\\ &= T(a)\gamma\theta_1(a).\end{aligned}$$

Therefore, T is a Jordan left θ_1-centralizer which is not a left θ-centralizer.

Let M be a Γ-ring and let θ be an endomorphism on M. An additive mapping $T : M \longrightarrow M$ is called a Jordan θ-centralizer if $T(x\alpha y+y\alpha x)=T(x)\alpha\theta(y)+\theta(y)\alpha T(x)$, for all $x,y \epsilon M$and $\alpha \epsilon \Gamma$. It is clear that every θ-centralizer is a Jordan θ-centralizer but the converse is not in general a θ-centralizer.

An additive mapping $D : M \longrightarrow M$ is a (θ,θ)-derivation if $D(x\alpha y)=D(x)\alpha\theta(y)+\theta(x)\alpha D(y)$ holds for all $x,y \epsilon M$and $\alpha \epsilon \Gamma$ and is called a Jordan (θ,θ)-derivation if $D(x,x)=D(x)\alpha\theta(x)+\theta(x)\alpha D(x)$ holds for all $x \epsilon M$ and $\alpha \epsilon \Gamma$.Now we begin with two examples which are ensure that a θ-centralizer and a Jordan θ-centralizer exist in Γ-ring.

Example 2: Let M be a Γ-ring satisfying the assumption (A) and let a be a fixed element of M such that $a \epsilon Z(M)$, the centre of M. Define a mapping $T : M \longrightarrow M$by$(x)=a\alpha\theta(x)+\theta(x)\alpha a$, where $\theta : M \longrightarrow M$ is an endomorphism and $\alpha \epsilon \Gamma$is a fixed element.Then for all $x,y \epsilon M$and Γ , we have:

$$\begin{aligned}T(x\beta y) &= a\alpha\theta(x\beta y)+\theta(x\beta y)\alpha a\\ &= a\alpha\theta(x)\beta\theta(y)+\theta(x)\beta\theta(y)\alpha a\\ &= a\alpha\theta(x)\beta\theta(y)+\theta(x)\alpha\theta(y)\beta a, \{ using\ (A)\}\\ &= a\alpha\theta(x)\beta\theta(y)+\theta(x)\alpha a\beta\theta(y), \{a\epsilon Z(M)\}\\ &= (a\alpha\theta(x)+\theta(x)\alpha a)\beta\theta(y)\end{aligned}$$

$$\begin{aligned}\text{Also, } T(x\beta y) &= a\alpha\theta(x)\beta\theta(y)+\theta(x)\beta\theta(y)\alpha a\\ &= \theta(x)\alpha a\beta\theta(y)+\theta(x)\beta\theta(y)\alpha a, \{a\epsilon Z(M)\}\\ &= \theta(x)\beta\alpha a\theta(y)+\theta(x)\beta\theta(y)\alpha a, \{using\ (A)\\ &= \theta(x)\beta(\alpha a\theta(y)+\theta(y)\alpha a)\\ &= \theta(x)\beta T(y).\end{aligned}$$

Therefore, T is a left and right θ $-$centralizer, so T is a $\theta - centralizer.$

Example 3: Let M be a Γ-ring and let $d : M \to M$ be a θ-centralizer, where $\theta : M \to M$ is an endomorphism. Define $M_1 = \{(x,x) : x \in M\}$ and $\Gamma_1 = \{(\alpha,\alpha) : \alpha \in \Gamma\}$. Define addition and multiplication on M_1 as follows: $(x,x)+(y,y)=(x+y,x+y)$ and $(x,x)(\alpha,\alpha)(y,y)=(x\alpha y,x\alpha y)$. Under these addition and multiplication M_1 is a Γ_1-ring.Define $T\big((x,x)\big)=\big(d(x),d(x)\big)$and $\theta_1((x,x))=\big(\theta(x),\theta(x)\big)$. Then T is additive mapping and θ_1 is an endomorphism on M_1. Let $a=(x,x), \gamma=(\alpha,\alpha)$, $b=(y,y)$. Then we have:

$$\begin{aligned}T(a\gamma b+b\gamma a) &= T(x\alpha y+y\alpha x\ ,y\alpha x+x\alpha y)\\ &= (d(x\alpha y+y\alpha x),d(y\alpha x+x\alpha y))\\ &= (d(x\alpha y)+d(y\alpha x),d(y\alpha x)+d(x\alpha y))\\ &= \begin{pmatrix}d(x)\alpha\theta(y)+\theta(y)\alpha d(x),\theta(y)\alpha d(x)\\ +d(x)\alpha\theta(y)\end{pmatrix}\\ &= \big(d(x)\alpha\theta(y),d(x)\alpha\theta(y)\big)\\ &\quad+\big(\theta(y)\alpha d(x),\theta(y)\alpha d(x)\big)\\ &= \big(d(x),d(x)\big)(\alpha,\alpha)\big(\theta(y),\theta(y)\big)\\ &\quad+\big(\theta(y),\theta(y)\big)(\alpha,\alpha)\big(d(x),d(x)\big)\\ &= T\big((x,x)\big)(\alpha,\alpha)\theta_1\big((y,y)\big)\\ &\quad+\theta_1\big((y,y)\big)(\alpha,\alpha)T\big((x,x)\big)\\ &= T(a)\gamma\theta_1(b)+\theta_1(b)\gamma T(a)\end{aligned}$$

Therefore, T is a Jordan θ_1-centralizer on M_1 which is not a θ-centralizer.

For proving our main results, we need the following Lemmas:

Lemma 1: (7, Lemma 1) Suppose$M$ is a semiprime Γ-ring satisfying the assumption (A). Suppose that the relation $a\alpha x\beta b+b\alpha x\beta c=0$ holds for all $x \in M$, some $a,b,c \in M$ and $\alpha,\beta \in \Gamma$. Then $(a+c)\alpha x\beta b=0$ is satisfied for all $x \in M$ and $\alpha,\beta \in \Gamma$.

Lemma 2: Let M be a 2-torsion free semiprimeΓ –ring satisfying the assumption (A) and let $T : M \to M$ be an additive mapping. Suppose that $T(x\alpha y\beta x)=\theta(x)\alpha T(y)\beta\theta(x)$holds for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. Then:

(i) $[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta=0$
(ii) $\theta(x)\beta[T(x),\theta(x)]_\alpha\,\gamma\theta(x)=0$
(iii) $\theta(x)\beta[T(x),\theta(x)]_\alpha=0$
(iv) $[T(x),\theta(x)]_\alpha\,\beta\theta(x)=0$
(v) $[T(x),\theta(x)]_\alpha=0.$

Proof: We prove (i)

$$[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta=0 \tag{2}$$

For linearization, we put $x+z$ for x in relation(1),we obtain:

$$T(x\alpha y\beta z + z\alpha y\beta x) = \theta(x)\alpha T(y)\beta\theta(z) + \theta(z)\alpha T(y)\beta\theta(x) \quad (3)$$

Replacing y for x and z for y in (3), we have:

$$T(x\alpha x\beta y + y\alpha x\beta x) = \theta(x)\alpha T(x)\beta\theta(x) + \theta(y)\alpha T(x)\beta\theta(x) \quad (4)$$

For $z = (x\alpha)^2 x$ in relation (3) reduces to:

$$T\left(x\alpha y\beta(x\alpha)^2 x + (x\alpha)^2 x\alpha y\beta x\right) = \theta(x)\alpha T(y)\beta\theta\left((x\alpha)^2\right)\theta(x) + \theta\left((x\alpha)^2\right)\theta(x)\alpha T(y)\beta\theta(x) \quad (5)$$

Putting $y = x\alpha y\beta x$ in (4), we obtain:

$$T\left((x\alpha)^2 x\beta y\beta x + x\beta y\beta(x\alpha)^2\right) = \theta(x)\alpha T(x)\beta\theta(x)\alpha\theta(y)\beta\theta(x) + \theta(x)\alpha\theta(y)\beta\theta(x)\alpha T(x)\beta\theta(x) \quad (6)$$

The substitution $x\alpha x\beta y + y\alpha x\beta x$ for y in the relation (1) gives:

$$T\left((x\alpha)^2 x\beta y\beta x + (x\alpha)^2 x\beta y\beta x\right) = \theta(x)\alpha T(x\alpha x\beta y + y\beta x\alpha x)\beta\theta(x)$$

which gives because of (4):

$$T\left((x\alpha)^2 x\beta y\beta x + x\beta y\beta(x\alpha)^2 x\right) = \theta(x\alpha)^2 T(x)\beta\theta(y)\beta\theta(x) + \theta(x)\alpha\theta(y)\beta T(x)\alpha\theta(x)\beta\theta(x) \quad (7)$$

Combining (6) and (7), we arrive at:

$$T(x)\alpha[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x) - \theta(x)\alpha\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x) = 0 \quad (8)$$

Using Lemma 2 in the above relation, we have:

$$\theta(x)\alpha\theta(y)\beta T(x)\alpha\theta(x)\beta\theta(x) - \theta(x)\alpha\theta(y)\beta\theta(x)\alpha T(x)\beta\theta(x) - \theta(x)\alpha[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x) = 0$$

$$T(x)\alpha\theta(x)\alpha\theta(y)\beta\theta(x)\beta\theta(x) - \theta(x)\alpha T(x)\beta\theta(x)\alpha\theta(y)\beta\theta(x) - \theta(x)\alpha[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x) = 0$$

$$T(x)\alpha\theta(x)\beta\theta(x)\alpha\theta(y)\beta\theta(x) - \theta(x)\alpha T(x)\beta\theta(x)\alpha\theta(y)\beta\theta(x) - \theta(x)\alpha[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x) = 0$$

$$(T(x)\alpha\theta(x) - \theta(x)\alpha T(x))\beta\theta(x)\alpha\theta(y)\beta\theta(x) - \theta(x)\alpha[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x) = 0$$

$$[T(x),\theta(x)]_\alpha\beta\theta(x)\alpha\theta(y)\beta\theta(x) - \theta(x)\beta[T(x),\theta(x)]_\alpha\alpha\theta(y)\beta\theta(x) = 0$$

$$([T(x),\theta(x)]_\alpha\beta\theta(x) - \theta(x)\beta[T(x),\theta(x)]_\alpha)\alpha\theta(y)\beta\theta(x) = 0$$

$$[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta\alpha\theta(y)\beta\theta(x) = 0 \quad (9)$$

Let $y = y\alpha[T(x),\theta(x)]_\alpha$ in (9), we have

$$[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta\alpha\theta(y)\alpha[T(x),\theta(x)]_\alpha\beta\theta(x) = 0 \quad (10)$$

Right multiplication of (9) by $\alpha[T(x),\theta(x)]_\alpha$ gives:

$$[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta\alpha\theta(y)\beta\theta(x)\alpha[T(x),\theta(x)]_\alpha = 0 \quad (11)$$

Subtracting (11) from (10) one obtains:

$$[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta\alpha\theta(y)\alpha[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta = 0$$

Since M is semiprime, so (2) follows, i.e., $[[T(x),\theta(x)]_\alpha,\theta(x)]_\beta = 0$

Now, we prove the relation (ii) :

$$\theta(x)\beta[T(x),\theta(x)]_\alpha\gamma\theta(x) = 0 \quad (12)$$

The linearization of (2) gives:

$$[[T(x),\theta(x)]_\alpha\theta(y)]_\beta + [[T(y),\theta(x)]_\alpha,\theta(x)]_\beta + [[T(y),\theta(x)]_\alpha,\theta(y)]_\beta + [[T(x),\theta(y)]_\alpha,\theta(x)]_\beta + [[T(x),\theta(y)]_\alpha,\theta(y)]_\beta + [[T(y),\theta(y)_\alpha,\theta(x)]_\beta = 0$$

Putting $x = -x$ in the above relation, we have:

$$[[T(x),\theta(x)]_\alpha,\theta(y)]_\beta + [[T(y),\theta(x)]_\alpha,\theta(x)]_\beta - [[T(y),\theta(x)]_\alpha,\theta(y)]_\beta + [[T(x),\theta(y)]_\alpha,\theta(x)]_\beta - [[T(x),\theta(y)]_\alpha,\theta(y)]_\beta - \left[[T(y),\theta(y)]_\beta,\theta(x)\right]_\beta = 0$$

Adding the above two relations, we have:

$$2[[T(x),\theta(x)]_\alpha,\theta(y)]_\beta + 2[[T(x),\theta(y)_\alpha,\theta(x)]_\beta + 2[[T(y),\theta(x)]_\alpha,\theta(x)]_\beta = 0$$

Since M is 2-torsion free semiprime Γ-ring, so, we have:

$$[[T(x),\theta(x)]_\alpha,\theta(y)]_\beta + [[T(x),\theta(y)]_\alpha,\theta(x)]_\beta + [[T(y),\theta(x)]_\beta = 0 \quad (13)$$

Putting $x\beta y\gamma x$ for y in (13) and using (1), (2), (13) and assumption (A), we have:

$$
\begin{aligned}
&0\\
&= [[T(x), \theta(x)]_\alpha, \theta(x)\beta\theta(y)\gamma\theta(x)]_\beta\\
&+ [[T(x), \theta(x)\beta\theta(y)\gamma\theta(x)]_\alpha, \theta(x)]_\beta\\
&+ \left[[\theta(x)\beta T(y)\gamma\theta(x), \theta(x)]_\beta, \theta(x)\right]_\beta\\
&= \theta(x)\beta[[T(x), \theta(x)]_\alpha, \theta(y)]_\beta\gamma\theta(x)\\
&+[[T(x), \theta(x)]_\alpha\beta\theta(y)\gamma\theta(x)\\
&+\theta(x)\beta[T(x), \theta(y)]_\alpha\gamma\theta(x)\\
&+ \theta(x)\beta\theta(y)\gamma[T(x), \theta(x)]_\alpha, \theta(x)]_\beta\\
&\quad\theta(x)\beta[[T(x), \theta(x)]_\alpha, \theta(y)]_\beta\gamma\theta(x)\\
&+ [T(x), \theta(x)]_\alpha\beta[\theta(y), \theta(x)]_\beta\gamma\theta(x)\\
&+ \theta(x)\beta[[T(x), \theta(y)]_\alpha, \theta(x)]_\beta\gamma\theta(x)\\
&+ \theta(x)\gamma[\theta(y), \theta(x)]_\beta\beta[T(x), \theta(x)]_\alpha\\
&+ \theta(x)\beta[[T(y), \theta(x)]_\alpha, \theta(x)]_\beta\gamma\theta(x)\\
&= [T(x), \theta(x)]_\alpha\beta[\theta(y), \theta(x)]_\beta\gamma\theta(x)\\
&+ \theta(x)\gamma[\theta(y), \theta(x)]_\beta\beta[T(x), \theta(x)]_\alpha\\
&= [T(x), \theta(x)]_\alpha\beta\theta(y)\beta\backslash\theta(x)\gamma\theta(x)\\
&-\theta(x)\gamma\theta(x)\beta\theta(y)\beta[T(x), \theta(x)] +\\
&\theta(x)\gamma\theta(y)\beta\theta(x)\beta[T(x), \theta(x)]\_\alpha\\
&- [T(x), \theta(x)]\_\alpha\ \beta\theta(x)\beta\theta(y)\gamma\theta(x).
\end{aligned}
$$

Therefore, we have:

$$
\begin{aligned}
&[T(x), \theta(x)]_\alpha\beta\theta(y)\beta\backslash\theta(x)\gamma\theta(x)\\
&-\theta(x)\gamma\theta(x)\beta\theta(y)\beta[T(x), \theta(x)]_\alpha\\
&+\theta(x)\gamma\theta(y)\beta\theta(x)\beta[T(x), \theta(x)]_\alpha\\
&-[T(x), \theta(x)]_\alpha\beta\theta(x)\beta\theta(y)\gamma\theta(x) = 0
\end{aligned}
$$

For all $x,y \in M, \beta \in T$, which reduces because of (3) and (8) to:

Left multiplication of the above relation by xβ gives:

$$\theta(x)\beta[T(x),\theta(x)]_\alpha\beta\theta(y)\beta\theta(x)\gamma\theta(x)\beta\theta(y)\beta[T(x),\theta(x)]_\alpha = 0$$

One can replace in the above relation according to (15), $\theta(x)\beta[T(x), \theta(x)]_\alpha\ \beta\theta(y)\beta\theta(x)$ by $\theta(x)\beta\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x)$ which gives:

$$
\begin{aligned}
&\theta(x)\beta\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x)\gamma\theta(x)\\
&-\theta(x)\beta\theta(x)\beta\theta(x)\gamma\theta(y)\beta[T(x),\theta(x)]_\alpha = 0
\end{aligned}
\tag{14}
$$

Left multiplication of the above relation by T(x)α gives:

$$
\begin{aligned}
&T(x)\alpha\theta(x)\beta\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x)\gamma\theta(x)\\
&-T(x)\alpha\theta(x)\beta\theta(x)\beta\theta(x)\gamma\theta(y)\beta[T(x),\theta(x)]_\alpha = 0
\end{aligned}
\tag{15}
$$

The substitution T(x)αθ(y) for y in (14), we have:

$$
\begin{aligned}
&\theta(x)\beta T(x)\alpha\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x)\gamma\theta(x)\\
&-\theta(x)\beta\theta(x)\beta\theta(x)\gamma T(x)\alpha\theta(y)\beta[T(x),\theta(x)]_\alpha = 0
\end{aligned}
\tag{16}
$$

Subtracting (16) from (15), we obtain:

$$
\begin{aligned}
&[T(x),\theta(x)]_\alpha\beta\theta(y)\beta[T(x),\theta(x)]_\alpha\beta\theta(x)\gamma\theta(x)\\
&-[T(x),\theta(x)\beta\theta(x)\gamma\theta(x)]_\alpha\beta\theta(y)\beta[T(x),\theta(x)]_\alpha = 0
\end{aligned}
$$

From the above relation and Lemma-2.1, it follows that:

$$
\begin{aligned}
&([T(x),\theta(x)\beta\theta(x)\gamma\theta(x)]_\alpha - [T(x),\theta(x)]_\alpha\beta\theta(x)\\
&\gamma\theta(x))\beta\theta(y)\beta[T(x),\theta(x)]_\alpha = 0
\end{aligned}
$$

which reduces to:

$$
\begin{aligned}
&(\theta(x)\beta[T(x),\theta(x)]_\alpha\gamma\theta(x) + \theta(x)\beta\theta(x)\\
&\gamma[T(x),\theta(x)]_\alpha)\beta\theta(y)\beta[T(x),\theta(x)]\alpha = 0
\end{aligned}
$$

Relation (2) makes it possible to write $[T(x),\theta(x)]_\alpha\gamma\theta(x)$ instead of $\theta(x)\gamma[T(x),\theta(x)]_\alpha$, which means that $\theta(x)\beta\theta(x)\gamma[T(x),\theta(x)]_\alpha$ can be replaced by $\theta(x)\beta[T(x),\theta(x)]_\alpha\gamma\theta(x)$ in the above relation. Thus we have:

$$\theta(x)\beta[T(x),\theta(x)]_\alpha\gamma\theta(x)\beta\theta(y)\beta[T(x),\theta(y)]_\alpha = 0$$

Right multiplication of the above relation by $\gamma\theta(x)$ and substitution $\theta(y)\beta\theta(x)$ for $\theta(y)$ gives finally, $\theta(x)\beta[T(x), \theta(x)]_\alpha\gamma\theta(x)\beta\theta(y)\beta\theta(x)$ $\beta[T(x), \theta(x)]_\alpha\gamma\theta(x) = 0$

Hence by semiprimeness of M, we have $\theta(x)\beta[T(x), \theta(x)]_\alpha\gamma\theta(x) = 0$.

Next we prove the relation (iii):

$$\theta(x)\beta[T(x), \theta(x)]_\alpha = 0, x \in M, \alpha \in \Gamma \tag{17}$$

First, we putting $\theta(y)\alpha\theta(x)$ for $\theta(y)$ in (8), gives because of (12):

$$\theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha\theta(x)\beta\theta(x) = 0 \tag{18}$$

The substitution $\theta(y)\alpha T(x)$ for $\theta(y)$ in (25), we have:

$$\theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha T(x)\alpha\theta(x)\beta\theta(x) = 0 \tag{19}$$

Right multiplication of (18) by $\alpha T(x)$,

$$\theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha\theta(x)\beta\theta(x)\alpha T(x) = 0 \tag{20}$$

Subtracting (20) from (19), we have:

$$
\begin{aligned}
&\theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha(T(x)\alpha\theta(x)\beta\theta(x)\\
&\qquad - \theta(x)\beta\theta(x)\alpha T(x)) = 0\\
&\Rightarrow \theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha[T(x), \theta(x)\beta\theta(x)]_\alpha\\
&\qquad = 0\\
&\Rightarrow \theta(x)\alpha[T(x), \theta(x)]_\alpha\beta\theta(y)\alpha([T(x), \theta(x)]_\alpha\beta\theta(x)\\
&\qquad + \theta(x)\beta[T(x), \theta(x)]_\alpha) = 0
\end{aligned}
$$

$$\Rightarrow \theta(x)\beta[T(x),\theta(x)]_\alpha\alpha\theta(y)\alpha([T(x),\theta(x)]_\alpha\beta\theta(x) + \theta(x)\beta[T(x),\theta(x)]_\alpha) = 0$$

According to (2), one can replaced $[T(x),\theta(x)]_\alpha\beta\theta(x)$ by $\theta(x)\beta[T(x),\theta(x)]_\alpha$ which gives:

$$\theta(x)\beta[T(x),\theta(x)]_\alpha\alpha\theta(y)\alpha\theta(x)\beta[T(x),\theta(x)]_\alpha = 0, x,y \in M, \alpha,\beta \in \Gamma$$

Hence by semiprimenessof M, $\theta(x)\beta[T(x),\theta(x)]_\alpha = 0$, $x,y \in M$, $\alpha,\beta \in \Gamma$.

Finally, we prove the equation (u) :

$$[T(x),\theta(x)]_\alpha = 0 \tag{21}$$

From (2) and (17), it follows that:

$$[T(x),\theta(x)]_\alpha\beta\theta(x) = 0, x \in M, \alpha,\beta \in \Gamma.$$

The linearization of the above relation gives (see how relation (13) was obtained from (2)):

$$[T(x),\theta(x)]_\alpha\beta\theta(y) + [T(x),\theta(y)]_\alpha\beta\theta(x) + [T(y),\theta(x)]_\alpha\beta\theta(x) = 0$$

Right multiplication of the above relation by $\beta[T(x),x]_\alpha$ gives because of (17):

$$[T(x),\theta(x)]_\alpha\beta\theta(y)\beta[T(x),\theta(x)]_\alpha = 0$$

which implies $[T(x),\theta(x)]_\alpha = 0$.

Lemma 3: Let M be Γ- ring satisfying the assumption (A) and let $T: M \to M$ be an additive mapping such that $T(x\alpha y\beta x) = \theta(x)\alpha T(y)\beta\theta(x)$ holds for all $x,y,\in M$ and $\alpha,\beta \in \Gamma$. Then:

$$\theta(x)\alpha(T(x\alpha y + y\alpha x) - T(y)\alpha\theta(x) - \theta(x)\alpha T(y))\beta\theta(x) = 0 \tag{22}$$

Proof: The substitution $x\alpha y + y\alpha x$ for y in (1) gives:

$$T(x\alpha x\alpha y\beta x + x\alpha y\alpha x\beta x) = \theta(x)\alpha T(x\alpha y + y\alpha x)\beta\theta(x) \tag{23}$$

On the other hand, we obtain by putting $z = x\alpha x$ in (3), we have:

$$T(x\alpha x\alpha y\beta x + x\alpha y\beta x\alpha x) = \theta(x)\alpha T(y)\alpha\theta(x)\beta\theta(x) + \theta(x)\alpha\theta(x)\alpha T(y)\beta\theta(x)$$

$$T(x\alpha x\alpha y\beta x + x\alpha y\alpha x\beta x) = \theta(x)\alpha T(y)\alpha\theta(x)\beta\theta(x) + \theta(x)\alpha\theta(x)\alpha T(y)\beta\theta(x) \tag{24}$$

By comparing (23) and (24), we have:

$$\theta(x)\alpha(T(x\alpha y + y\alpha x) - T(y)\alpha\theta(x) - \theta(x)\alpha T(y))\beta\theta(x) = 0.$$

We define:

$$G_\alpha(\theta(x),\theta(y)) = T(x\alpha y + y\alpha x) - T(y)\alpha\theta(x) - \theta(x)\alpha T(y).$$

Then it is clear that:

$$\theta(x)\alpha G_\alpha(\theta(x),\theta(y))\beta\theta(x) = 0 \textit{ and } G_\alpha(\theta(x),\theta(y)) = G_\alpha(\theta(y),\theta(x)).$$

Replacing x for y and using (22), we have:

$\theta(y)\alpha G_\alpha(\theta(x),\theta(y))\beta\theta(y) = 0.$ *We can also easily prove the following results :–*

(1) $G_\alpha(\theta(x)+\theta(z),\theta(y)) = G_\alpha(\theta(x),\theta(y)) + G_\alpha(\theta(z),\theta(y))$

(2) $G_\alpha(\theta(x),\theta(y)+\theta(z)) = G_\alpha(\theta(x),\theta(y)) + G_\alpha(\theta(x),\theta(z))$

(3) $G_{\alpha+\beta}(\theta(x),\theta(y)) = G_\alpha(\theta(x),\theta(y)) + G_\beta(\theta(x),\theta(y))$

(4) $G_\alpha(-\theta(x),\theta(y)) = -G_\alpha(\theta(x),\theta(y))$

(5) $G_\alpha(\theta(x),-\theta(y)) = -G_\alpha(\theta(x),\theta(y))$.

Lemma 4: Let M be a 2-torsion free semiprime Γ-ring satisfying the assumption (A) and let $T: M \to M$ be an additive mapping. Suppose that $T(x\alpha y\beta x) = \theta(x)\alpha T(y)\beta(x)$ holds for all $x,y \in M$ and $\alpha,\beta \in \Gamma$. Then:

(a) $[G_\alpha(\theta(x),\theta(y)),\theta(x)]_\alpha = 0$

(b) $G_\alpha(\theta(x),\theta)) = 0$

Proof: First we prove the relation (a):

$$[G_\alpha(\theta(x),\theta(y)),\theta(x)]_\alpha = 0 \tag{25}$$

The linearization of (21) gives:

$$[T(x),\theta(y)]_\alpha + [T(y),\theta(x)]_\alpha = 0 \tag{26}$$

Putting $x\alpha y + y\alpha x$ for y in the above relation and using (21), we obtain:

$$[T(x),\theta(x)\alpha\theta(y) + \theta(y)\alpha\theta(x)]_\alpha + [T(x\alpha y + y\alpha x),\theta(x)]_\alpha = 0$$

$$\Rightarrow \theta(x)\alpha[T(x),\theta(y)]_\alpha + [T(x),\theta(y)]_\alpha \alpha\theta(x) + [T(x\alpha y + y\alpha x),\theta(x)]_\alpha = 0$$

$$\Rightarrow [T(x\alpha y + y\alpha x),\theta(x)]_\alpha + \theta(x)\alpha[T(x),\theta(y)]_\alpha + [T(x),\theta(y)]_\alpha \alpha\theta(x) = 0$$

According to (26) one can replace $[T(x),\theta(y)]_\alpha$ by $-[T(y),\theta(x)]_\alpha$ in the above relation. We have therefore:

$$[T(x\alpha y + y\alpha x),\theta(x)]_\alpha - \theta(x)\alpha[T(y),\theta(x)]_\alpha$$
$$-[T(y),\theta(x)]_\alpha \alpha\theta(x) = 0$$

i.e., $[G_\alpha(\theta(x),\theta(y)),\theta(x)]_\alpha = 0$

The proof is therefore complete.
Finally, we prove the relation (b):

$$G_\alpha(\theta(x),\theta(y)) = 0 \qquad (27)$$

From (22) one obtains (see how (13) was obtained from (2)), we have:

$$\theta(x)\alpha G_\alpha(\theta(x),\theta(y))\beta\theta(z) + \theta(x)\alpha G_\alpha$$
$$(\theta(z),\theta(y))\beta\theta(x) + \theta(z)\alpha G_\alpha(\theta(x),\theta(y)) = 0$$

Right multiplication of the above relation by $G_\alpha(\theta(x),\theta(y))\alpha\theta(x)$ gives because of (22):

$$\theta(x)\alpha G_\alpha(\theta(x),\theta(y))\beta\theta(z)\beta G_\alpha(\theta(x),\theta(y))\alpha\theta(x) = 0 \qquad (28)$$

Relation (25) makes it possible to replace in (28), $\theta(x)\alpha G_\alpha(\theta(x),\theta(y))$ by $G_\alpha(\theta(x),\theta(y))\alpha\theta(x)$. Thus we have:

$$G_\alpha(\theta(x),\theta(y))\alpha\theta(x)\beta\theta(z)\beta G_\alpha(\theta(x),\theta(y))\alpha\theta(x) = 0 \qquad (29)$$

Therefore by semiprimeness of M:

$$G_\alpha(\theta(x),\theta(y))\alpha\theta(x) = 0 \qquad (30)$$

Of course, we have also:

$$\theta(x)\alpha G_\alpha(\theta(x),\ \theta(y)) = 0$$

The linearization of (30) with respect to x gives:

$$G_\alpha(\theta(x),\ \theta(y))\alpha\theta(z) + G_\alpha(\theta(z),\ \theta(y))\alpha\theta(x) = 0$$

Right multiplication of the above relation by $\alpha G_\alpha(\theta(x),\ \theta(y))$ gives because of (31):

$$G_\alpha(\theta(x),\ \theta(y))\alpha\theta(z)\alpha G_\alpha(\theta(x),\ \theta(y)) = 0 \qquad (31)$$

which gives $G_\alpha(\theta(x),\ \theta(y)) = 0$ i.e.,

$$T(x\alpha y + y\alpha x) = T(y)\alpha\theta(x) + \theta(x)\alpha T(y)$$

Hence the proof is complete.

Theorem 1: Let M be a 2-torsion free semiprime Γ-ring satisfying the assumption (A) and let $T : M \to M$ be an additive mapping. Suppose that T(xαxβx) = θ(x)αT(x)βθ(x) holds for all $x \in M$ and $\alpha, \beta \in \Gamma$. Then T is a θ-centralizer.

Proof: In particular for $y = x$, the relation (32) reduces to:

$$2T(x\alpha x) = T(x)\alpha\theta(x) + \theta(x)\alpha T(x)$$

Combining the above relation with (21), we arrive at:

$$2T(x\alpha x) = 2T(x)\alpha\theta(x), \qquad x \in M,\ \alpha \in \Gamma$$

And

$$2T(x\alpha x) = 2\theta(x)\alpha T(x), \qquad x \in M,\ \alpha \in \Gamma\ .$$

Since M is 2-torsion free, so we have:

$$T(x\alpha x) = T(x)\alpha\theta(x), \qquad x \in M,\ \alpha \in \Gamma$$
$$T(x\alpha x) = \theta(x)\alpha T(x), \qquad x \in M, \alpha \in \Gamma$$

By theorem 3.5 in Ullah and Chaudhary (2012), it follows that T is a left and also right θ–centralizer which completes the proof of the theorem.
Putting y = x in relation (1), we obtain:

$$T(x\alpha x\beta x) = \theta(x)\alpha T(x)\beta\theta(x), x \in M, \alpha, \beta \in T \qquad (33)$$

The question arises whether in a2-torsion free semi-prime Γ–ring the above relation implies that T is a θ-centralizer. Unfortunately we were unable to answer affirmative if M has an identity element.

Theorem 2: Let M be a 2-torsion free semi-prime Γ–ring with identity element 1 satisfying the assumption (A) and let $T : M \to M$ be an additive mapping. Suppose that $T(x\alpha x\beta x) = \theta(x)\alpha T(x)\beta\theta(x)$ holds for all $x \in M$ and $\alpha, \beta \in \Gamma$. Then T is a θ-centralizer.

Proof: Putting x+1 for x in relation (33), one obtains after some calculations:

$$3T(x\alpha x) + 2T(x) = T(x)\beta\theta(x)\theta(x)\alpha T(x)$$
$$+\theta(x)\alpha a\beta\theta(x) + a\alpha\theta(x) + \theta(x)\beta a$$

where a stands for T(1). Putting –x for x in the relation above and comparing the relation so obtain with the above relation we have:

$$6T(x\alpha x)=2T(x)\beta\theta(x)+2\theta(x)\alpha T(x)+2\theta(x)\alpha a\beta\theta(x) \quad (34)$$

and

$$2T(x)=a\alpha\theta(x)+\theta(x)\beta\alpha \quad (35)$$

We shall prove that $a\in Z(M)$.According to (35) one can replace $2T(x)$ on the right side of (41) by $a\alpha\theta(x)+\theta(x)\beta\alpha$ and $6T(x\alpha x)$ on the left side by $3a\alpha\theta(x)\beta\theta(x)+3\theta(x)\beta\theta(x)\alpha a$, which gives after some calculation:

$$a\alpha\theta(x)\beta\theta(x)+\theta(x)\beta\theta(x)\alpha a-2\theta(x)\alpha a\beta\theta(x)=0$$

The above relation can be written in the form:

$$[[a,\theta(x)]_\alpha,\theta(x)]_\beta=0; x\in M,\alpha,\beta\in\Gamma \quad (36)$$

The linearization of the above relation gives:

$$[[a,\theta(x)]_\alpha,\theta(y)]_\beta+[[a,\theta(y)]_\alpha,\theta(x)]_\beta=0 \quad (37)$$

Putting $y=x\alpha y$ in (37), we obtain because of (36) and (37):

$$\begin{aligned}0&=[[a,\theta(x)]_\alpha,\theta(x)\alpha\theta(y)]_\beta+[[\alpha,\theta(x)\alpha\theta(y)]_\alpha,\theta(x)]_\beta\\&=[[a,\theta(x)]_\alpha,\theta(x)]_\beta\beta\theta(y)]+\theta(x)\\&\alpha[[a,\theta(x)],\theta(y)]_\beta+[[a,\theta(x)]_\alpha\alpha\theta(y)\\&+\theta(x)\alpha[a,\theta(y)]_\alpha,\theta(x)]_\beta\\&=\theta(x)\alpha[[a,\theta(x)]_\alpha,\theta(y)]_\beta+[[a,\theta(x)]_\alpha\\&\alpha\theta(y),\theta(x)]_\beta+[\theta(x)\alpha[a,\theta(y)]_\alpha,\theta(x)]_\beta\\&=\theta(x)\alpha[[a,\theta(x)]_\alpha,\theta(y)]_\beta+[[a,\theta(x)_\alpha\\&\theta(x)]_\beta\beta\theta(y)+[a,\theta(x)]_\alpha\beta[\theta(y),\theta(x)]_\alpha\\&+[\theta(x)\alpha[a,\theta(y)]_\alpha,\theta(x)]_\beta\\&=[a,\theta(x)]_\alpha\beta[\theta(y),\theta(x)]_\alpha\end{aligned}$$

The substitution $\theta(y)\beta a$ for $\theta(y)$ in the above relation gives:

$$[a,\theta(x)]_\alpha\beta\theta(y)\beta[a,\theta(x)]_\alpha=0$$

Hence it follows $a\in Z(M)$, which reduces (35) to the form $T(x)=a\alpha\theta(x),\ \ x\in M,\alpha\in\Gamma$. The proof of the theorem is complete.

We conclude with the following conjecture: let M be a semiprime Γ-ring with suitable torsion restrictions and θ be an endomorphism of M. Suppose there exists an additive mapping $T:M\to M$ such that $T((x\alpha)^m(x\beta)^n x)=\theta(x\alpha)^m T(x)\theta(\beta x)^n$ holds for all $x\in M,\alpha,\beta\in\Gamma$, where $m\geq 1, n\geq 1$ are some integers. Thus T is a θ-centralizer.

CONCLUSION

In this study, we have given some examples which have shown that θ-centralizer exists in Γ-rings. We proved that if T is an additive mapping on a 2-torsion free semiprime Γ-ring M satisfying the assumption (A) such that $T(x\alpha y\beta x)=\theta(x)\alpha T(y)\beta\theta(x)$ for all $x,y\in M,\alpha,\beta\in\Gamma$ and θ an endomorphism on M,then T is a θ-centralizer. We have also showed that T is a θ-centralizer if M contains a multiplicative identity 1.

REFERENCES

Bernes, W.E., 1966. On the Γ-rings of nobusawa. Pacific J. Math., 18: 411-422.

Ceven, Y., 2002. Jordan left derivations on completely prime Γ-rings. Ç.U. Art. Sci. J. Sci. Technol. (Turkish), 23(2): 39-43.

Hoque, M.F. and A.C. Paul, 2011. On centralizers of semiprime gamma rings. Int. Math. Forum, 6(13): 627-638.

Hoque, M.F. and A.C. Paul, 2012. Centralizers of semiprime gamma rings. Italian J. Pure Appl. Math., 30(To Appear). Retrieved from: http:// www. researchmathsci. org/ apamart/ apam- v1n1-7. pdf.

Hoque, M.F., H.O. Roshid and A.C. Paul, 2012. An equation related to θ-centralizers in semiprime gamma rings. Int. J. Math. Combin., 4: 17-26.

Kyuno, S., 1978. On prime gamma ring. Pacific J. Math., 75: 185-190.

Luh, L., 1969. On the theory of simple gamma rings. Michigan Math. J., 16: 65-75.

Mayne, J.H., 1984. Centralizing mappings of prime rings. Canad. Math. Bull., 27(1): 122-126.

Nobusawa, N., 1964. On the generalization of the ring theory. Osaka J. Math., 1: 81-89.

Ullah, Z. and M.A. Chaudhary, 2010. θ-centralizers of rings with involution. Int. J. Algera, 4(17): 843-850.

Ullah, Z. and M.A. Chaudhary, 2012. On K-centralizers of semiprime gamma rings. Int. J. Algera, 6(21): 1001-1010.

Vukman, J., 1997. Centralizers in prime and semiprime comment. Math. Univ., Carolinae, 38: 231-240.

Vukman, J., 1999. An identity related to centralizers in semiprime rings. Comment. Math. Univ., Carolinae, 40(3): 447-456.

Vukman, J., 2001. Centralizers on Semiprime rings. Comment. Math. Univ., Carolinae, 42(2): 237-245.

Zalar, B., 1991. On centralizers of semiprime rings. Comment. Math. Univ., Carolinae, 32: 609-614.