

## Research Article

### An Improved Point-track Optimal Assignment Algorithm

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**Abstract:** In order to improve the accuracy of data association of the Optimal Assignment (OA) algorithm based on dynamic information, an improved Point-Track Optimal Assignment (IPTOA) algorithm based on multi-source information is proposed. The improved algorithm gets valid 3-tuple of measurement set by solving 3-Dimensional (3-D) assignment problem which is based on dynamic information. Then fuses multi-source information by combination rule of D-S evidence theory and constructs the point-track correlation matrix between valid 3-tuple of measurement and target track. Compared with the optimal assignment algorithm based on dynamic information, the new algorithm effectively fuses multi-source information to correlate measurement data, which improves the performance of multi-target tracking in different degrees. Simulation results verify the feasibility and effectiveness of the new algorithm.

**Keywords:** Combination rule of D-S evidence theory, data correlation, multi-source information, the optimal assignment algorithm

## INTRODUCTION

The multi-target tracking algorithm based on state estimate only takes advantage of state measurement to correlate and track target, so the tracking performance has some limitations. In dense target and clutter scenario, the uncertainty of state measurement source increases, then wrong correlation will occur, which will decrease the correct correlation rate and further cause wrong tracking or target losing (He *et al.*, 2006; Pan *et al.*, 2009). With the development of science technology, especially the sensor detection and information processing technology, in practice, sensor can not only get state measurement of target, but also get other feature information, such as emitter carrier frequency, pulse repetition period, impulse width etc. These characters reflect the inherent property information of target, if these feature information can be effectively fused and used in target tracking, the uncertainty of state measurement and the number of candidate measurements will be decreased respectively and the target tracking performance will be improved (Zhao, 2010; Zhang and Zhang, 2011).

Cheng *et al.* (1999) proposes a conception of quality function on the basis of Nearest Neighbor (NN) (Johns, 1987) method and defines a new correlation degree function between measurement and target, which fuses doppler frequency, echo amplitude of the signal and state measurement of target, this in some extent improves the target tracking performance. Wang

and Luo (2004) fuses multiple feature information belongs to single target by combination rule of D-S evidence theory, then correlates fused measurement and multi-target comprehensively and tracks multi-target by using extended Kalman filter algorithm. Compared with the traditional NN algorithm, this algorithm possesses better multi-target tracking effects. Wang *et al.* (2006) takes advantage of Grey relational analysis method on the basis of PDA algorithm to define absolute difference and grey relational coefficients of various feature information, then calculates the grey relational degree, fuses multiple feature information with state measurement and gets a new correlation probability, which is used to update the state of target by using Kalman filter algorithm. Study results show that this algorithm is superior to the traditional PDA algorithm and has lower time complexity.

It can be seen that the target tracking algorithm fusing feature information can improve the target tracking performance in different degrees. As the optimal assignment algorithm based on dynamic information only takes advantage of state information to correlate measurement data, in dense target and clutter environment, the data correlation results don't correspond entirely with correct correlations, this will lead to a deteriorating tracking performance. To solve this problem, one considers improving the tracking effect of OA algorithm by fusing multiple feature information with state measurement in the process of point-track correlation and this will be implemented by

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a new algorithm which is called improved point-track optimal assignment algorithm. The new algorithm uses several good solutions of the optimal assignment problem based on dynamic information to get valid 3-tuple of measurement set, fuses the same kind of feature information from three sensors and point-track correlation probability based on different feature information by using combination rule of D-S evidence theory, respectively. Then the correlation matrix based on state and multi-feature information fusion between the valid 3-tuple of measurement and target track can be got by using probability multiplication method. And by solving a maximizing assignment problem with effective matrix as above fusion correlation matrix, one can get the optimal point-track matches. Then Kalman filter is used to update the state of target. Simulation results show that IPTOA is superior to OA algorithm and the increasing amplitude of time spent of IPTOA algorithm is lower, so it is an effective multi-target tracking algorithm.

**SYSTEM DESCRIPTIONS**

Let us suppose that the state equation of discrete-time system is as follows:

$$X^t(k+1) = F^t(k)X^t(k) + V^t(k) \quad t = 1, 2, \dots, T \quad (1)$$

where,  $F^t(k)$  is the state transition matrix of target  $t$  at time  $k$ ,  $X^t(k)$  is the state vector of target  $t$  at time  $k$ ,  $V^t(k)$  is the zero mean and white Gaussian process noise sequence of target  $t$  at time  $k$ , its corresponding covariance matrix is  $Q^t(k)$  and:

$$E[V^t(k)] = 0, \quad E[V^t(k)V^t(l)^T] = Q^t(k)\delta_{kl}$$

The measurement equation is:

$$z_j(k) = H(k)X^t(k) + W(k), \quad (2)$$

$$j = 1, 2, \dots, m_k; \quad t = 1, 2, \dots, T$$

where,  $p_{00}$  is the  $j$ th measurement of target  $t$  at time  $k$ ,  $m_k$  is the number of effective measurement at time  $k$ ,  $H(k)$  is measurement matrix,  $W(k)$  is the zero mean and white Gaussian process noise sequence whose covariance matrix is  $R(k)$  and

$$E[W(k)] = 0, \quad E[W(k)W(l)^T] = R(k)\delta_{kl}$$

**THE OPTIMAL ASSIGNMENT ALGORITHM**

Let us suppose that the state variable of target  $t$  follows a normal distribution and then the probability density function of target  $t(t = 1, 2, \dots, T)$

corresponding to measurement  $j(j = 1, 2, \dots, n_s)$  can be denoted as:

$$p_{jt} = P_G^{-1} N[z_{jt}(k); \hat{Z}(k|k-1), S_t(k)] \quad (3)$$

where,  $P_G$  is gate probability,  $v_{jt}(k) = z_{jt}(k) - Z_t(k|k-1)$  is filtering residual vector,  $S_t(k)$  is residual covariance matrix. If  $j = 0, t \neq 0$ , then  $p_{0t}$  represents the probability that target  $t$  is not detected by any correct measurement; if  $j \neq 0, t = 0$ , then  $p_{j0}$  represents the probability that measurement  $j$  is false alarm; if  $j = 0, t = 0$ , then the correlation of measurement  $j$  and target  $t$  is meaningless, now one lets  $p_{00} = 0$ . In this way, the marshal probabilistic statistical distance matrix between measurement  $j(j = 0, 1, \dots, m_s)$  and target track  $t(t = 0, 1, 2, \dots, T)$  of sensor  $s$  can be expressed as:

$$P_s = (p_{jt}), \quad j = 0, 1, \dots, n_s; \quad (4)$$

$$t = 0, 1, \dots, T; \quad s = 1, 2, 3$$

Through normalizing the element of each row of above matrix, one can get the basic probability assignment matrix of measurement of target  $t$  from sensor  $s$  as follows:

$$M_s = (m_{jt}), \quad j = 0, 1, \dots, n_s; \quad (5)$$

$$t = 0, 1, \dots, T; \quad s = 1, 2, 3$$

Then the inconsistent measure of each component of 3-tuple of measurement  $Z_{i_1 i_2 i_3} = \{Z_{1i_1}, Z_{2i_2}, Z_{3i_3}\}$  can be expressed as:

$$k_{i_1 i_2 i_3} = \sum_{A_p \cap B_q = \phi} m_{i_1}(A_p) \cdot m_{i_2}(B_q) \quad (6)$$

$$+ \sum_{B_q \cap C_r = \phi} m_{i_2}(B_q) \cdot m_{i_3}(C_r)$$

where,  $A_p, B_q, C_r \subseteq U$ ,  $U$  is identification frame of target  $t$ . So the measurement data correlation problem of three sensors multi-target tracking system can be transformed into a 3-D assignment problem as follow:

$$\min_{\rho_{i_1 i_2 i_3}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} k_{i_1 i_2 i_3} \cdot \rho_{i_1 i_2 i_3} \quad (7a)$$

Subject to:

$$\begin{cases} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} \rho_{i_1 i_2 i_3} = 1; & i_1 = 1, 2, \dots, n_1 \\ \sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \rho_{i_1 i_2 i_3} = 1; & i_2 = 1, 2, \dots, n_2 \\ \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \rho_{i_1 i_2 i_3} = 1; & i_3 = 1, 2, \dots, n_3 \end{cases} \quad (7b)$$

where,  $\rho_{i_1 i_2 i_3}$  are binary variables, if a 3-tuple of measurement originates from a real target, it is 1. Otherwise, it is 0.

One can get the optimal solution of assignment problem by solving (7a-7b) using Lagrange relaxation algorithm and each solution component exactly corresponds to a 3-tuple of measurement. By constructing correlation matrix between above 3-tuple of measurement and target track, one can get the optimal point-track correlations by solving a 2-D assignment problem which takes above matrix as correlation effective matrix and further update the state of target with Kalman filter.

### IMPROVED POINT-TRACK OPTIMAL ASSIGNMENT ALGORITHM

Studies have shown that in relatively good detection environment, one can get satisfactory solution and relatively good multi-target tracking performance in polynomial time by solving 3-D assignment problem (7a-7b) using Lagrange relaxation algorithm. But with the changing of detection environment, when the density of target and clutter is increased or the measurement error is relatively big, there inevitably exists a certain model error in the 3-D assignment problem. Right now, even if one uses Lagrangian relaxation algorithm to obtain the optimal solution of the 3-D assignment problem, the solution component may not exactly correspond to a 3-tuple of measurement from the same target. Under the condition of not increasing the number of dimension of the assignment problem, the main method to decrease the error of the model is to increase the amount of effective information. So one considers using components of several good solutions of (7a-7b) to compose of valid 3-tuple of measurements set, then improving the correlation degree expression between effective 3-tuple of measurement and target track by taking advantage of multi-source information, so as to better distinct which 3-tuple of measurement really originates from the same target and get the goal of improving the accuracy of target tracking.

**Fusion of multi-source information:** By solving (7a-7b), one can get valid 3-tuple of measurement set, the position  $(x_t, y_t)$  of the corresponding target

$t (t = 0, 1, 2, \dots, T)$  of the  $i$ th ( $i = 1, 2, \dots, m_k$ ) fusion measurement  $z_i^t$  can be estimated by the following equations:

$$\begin{cases} \hat{x}_t = \sum_{s=1}^3 \frac{x_s}{\sigma_{x_s}^2} / \sum_{s=1}^3 \frac{1}{\sigma_{x_s}^2} \\ \hat{y}_t = \sum_{s=1}^3 \frac{y_s}{\sigma_{y_s}^2} / \sum_{s=1}^3 \frac{1}{\sigma_{y_s}^2} \end{cases} \quad (8)$$

where,  $\sigma_x^2, \sigma_y^2$  are the location variances in  $x, y$  direction, respectively;  $x_s = x_t + u_s, y_s = x_t + v_s$  are, respectively the position components in  $x, y$  direction of target  $t (t = 0, 1, 2, \dots, T)$  detected by sensor  $s$ .

Take the estimate point as the integrated measurement  $z_{it}^1, i = 1, 2, \dots, m_k$  of the  $i$ th 3-tuple of measurement and use it to construct the point-track correlation probability based on state estimate at time  $k$  as follows:

$$\lambda_{it}^1(k) = \exp\left\{-\frac{1}{2} v_{it}^1(k) \cdot [S_{it}(k)]^{-1} \cdot v_{it}^1(k)\right\} / \sqrt{2\pi S_{it}(k)} \quad (9)$$

where,  $i = 1, 2, \dots, m_k; t = 1, 2, \dots, T$ .  $v_{it}$  is the residual vector that target  $t$  originates from the fused measurement  $z_i^t, S_{it}$  is the residual covariance matrix. Then the point-track correlation probability matrix between integrated measurement  $z_{it}^1$  and target  $t$  can be expressed as:

$$A^1(k) = [\lambda_{it}^1(k)] \quad (10)$$

Let us suppose that one can get other 1 kinds of feature information of target  $t$  except the state information. Firstly, one constructs the correlation degree between each kind of feature measurements and target track according to the type of the feature information. Furthermore, through normalizing the correlation degree between each measurement of the sensor and different targets, one gets the basic probability assignment between each measurement and different targets and one uses the combination rule of D-S evidence theory:

$$\begin{cases} m(\phi) = 0 \\ m(A) = \frac{\sum_{A_i \cap \dots \cap A_j = A} \prod_{j=1}^l m_j(A_j)}{1 - \sum_{A_i \cap \dots \cap A_j = \phi} \prod_{j=1}^l m_j(A_j)} = \frac{\sum_{A_i \cap \dots \cap A_j = A} \prod_{j=1}^l m_j(A_j)}{\sum_{A_i \cap \dots \cap A_j \neq \phi} \prod_{j=1}^l m_j(A_j)} \end{cases} \quad (11)$$

to fuse the same kind of feature information measured by three sensors, then the basic probability between each fusion measurement of three sensors and different targets are obtained. To comprehensively process the correlation degrees based on different information source, one continues to use the combination rule of D-S evidence theory to fuse various kinds of feature information. And then one can get the point-track correlation degree based on different kinds of feature information and turn it into the point-track correlation probability. Let the correlation probability between fusion feature measurement  $z_{it}^2$  ( $i=1,2,\dots,m_k$ ) and target track  $t$  is  $\lambda_{it}^2$ , then the corresponding point-track correlation probability matrix can be expressed as:

$$A^2(k)=[\lambda_{it}^2(k)] \quad (12)$$

Let:

$$\lambda_{it}(k) = \lambda_{it}^1(k) \cdot \lambda_{it}^2(k) \quad (13)$$

then  $\lambda_{it}(k)$  represents the correlation probability of integrated measurement  $z_{it}$  ( $i=1,2,\dots,m_k$ ) based on multi-source information and target track  $t$ , denote the corresponding point-track correlation probability matrix as:

$$A(k)=[\lambda_{it}(k)] \quad (14)$$

then under one-to-one feasible rule, by solving a maximizing 2-D assignment problem whose correlation effective matrix is (14), one can get the optimal point-track correlation matches based on multi-source information fusion.

**Steps of the improved algorithm:** The concrete steps of the improved point-track optimal assignment algorithm are described as follows:

**Step 1:** Construct the valid 3-tuple of measurement with the solution components of several good solutions of (7a-7b). For each valid 3-tuple measurement  $z_{it}^1$ ,  $i=1,2,\dots,m_k$ , estimate the position of the corresponding target by (8), take the estimate as an integrated measurement of the 3-tuple measurement and calculate the correlation probability  $\lambda_{it}^1$  between the integrated measurement and target track. Then the correlation degree matrix between integrated measurement and target track is given as  $A^1(k)=[\lambda_{it}^1(k)]$ .

**Step 2:** Fuse the same kind of feature information detected by three sensors by using the combination rule of D-S evidence theory and get the basic probability assignments of different targets for each integrated

measurement of three sensors. Further, one calculates the point-track correlation probability  $\lambda_{it}^2(k)$  based on multi-feature information continuing to use the combination rule of D-S evidence theory and denotes the corresponding correlation matrix as  $A^2(k)=[\lambda_{it}^2(k)]$ .

**Step 3:** Multiply the corresponding elements respectively originate from above two correlation probability matrixes by probability multiplication method, then one can get the point-track correlation probability matrix  $A(k)=[\lambda_{it}^1(k) \cdot \lambda_{it}^2(k)]$  based on state measurement and feature measurement fusion. Solve the 2-D assignment problem whose effective matrix is  $A(k)$ , then the point-track correlation result based on multi-source information can be obtained.

**Step 4:** Locate and track multi-target with Kalman filter.

## SIMULATIONS

Let us suppose that 8 target move in variable motion in a plane, the directions of movement of different targets are the same. The initial speeds of 8 target are respectively  $v_x = 20m/s$ ,  $v_y = 10m/s$ . The angle and range measurement errors of three sensors are respectively the same. Detection probability is  $P_D = 0.95$ ; gate probability is  $P_G = 1$ ; clutter coefficient is  $\lambda = 2$ . Select target state (ST), emitter carrier frequency (RF), pulse repetition period (PRI), pulse width (PW) as information sources, where, the types of RF, PRI and PW are all fixed. The central value of radiation source carrier frequency is  $5000Mhz$ , central value of pulse repetition frequency is  $3ms$ , and the central value of pulse width is  $8\mu s$ ; measurement errors of various feature information are  $e_{RF} = 1Mhz$ ,  $e_{PRI} = 0.03ms$  and  $e_{PW} = 0.08\mu s$ . Radar sampling interval is  $T = 2s$ .

In this study, one analyzes and compares performances of OA and IPTOA algorithms under conditions of different target interval and different measurement errors. The number of simulate steps is 150 and number of simulation times is 50. Simulations are implemented in the following scenarios:

- When target interval is 1000 m, under different range and angle measurement errors, the comparisons of position of Root-Mean-Square Error (RMSE) of OA and IPTOA algorithms are as follows:
- When target interval is 300 m, under different measurement errors, the comparisons of position of RMSE of OA and IPTOA algorithm are as follows:

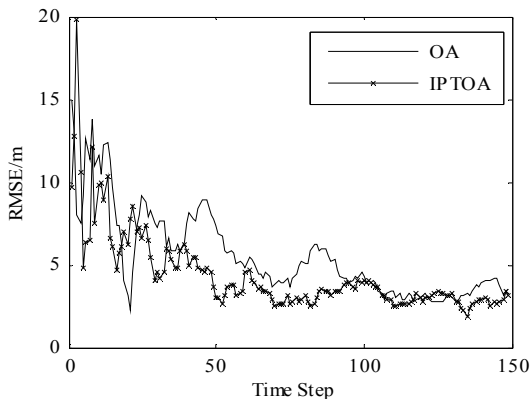


Fig. 1: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 100m$ ,  $e_\theta = 0.01rad$

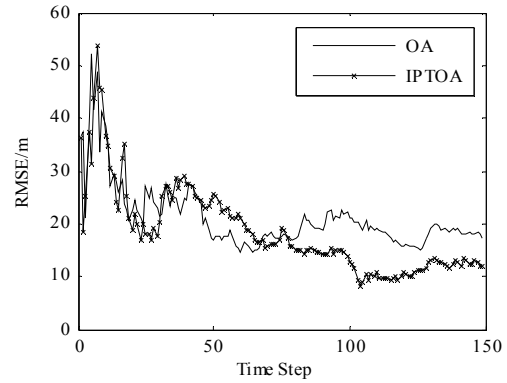


Fig. 4: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 250m$ ,  $e_\theta = 0.025rad$

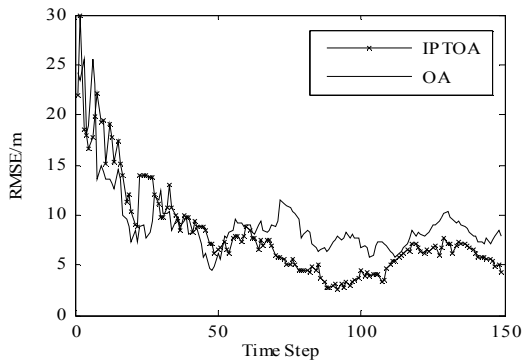


Fig. 2: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 200m$ ,  $e_\theta = 0.02rad$

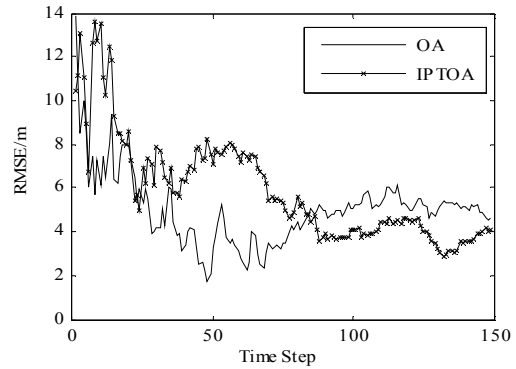


Fig. 5: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 100m$ ,  $e_\theta = 0.01rad$

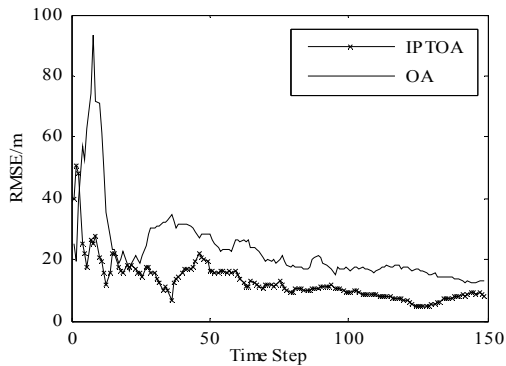


Fig. 3: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 200m$ ,  $e_\theta = 0.02rad$

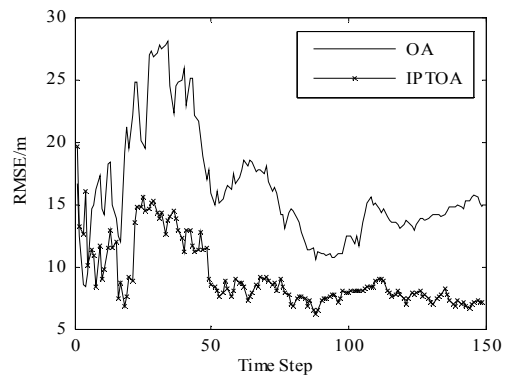


Fig. 6: Comparisons of RMSE of OA and IPTOA under condition of  $e_r = 200m$ ,  $e_\theta = 0.02rad$

- When target interval is 150 m, under different measurement errors, the comparisons of position of RMSE of OA and IPTOA algorithm are as follows:

One can see from Fig. 1 and 2 that when target interval is 1000 m, under different range and angle measurement errors, IPTOA algorithm is superior to OA algorithm, but the optimization of amplitude is not

big. This is because that when the density of target and clutter is not high, the performance of multi-target tracking has been very good, so, the IPTOA algorithm fusing multi-source feature information doesn't perform much better in respect of improving the accuracy of multi-target tracking.

Compare Fig. 3 to 6, one can see that when target interval becomes smaller and measurement errors

Table 1: Comparisons of time spent of different algorithms

Number of target	4	8	12	16
OA	0.73	1.55	3.31	5.01
IPTOA	0.81	1.70	3.88	5.93

become bigger, compared with OA algorithm, the improving amplitude of multi-target tracking accuracy of IPTOA algorithm becomes bigger. This is because that with the deterioration of detection environment, the multi-target tracking accuracy of OA algorithm becomes worse. In this condition, IPTOA algorithm fusing multiple feature information can better perform the advantage of multi-target tracking algorithm based on multi-source information fusion in respect of information complement; this improves the multi-target tracking performance in various degrees.

Table 1 is the comparisons of time spent of OA and IPTOA algorithm when target interval is 300 m, range measurement error is 200 m, angle measurement error is 0.02 rad and simulation steps are 150. One can see from Table 1 that, when the number of target is different, compared with OA algorithm, the increasing amplitude of time spent of IPTOA algorithm is relatively less.

### CONCLUSION

To improve the multi-target tracking performance of the optimal assignment algorithm based on dynamic information, a new point-track optimal assignment algorithm is proposed. The new algorithm improves the optimal assignment algorithm based on dynamic information in two respects: first, effectively choose the valid 3-tuple of measurement; second, improve correlation degree between candidate measurement and target track by using multi-source information fusion. Simulation results show that, compared with the optimal assignment algorithm based on dynamic information, the new algorithm improves multi-target tracking performance in different degrees. Since one

just fuses multi-source information on the valid 3-tuple of measurement whose number is less, so, compared with OA algorithm, the increasing amplitude of time spent of IPTOA algorithm is lower. Therefore, IPTOA algorithm is an effective multi-target tracking algorithm.

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