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Research Article

Multiple Time-Varying Delays Analysis and Coordinated Control for Power System Wide Area Measurement

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Abstract: The stability of wide-area power system with multiple time-varying delays is investigated in this study. It is known for all that power system is a delay system for wide area measurement information of power system. Therefore, transmission time delay can not be ignored and it has great impact on the controller design. The steady stability analysis model of power system with multiple time-varying delays is constructed. The criterion of stability is formulated as feasibility problems of linear matrix inequality. According to the characteristics of the time-varying delays, the feedback controller is separated into two parts: The first level of control is derived from local signals without time delay and the second level of control is supplied from other generators with different time-varying delays. Taking a four-machine power system as example, the performance of the proposed controller is validated by the results of simulation in time-domain and it is proved that the proposed method is effective.

Keywords: Feedback controller, multiple time-varying delays, power system, Wide-Area Measurement System (WAMS)

INTRODUCTION

Wide-Area Measurement System (WAMS) technology is widely applied to monitor and control the large interconnected power systems to improve the stability of systems. WAMS has the technology of Phase Measurement Unit (PMU), it was used to analyze power system performance such as damping inter-area oscillation (Hauer *et al*., 2007, Far *et al*., 2009) between different control areas, load shedding (Seethalekshmi *et al*., 2009) and state estimation (Hong and Weiguo, 2009). PMUs are units that measure dynamic data of power systems, such as voltage, current, angle and frequency (speed). The technology using accurate PMUs becomes a powerful source of wide-area dynamic information. It is found that dynamic performance of system can be enhanced if remote signals from one or more distant locations of the power system applied to local controller design (Aboul-Ela *et al*., 1996). With the development of WAMS, it is possible to take the global signals as the feedback for wide-area stability control in large power systems, which breaks through the limit that the traditional damping controllers could only adopt local information as the feedback signals (Abido *et al*., 2001). Those signals are derived from synchronized measurements provided by PMUs installed at system (typically generator) buses. The signals from a wide-area controller are synchronized by using the Global Positioning System (GPS) technology. It is found that the dynamic performance of system can be increased with respect to inter-area oscillations if measured signals from remote locations are applied to the controller (Hui *et al*., 2002). In this trend, the impact of time delays introduced by remote signal transmission and processing in WAMS has not been ignored (Hongxia *et al*., 2004).

Time delays are often the sources of poor performance and instability of wide-area power systems. Recently, improved performances have been reported by using Lyapunov-Krasovskii theorems and Linear Matrix Inequality (LMI) techniques (Wu *et al*., 2004, He *et al*., 2006). One of the difficulties in applying Lyapunov methods is the lack of efficient algorithms for constructing the Lyapunov functional. In general, the use of reduced functional may result in conservatism. For instances, a condition of improved asymptotic stability for constant time delay systems is provided in Shengyuan and Lam (2007) and it avoids bounding certain cross terms which often leads to conservatism.

In addition, two-level structure controller was firstly proposed in Yang and Bose (2008), in which a local signal is used to damp local mode and a wide-area signal feedback to damp inter-area modes. During the design of these controllers mentioned above, some of them did not consider the effect of time delays and the

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most of them use robust control methods to handle the time delay as part of system uncertainties.

At present, on the field of wide-area damping control, the researches focus on linear and constant time-delay (Yao *et al*., 2011). Based on the Lyapunov approach, the criteria of time-delay stability could guarantee the system to be asymptotic stability within limits, but it is at the expense of the control performance of damping controllers on a certain extent (He *et al*., 2009).

From this perspective, in this study, a steady stability analysis model of power system with multiple time delays is constructed which uses the state variable of the local generator as the first level of control input signal and uses the signal difference between regions with different time delays as the second level control input signal to deal with the poorly damped inter-area modes. The closed-loop system with two levels of damping controller is modeled as a linear system with time delays. Then the maximal delays and parameters of the damping controller which allow the closed-loop power system to retain stable, are calculated to use the multiple time-varying delays stability criterion based on the Lyapunov theorems and using the LMI technique.

Model of multiple time delays power system: Studies in this study are based on multiple machine power system. The system model can be described as follows:

$$\frac{d\delta_i}{dt} = \omega_0(\omega_i - 1) \tag{1a}$$

$$\frac{d\omega_i}{dt} = \frac{1}{T_{Ji}}[P_{mi} - P_{ei} - D_i(\omega_i - 1)] \tag{1b}$$

$$\frac{dE'_{qi}}{dt} = \frac{1}{T'_{doi}}[-E'_{qi} - (x_{di} - x'_{di})I_{di} + E_{fdi}] \tag{1c}$$

$$\frac{dE'_{fdi}}{dt} = \frac{1}{T_{ai}}(-E'_{fdi} - K_{Ai}U_{ti} + K_{Ai}V_{si}) \tag{1d}$$

where, i = 1, L, n, n stands for the number of the generators:

$$P_{ei} = E'_{qi}\sum_{j=1}^{n} E'_{qj}(B_{ij}\sin\delta_{ij} + G_{ij}\cos\delta_{ij}) \tag{2a}$$

$$P_m = const. \tag{2b}$$

$$i_{di} = -E'_{qi}B_{ii} + \sum_{j=1, j\neq i}^{n} E'_{qj}Y_{ij}\sin(\delta_{ij} - \beta_{ij}) \tag{2c}$$

$$i_{qi} = E'_{qi}G_{ii} + \sum_{j=1, j\neq i}^{n} E'_{qj}Y_{ij}\cos(\delta_{ij} - \beta_{ij}) \tag{2d}$$

Defining $x_i = (\Delta\delta_i, \Delta\omega_i, \Delta E'_{qi}, E_{fdi})^T$ and it is at the neighborhood of equilibrium point $x_{i0} = (\Delta\delta_{i0}, \Delta\omega_{i0}, \Delta E'_{qi0}, E_{fdi0})^T$. The multiple machine dynamic Eq. (1) is linearized at x_{i0}.

We have the differential equation of whole the multiple machine power system as follow:

$$\dot{X}(t) = AX(t) + BU(t) \tag{3}$$

where, $X(t) = (\Delta\delta^T, \Delta\omega^T, \Delta E_q^T, \Delta E_{fd}^T)^T \epsilon R^{4n}$, $A \epsilon R^{4n\times 4n}$, $B \epsilon R^{4n\times p}$, $U \epsilon R^p$, p is the number of the feedback controller adding at the generator ($p \le n$).

Generally, the application of wide-area signals is mainly to damp the local mode in area for the design of the coordinated control in which the controller is located using the generator rotor speed as an input signal and the delay time $\tau_{ij}(t)$ is very small when i = j, thus the local time delay can be defined as $\tau_{ii}(t) = 0$. On the other hand, The signal $\tau_{ij}(t)$ is an additional global signal for damping inter-area modes when $i \neq j$. It can be assumed that the time delay can be taken as $\tau_{ij}(t)$, $0 \le \tau_{ij}(t) \le d_{ij}$, i, …, p; j = 1, …, n, i = 1, …, p; if the transmission delay and processing delay are considered, then the feedback law can be given as the following representation:

$$U(t) = \sum_{i=1}^{p} K_{ii}X(t) + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} K_{ij}X(t - \tau_{ij}(t)), \ i = 1, \ldots, p, \ j = 1, \ldots, n. \tag{4}$$

To identify the local and inter-area modes of *n* generators in system, K_{ii} is local controller without time delay, K_{ij} is local controller with multiple time-varying delays, then system (3) can be rewritten as follows:

$$\dot{\boldsymbol{X}}(t) = \boldsymbol{A'}\boldsymbol{X}(t) + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} \boldsymbol{B}K_{ij}\boldsymbol{X}(t - \tau_{ij}(t)) \tag{5}$$

where, $A' = \left(A + \sum_{i=1}^{p} K_{ii}\right)$, $0 \le \tau_{ij}(t) \le d_{ij}$; $\left|\dot{\tau}_{ij}(t)\right| \le \mu_{ij} \le 1; i = 1, \ldots, p, j = 1.$

The stability criteria for multiple time-varying delays systems: The purpose of this study is to formulate practically computable criterions to check the stability of the power system (5) with multiple delays in different cases. The following theorem can be used to prove asymptotic stability of the system above.

Theorem 1: The multiple time-varying delays system (5) is asymptotically stable if there exist symmetric positive definite matrix $P \in R^n$, positive semi-definite matrices S_{ij} and $R_{ij} \in R^n$, matrices Y_{ij} and T_{ij} of any number of dimensions, i = 1, …, p; j = 1, …, n; and positive semi-definite matrix X satisfy:

$$X = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1\times(pn-p)} \\ X_{12}^T & X_{22} & \cdots & X_{2\times(pn-p)} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1\times(pn-p)}^T & X_{2\times(pn-p)}^T & \cdots & X_{(pn-p)\times(pn-p)} \end{pmatrix} \ge 0 \tag{6}$$

such that the following LMI holds:

$$\Xi = \begin{pmatrix} F_1 & F_2 & \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} d_{ij}\boldsymbol{A}'^T\boldsymbol{R}_{ij} \\ F_2^T & F_3 & G_1 \\ \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} d_{ij}\boldsymbol{R}_{ij}\boldsymbol{A}' & G_1^T & -\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} d_{ij}\boldsymbol{R}_{ij} \end{pmatrix} < 0 \tag{7}$$

$$\Pi = \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1\times(pn-p)} & \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} Y_{ij} \\ X_{12}^T & X_{22} & \cdots & X_{2\times(pn-p)} & T_{12} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ X_{1\times(pn-p)}^T & X_{2\times(pn-p)}^T & \cdots & X_{(pn-p)\times(pn-p)} & T_{(pn-p)\times(pn-p)} \\ \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} Y_{ij}^T & T_{12}^T & \cdots & T_{(pn-p)\times(pn-p)}^T & \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} \boldsymbol{R}_{ij} \end{pmatrix} \geq 0 \tag{8}$$

where,

$$F_1 = A'^T P + PA' + \sum_{i=1}^{p}\sum_{j=1}^{n}(S_{ij} + Y_{ij} + Y_{ij}^T) + \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} d_{ij}X_{11}$$

$$F_2 = \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\left\{PA_{ij} + T_{ij} - Y_{ij} + d_{ij}X_{ij}\times(\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\right\}$$

$$F_3 = \sum_{i=1}^{p}\sum_{j=1}^{n}\left\{(-T_{ij} - T_{ij}^T - (1-\mu_{ij})S_{ij} + d_{ij}X_{ij}\times\begin{pmatrix} 0 & \cdots & 0 \\ \vdots & I_{(i-1)n+j-a_k,(i-1)n+j-a_k} & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_{(pn-p)\times(pn-p)}\right\}$$

$$G_1 = \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\left\{A_{ij}^T R_{ij}(\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\right\},$$ in which

$$\{a_k\}_{k=1}^{pn-p} = \overbrace{\underbrace{1,1,\cdots 1,}_{n} \ \underbrace{2,2,\cdots 2,}_{n} \ \underbrace{3,3,\cdots 3,}_{n} \underbrace{4,4\cdots}_{n} \cdots}^{pn-p}.$$

Proof: Choose the Lyapunov-Krasovskii functional for the system (5) with time-varying delays as follows:

$$\boldsymbol{V}_o(x_t) = \boldsymbol{V}_{o1}(\boldsymbol{x}_t) + \boldsymbol{V}_{o2}(\boldsymbol{x}_t) + \boldsymbol{V}_{o3}(\boldsymbol{x}_t) \tag{9}$$

where,

$$V_{o1}(\boldsymbol{x}_t) = \boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t) \tag{10a}$$

$$\boldsymbol{V}_{o2}(x_t) = \sum_{i=1}^{p}\sum_{\substack{j=1\\ j\neq i}}^{n}\int_{-d_{ij}}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^T(s)\boldsymbol{R}_{ij}\dot{\boldsymbol{x}}(s)dsd\theta \tag{10b}$$

$$\boldsymbol{V}_{o3}(x_t) = \sum_{i=1}^{p}\sum_{\substack{j=1\\ j\neq i}}^{n}\int_{t-\tau_{ij}(t)}^{t}\boldsymbol{x}^T(\sigma)\boldsymbol{S}_{ij}\boldsymbol{x}(\sigma)d\sigma \tag{10c}$$

According to following Newton-Leibniz formula:

$$x(t-\tau(t)) = x(t) - \int_{t-\tau(t)}^{t}\dot{x}(s)ds \tag{11}$$

we have $\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\left\{2\left[x^T(t)Y_{ij} + x^T(t-d)(t)T_{ij}\right]\left[x^T(t) - \int_{t-\tau_{ij(t)}}^{t}\dot{x}(s)ds - x(t-\tau_{ij}(t))\right]\right\} = 0$, the detailed proof of the LMI above can be shown in Wu *et al*. (2004).

Since P>0 ($\exists > \varepsilon 0: P \geq \varepsilon I$), R_{ij} and $S_{ij}\geq 0$, i = 1,…, p; j = 1, …, n, the first Lyapunov-Krasovskii condition holds:

$$\boldsymbol{V}_o(x_t) \geq x^T(t)Px(t) \geq \varepsilon\|x(t)\|^2 \tag{12}$$

Then, there exists a positive ε' such that the derivative of functional $\dot{V}_o(x_t) \leq -\varepsilon'\|x(t)\|^2$ along the trajectories of the system.

Consider the derivative of $V_0(x_t)$ along the trajectories of (5), one has:

$$\dot{\boldsymbol{V}}_o(x_t) = \dot{\boldsymbol{V}}_{o1}(x_t) + \dot{\boldsymbol{V}}_{o2}(x_t) + \dot{\boldsymbol{V}}_{o3}(x_t) \tag{13}$$

$$\begin{aligned}\dot{V}_o(x_t) \leq{}& x^T(PA' + A'^T P)x + x^T P\sum_{i=1}^{p}\sum_{j=1,j\neq 1}^{n} A_{ij}x(t-\tau_{ij}(t)) \\ &+ \sum_{i=1}^{p}\sum_{j=1,j\neq 1}^{n} A_{ij}x(t-\tau_{ij}(t))A_{ij}\,Px \\ &+ \sum_{i=1}^{p}\sum_{j=1,j\neq 1}^{n}\{x^t S_{ij}x \\ &-(1-\mu_{ij})x^T(t-\tau_{ij}(t))S_{ij}x(t-\tau_{ij}(t))\} \\ &+ \sum_{i=1}^{p}\sum_{j=1,j\neq 1}^{n}\{d_{ij}x^T A'^T R_{ij}A'x \\ &+ d_{ij}x^T A'^T R_{ij}\sum_{k=1}^{p}\sum_{l=1,l\neq k}^{n} A_{kl}x(t - \tau_{kl}(t)) \\ &+ d_{ij}\boldsymbol{x}^T(t-\tau_{kl}(t))\sum_{k=1}^{p}\sum_{\substack{l=1\\ l\neq k}}^{n} A_{kl}^T\boldsymbol{R}_{ij}A'\boldsymbol{x} - \int_{t-\tau_{ij}(t)}^{t}\dot{\boldsymbol{x}}^T(s)\boldsymbol{R}_{ij}\dot{\boldsymbol{x}}(s)ds \\ &+ d_{ij}\boldsymbol{x}^T(t-\tau_{kl}(t))\sum_{k=1}^{p}\sum_{l=1,l\neq k}^{n} A_{kl}^T\boldsymbol{R}_{ij}\sum_{k=1}^{p}\sum_{l=1,l\neq k}^{n} A_{kl}\,\boldsymbol{x}(t-\tau_{kl}(t))\ \} \\ &+ \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\left\{2[x^T(t)Y_{ij} + x^T(t-d(t))T_{ij}]\times[x^T(t) - \int_{t-\tau_{ij}(t)}^{t}\dot{x}(s)ds - x(t-\tau_{ij}(t))]\right\} \\ &+ \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\left\{d_{ij}\xi^T(t)X\xi(t) - \int_{t-\tau_{ij}(t)}^{t}\xi^T(t)X\xi(t)ds\right\},\end{aligned} \tag{14}$$

where,

$$\begin{aligned}&\xi(t) = \left[x^T(t) \quad x^T(t-\tau_{12}(t)) \quad \cdots \quad x^T(t-\tau_{pn}(t))\right], \\ &X = X^T \geq 0,\ \xi^T(t,s) \neq 0,\end{aligned} \tag{15}$$

then we have:

$$\begin{aligned}&\dot{\boldsymbol{V}}_o(x_t) \leq \xi^T(t)\Xi\xi(t) - \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n}\int_{t-\tau_{ij}(t)}^{t}\varsigma^T(t,s)\,\Pi\,\varsigma(t,s), \\ &i = 1,\ldots,p;\ j = 1,\ldots n,\end{aligned} \tag{16}$$

$$\varsigma^T(t,s) = \left[x^T(t) \quad x^T(t-\tau_{12}(t)) \quad \cdots \quad x^T(t-\tau_{pn}(t)) \quad \dot{x}^T(s)\right] \tag{17}$$

$$\Xi = \begin{pmatrix} F_1 & F_2 \\ F_2^T & F_3 \end{pmatrix} \tag{18}$$

$$F_1 = \boldsymbol{A'}^T \boldsymbol{P} + \boldsymbol{PA'} + \sum_{i=1}^{p}\sum_{j=1}^{n}(\boldsymbol{S}_{ij} + Y_{ij} + Y_{ij}^T) + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}X_{11} + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}\boldsymbol{A'}^T\boldsymbol{R}_{ij}\boldsymbol{A'};$$

$$F_2 = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}\{PA_{ij} + T_{ij} - Y_{ij} + d_{ij}X_{ij} + d_{ij}A^T R_{ij}\sum_{k=1}^{p}\sum_{l=1, l\neq k}^{n} A_{kl} \times (\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\};$$

$$F_3 = \sum_{i=1}^{p}\sum_{j=1}^{n}\{(-T_{ij} - T_{ij}^T - (1-\mu_{ij})S_{ij} + d_{ij}X_{ij} + d_{ij}\sum_{k=1}^{p}\sum_{l=1, l\neq k}^{n} A_{kl}^T R_{ij}\sum_{k=1}^{p}\sum_{l=1, l\neq k}^{n} A_{kl})$$

$$\times \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & I_{(i-1)n+j-a_k, (i-1)n+j-a_k} & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_{(pn-p)\times(pn-p)} \},$$

where, $\{a_k\}_{k=1}^{pn-p} = \overbrace{\underbrace{1,1,\cdots 1,}_{n} \ \underbrace{2,2,\cdots 2,}_{n} \ \underbrace{3,3,\cdots 3,}_{n} \underbrace{4,4\cdots}_{n} \cdots}^{pn-p}$.

By applying the standard Schur's formula (Boyd *et al*., 1994), the matrix Ξ can be denoted as (7). Therefore, the system (5) is asymptotically stable when the inequalities (7) and (8) hold. This completes the proof.

Feedback controller design: The Theorem 1 as the criterion standard of the system (5) with multiple time-varying delays and the Eq. (7) can be described as:

$$\Xi = \begin{pmatrix} F_1 & F_2 & \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}\boldsymbol{A'}^T\boldsymbol{R}_{ij} \\ F_2^T & F_3 & G_1 \\ \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}\boldsymbol{R}_{ij}\boldsymbol{A'} & G_1^T & -\sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}\boldsymbol{R}_{ij} \end{pmatrix} < 0, \quad i = 1, \cdots, p;$$

$$j = 1, \cdots, n, \tag{19}$$

where,

$$F_1 = (\boldsymbol{A} + \sum_{i=1}^{p} K_{ii})^T \boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \sum_{i=1}^{p} K_{ii}) + \sum_{i=1}^{p}\sum_{j=1}^{n}(\boldsymbol{S}_{ij} + Y_{ij} + Y_{ij}^T) + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}X_{11};$$

$$F_2 = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}\{P\boldsymbol{B}K_{ij} + T_{ij} - Y_{ij} + d_{ij}X_{ij} \times (\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\};$$

$$F_3 = \sum_{i=1}^{p}\sum_{j=1}^{n}\{(-T_{ij} - T_{ij}^T - (1-\mu_{ij})S_{ij} + d_{ij}X_{ij} \times \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & I_{(i-1)n+j-a_k, (i-1)n+j-a_k} & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_{(pn-p)\times(pn-p)} \};$$

$$G_1 = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}\{K_{ij}^T B^T R_{ij}(\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\};$$ in which

$$\{a_k\}_{k=1}^{pn-p} = \overbrace{\underbrace{1,1,\cdots 1,}_{n} \ \underbrace{2,2,\cdots 2,}_{n} \ \underbrace{3,3,\cdots 3,}_{n} \underbrace{4,4\cdots}_{n} \cdots}^{pn-p}$$

The matrices $\sum_{i=1}^{p} K_{ii}$ and $\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} K_{ij}$ are the two unknown matrix variables in the above formula and they are nonlinear form of this matrix inequalities, therefore, they can be replaced by an appropriate variable in order to translate nonlinear matrix inequalities into a new variable equivalent LMIs and to get the value of the new variables by solving LMI.

The left matrix of the inequality (19) is left multiplication and right multiplication the martrix $diag\ \overbrace{\left(P^{-1},\ P^{-1}, \ldots P^{-1}, \left(\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} K_{ij}\right)^{-1}\right)}^{pn-p+2}$, respectively. The time delays d_{ij} are given firstly and the range of the time delays is 20ms≤ d_{ij} ≤350ms. Then the inequality (19) can be transformed into the inequality as follows:

$$\begin{pmatrix} \boldsymbol{W}_{11} & \boldsymbol{W}_{12} & \boldsymbol{W}_{13} \\ \boldsymbol{W}_{12}^T & \boldsymbol{W}_{22} & \boldsymbol{W}_{23} \\ \boldsymbol{W}_{13}^T & \boldsymbol{W}_{23}^T & \boldsymbol{W}_{33} \end{pmatrix} < 0, \ i = 1, \cdots, p; \ j = 1, \cdots, n, \tag{20}$$

where, $Z = P^{-1}$; $M = \sum_{i=1}^{p} K_{ii} P^{-1}$; $N_{ij} = K_{ij}\ P^{-1}$; $Q_{ij} = P^{-1} S_{ij} P^{-1}$; $U_{ij} = P^{-1} R_{ij}\ P^{-1}$; $Y_{ij} = P^{-1} Y_{ij} P^{-1}$; $T_{ij} = P^{-1} T_{ij} P^{-1}$; $X_{ij} = P^{-1} X_{ij} P^{-1}$;

$$\boldsymbol{W}_{11} = \boldsymbol{ZA}^T + \boldsymbol{AZ} + \boldsymbol{Z}M^T + M\boldsymbol{Z} + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}(Q_{ij} + Y_{ij} + Y_{ij}^T) + \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}X_{11}$$

$$\boldsymbol{W}_{12} = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}\{\boldsymbol{B}N_{ij} + T_{ij} - Y_{ij} + d_{ij}X_{ij} \times (\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\}$$

$$\boldsymbol{W}_{13} = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n} d_{ij}(Z\boldsymbol{A}^T + ZM^T);$$

$$\boldsymbol{W}_{22} = \sum_{i=1}^{p}\sum_{j=1}^{n}\{(-T_{ij} - T_{ij}^T - (1-\mu_{ij})Q_{ij} + d_{ij}X_{ij}\begin{pmatrix} 0 & \cdots & 0 \\ \vdots & I_{(i-1)n+j-a_k, (i-1)n+j-a_k} & \vdots \\ 0 & \cdots & 0 \end{pmatrix}_{(pn-p)\times(pn-p)} \};$$

$$\boldsymbol{W}_{23} = \sum_{i=1}^{p}\sum_{j=1, j\neq i}^{n}\{N_{ij}^T B^T U_{ij}(\overbrace{\underbrace{0 \ \cdots \ I}_{(i-1)n+j-a_k} \ 0 \ \cdots \ 0}^{pn-p})\};$$

$$\boldsymbol{W}_{33} = -\sum_{i=1}^{p}\sum_{j=1}^{n} d_{ij}R_{ij}^{-1};$$

where, $\{a_k\}_{k=1}^{pn-p} = \overbrace{\underbrace{1,1,\cdots 1,}_{n} \ \underbrace{2,2,\cdots 2,}_{n} \ \underbrace{3,3,\cdots 3,}_{n} \underbrace{4,4\cdots}_{n} \cdots}^{pn-p}$,

we can obtain matrix M and N after the LMI (20) is solved and $\sum_{i=1}^{p} K_{ii} = MP$, $\sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} K_{ij} = \sum_{i=1}^{p}\sum_{j=1,j\neq i}^{n} N_{ij}\, P$.

Simulation: The two-area four-machine power system shown in Fig. 1 was considered for this study, the detailed description of the study system including network data and dynamic data for the generators can be found in Kundur (1994) and Table 2. It has been widely used for the study of the low frequency oscillation in power system. Each generator of the test

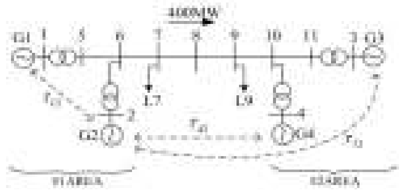

Fig. 1: The two-area four=machine power system

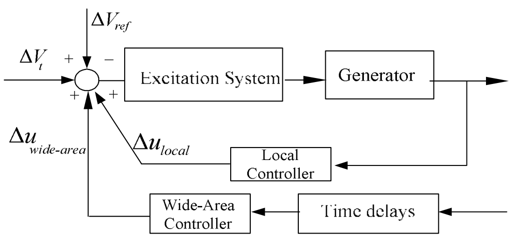

Fig. 2: Wide-area control configuration

Table 1: Eigen value analysis of 4-generator without controller

Mode	Eigen values	D.R.	Freq.(Hz)	Main generator
1	-0.6772+7.0470i	0.0957	1.1216	#1
2	-0.6663+7.2708i	0.0913	1.1572	#3
3	0.1082+4.0272i	-0.0269	0.6409	#1, #3

Table 2: System generator data

x_d (pu)	0.2	T'_{d0} (s)	8.0	T_a (s)	0.05
x_q (pu)	0.189	T'_{q0}(s)	0.4	T_b (s)	1.0
x'_d (pu)	0.0333	T_J (s)	117(G1,2) 111.15(G3,4)	T_c (s)	1.0
x'_q (pu)	0.0333	K_A (s)	50	D (s)	0

system is represented by a 4th-order differential equation and equipped with excitation system and no governor. All four generators are equipped with IEEE AC type excitation system. The open loop system has an inter area oscillation mode with negative damping ratio as shown in Table 1 and generator 2 and generator 3 are the main generators of this mode. It is found that a local controller installed at generator 2 is more effective in damping inter-area oscillations than other locations (Kussner *et al*., 2001). Three different delay times are considered in the test system and dij>0($i \neq j$).

The structure of the feedback controller is shown in Fig. 2. The V_t and V_{ref} denote the generator terminal voltage and its reference, respectively. The local mode is damped by local controller which uses the rotor speed of local generator as input and its parameters are determined based on the phase compensation of local mode frequency. The output of the wide-area controller, ΔV_{local} is added to the excitation system of the selected machine together with the output of local controller, $\Delta V_{wide\text{-}area}$ is provide to damping for the inter-area modes. The conventional wide-area controller uses a wide-area signal as feedback input and provides an additional control signal to the selected machine for the damping of the inter-area modes.

Local controller: The states of test system can be written as:

$$x_{(1\sim n)} = \left(\Delta\delta, \Delta\omega, \Delta E'_q, \Delta E_{fd}\right)^T,$$

where, n is the number of generators. Local controller is a conventional controller without time-delay. The local controller V_{local} is described as follows:

$$V_{local} = -\sum_{i=1}^{p} K_{ii} X_{local}(t - \tau_{ii}(t))$$

where, $\sum_{i=1}^{p} K_{ii}$ is the constant gain of the controller. The pole placement design method is applied to design local controller. After adding the local controller, all eigenvalues of the state matrix A are λ_k (k = 1, …, n) and Re(λ_k)>0 (k = 1, …, n) hold.

Wide-area controller: There are three groups of remote signal with different time delays (μ_{ij} = 0) as follows:

Group I: $d_{12} = 30ms$, $d_{32} = 70ms$, $d_{42} = 50ms$.

K_I=100*[0.474941 -0.000433 -0.001323 -2.28782 ; 0 0 0 0; -0.533433 -0.0000542 0.000564 0.4610376; -0.443698 0.000133 0.001444 2.305355].

Group II: $d_{12} = 160ms$, $d_{32} = 220ms$, $d_{42} = 200ms$.

K_{II}=1000*[0.063405 -0.000160 -0.000649 -1.165591; 0 0 0 0; -0.002015 -0.000009 0.000052 0.0461442; -0.015482 0.000012 0.000165 0.293769].

Group III: $d_{12} = 200ms$, $d_{32} = 350ms$, $d_{42} = 250ms$.

K_{III}=1000*[-0.266117 0.000073 0.000237 0.371280; 0 0 0 0; 0.110353 -0.000092 -0.001078 -1.704954; 0.183432 -0.000068 -0.000477 -0.341896].

The system response without wide-area controllers is shown in Fig. 3. By comparing the response with time delay, it is estimated that the system with a local controller can tolerate time delay up to 70 ms in Fig. 3, the system is towards unstable as the time-delay exceeds 100 ms.

With different time delays, there are three different wide-area controllers are designed by the method mentioned in this study. The parameters of wide-area controller shown as K_I, K_{II} and K_{III} above, the simulation curves of the system with wide-area controller response are shown in Fig. 4.

The step response of speed deviation of generator 3 with time-delay group 1 is shown in Fig. 5. The step in this case is the response due to a step change in V_{ref} of generator 3.

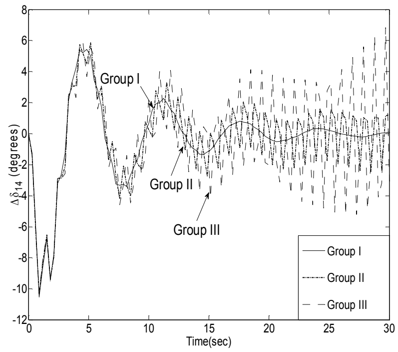

Fig. 3: Simulation curves without wide-area controller

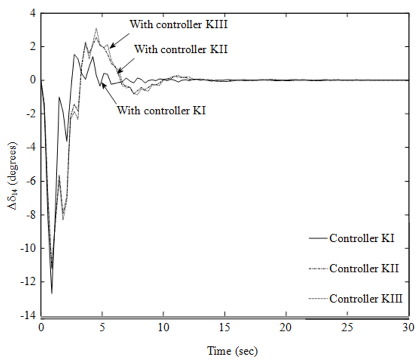

Fig. 4: Comparison of the performance of the system with wide-area feedback controller

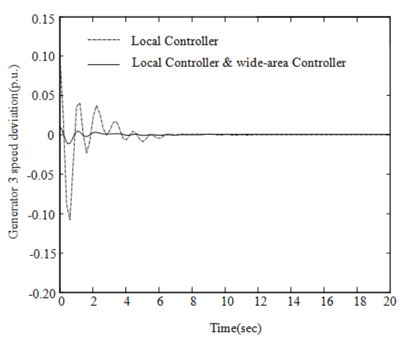

Fig. 5: Comparison of the performance of local controller and adding wide-area controller KI.

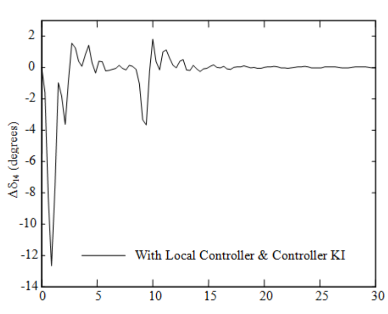

Fig. 6: Closed-loop system fault response with time-delay group I

When a three-phase fault occurs at bus 7, the deviation step response of rotor angle δ_{14} with time-delay group I is shown in Fig. 6. If the time-delay is considered, the closed-loop system can keep stable.

CONCLUSION

The wide-area signals with time delay will weak the system performance or even cause instability. To achieve maximum available transfer capability, such a time delay should be taken into account to design the centralized control for large power system. The analysis of the impact of time delay to wide-area power system control is addressed in this study by two levels of controller design. The first local control is derived from local signals without time delay and the second level control is supplied from a coordinator using global states with different time-varying delays. A criterion based on the Lyapunov-Krasovskii functional theorem is derived to design appropriate controller. This aim of design is to alleviate the vulnerability of interconnected power system. It is observed that the time-domain performances of the system are improved with the different time delays signals.

NOMENCLATURE

δ_i = The state variables are the rotor angle of generator (in radian)
ω_i = The rotor speed (in rad/s)
T_{Ji} = The mechanical starting time (in s)
T'_{doi} = The d -axis transient short circuit time constant of generator (in s)
P_{mi} = The mechanical active power
P_{ei} = The electrical active power
D_i = The damping coefficient
E'_{qi} = The q -axis transient voltage
E_{fdi} = The excitation voltage;
x_{di}, x_{qi} = x_{di} is d-axis synchronous reactance of generator
x_{qi} is q-axis synchronous reactance of generator
x'_{di} = x'_{di} is d-axis transient reactance of generator
i_d, i_q = i_d is d -axis circuit of generator; i_q is q -axis
circuit of generato
v_d, v_q = v_d is d -axis voltage of generator
v_q is q -axis voltage of generator
K_{Ai} = The compensation network gain
T_{ai} = lead time constant (in s)
V_{si} = The output signals of the feedback controller
G_{ij}, B_{ij} = The conductance between node i and j
The susceptance between node i and j
$T_{ij}(t)$ = Time-delays between node i and j
D_{ij} = The maximal time-delays between node i and j

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