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Research Article

Mathematical Model and Tool Path Calculation for Helical Groove Whirling

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Abstract: The helical groove shape plays a key role in ensuring the adequate flute space of many screw components. In many situations, the helical groove is machined through profiled grooving cutter, which brings a huge cost. This study establishes the mathematical model of helical groove based on cross-section and presents an approach to calculate tool path using the whirling process which machines helical groove through enwrapping movements with standard cutters. Finally, a case study and the error analysis are provided to illustrate the validity of the developed models and algorithms, which offers an alternative method for further computer aided manufacturing.

Keywords: Helical groove, toolpath calculation, whirling process

INTRODUCTION

Helical grooves are of importance in many screw components, especially those which form the flutes like screw shafts and extrusion screws. Generally, the process of helical groove machining is that a helical swept groove is obtained based on a cylindrical workpiece using a disk-type tool. Extensive work on helical groove has been carried out. Jung-Fa (2006) presented a mathematical model and sensitivity analysis for helical groove machining. Ivanov and Nankov (1998) provided a generalized analytical method for the profiling of rotation tools for forming helical grooves. Sun *et al*. (2008) presented a new simulation model for generation of the helical surface profiles on cutting tools. Zhang *et al*. (2006) presented a practical method of modeling and simulation for drill fluting. Onacea *et al*. (2010) proposed to use Bezier polynomials in approximating a cylindrical helical surface with constant pitch and in-plane generatrix known in discrete form. Wang *et al*. (2006) proposed the use of standard cutters (cutting blades), through whose enwrapping movements to produce the helical groove.

This study aims at presenting the mathematical model and the approach of calculating tool path for whirling the helical groove.

HELICAL GROOVE MATHEMATICAL MODEL

The helical groove can be considered as the result of the helical movement of a profile curve. In principle, the curve, as the generatrix of the helical groove, can be chosen arbitrarily. In practice however, a particular section profile is commonly used. As illustrated in Fig. 1, suppose the helical groove is formed by the helical motion of the cross section profile along the z axis in a cartesian coordinate system (o-x,y,z), whose versors are i, j, k, then the curve Γ is on xoy plane and can be expressed as:

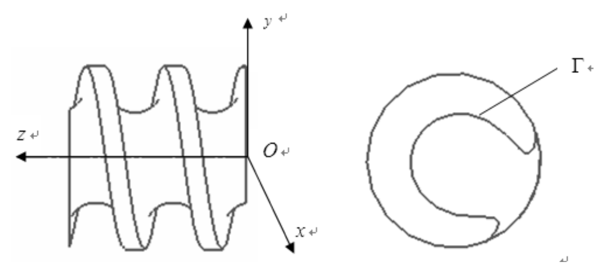

Fig. 1: Example of helical groove and its generatrix

$$r_0 = r_0(u) = x_0(u)\bar{i} + y_0(u)\bar{j} \tag{1}$$

where, u is a variable parameter. In this case the equation of the cylindrical helical groove can be written as:

$$r = \begin{bmatrix} x_0(u)cos\theta - \\ y_0(u)\sin\theta \end{bmatrix}\bar{i} + \begin{bmatrix} x_0(u)\sin\theta + \\ y_0(u)\cos\theta \end{bmatrix}\bar{j} + (p\theta)\bar{k} \tag{2}$$

where, θ is the rotation angle around z axis, $|p| = \frac{p_z}{2\pi}$, where p_z is the lead. For right handed helical grooves, p is positive; otherwise, p is negative.

WHIRLING PROCESS OF HELCIAL GROOVE

Whirling is an efficient process for producing helical grooves on three-axis CNC machines. It is a particular type of milling that the tool disk rotates at high speeds around a slowly turning workpiece. As

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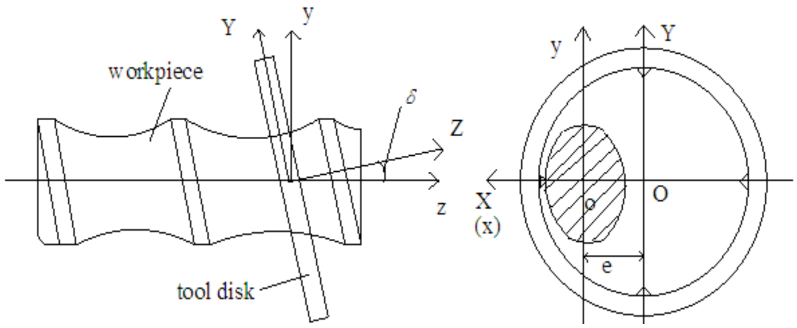

Fig. 2: Schematic diagram of whirling process

shown in Fig. 2, (o-x,y,z) is the workpiece coordinate system and (O-X,Y,Z) is the coordinate system of the tool ring (the whirling spindle) on which several cutters (the inserts) are evenly installed. There is a distance e between y and Y and an installation angle δ between z and Z. In machining, the tool disk rotates at a high speed making the cutting movement, in the mean time it moves along both z and x axes.

For a discrete screw shaft, the whirling process is usually conducted using specially profiled cutters that fit in with the grooves, with the feed rate being linked to the screw lead. While this is the typical method of thread whirling, it is also feasible to cut the helical groove into shape with standard cutters as long as their tool paths are well planned as to enwrap the desired surface. The latter method provides much flexibility for the whirling process as well as for its implementation. It is recognized there are still different ways to create the helical groove in terms of the machining strategies and tool path. In this study however, we focus on tipped cutters (tool bits with nose radius less than 1.5 mm) and choose to use the cross section profile as the envelope of the cutter locations. In the process, cutting tools are supposed to sweep along the workpiece while moving in and out according to its helical groove. In this case the tool path is a special-shaped helix, which needs three-axis (C, X, Z) linkage to realize the enveloping process.

TOOL PATH CALCULATION MODEL

Meshing motion model between workpiece and cutting tool: In machining the helical grooves, the surface of cutting tool is tangential to the surface of helical groove at any arbitrary contact point to ensure no interference. Furthermore, there is a common tangent plane and normal on each tangent point.

According to Eq. 1 and 2 and for simplicity, the equation of cross section profile can be rewritten as:

$$\begin{cases} x = x \\ y = f(x) \end{cases} \tag{3}$$

Similarly, the equation of helical groove can be expressed as:

$$\begin{cases} X = x\cos\theta - y\sin\theta \\ Y = x\sin\theta + y\cos\theta \\ Z = p\theta \end{cases} \tag{4}$$

where, x and θ are two values.

As shown in Fig. 3, the given point M (X,Y,Z) is an enwrapping point and point N (x_N, y_N, z_N) is the corresponding central point of cutting tool. Suppose O (x_T, y_T, z_T) is the central point of tool disk, which needs to be calculated and in this case, point P ($0,0,$ z_T) is on the negative X axis.

The definitions of some vectors are as follows:

$$\vec{\rho} = \overline{oM} = X\vec{i} + Y\vec{j} + Z\vec{k} \tag{5}$$

$$\overline{oN} = x_N\vec{i} + y_N\vec{j} + z_N\vec{k} \tag{6}$$

$$\overline{oP} = z_T\vec{k} \tag{7}$$

$$\vec{r} = \overline{MN} = (x_N - X)\vec{i} + (y_N - Y)\vec{j} + (z_N - Z)\vec{k} \tag{8}$$

$$\vec{R} = \overline{NO} = (x_T - x_N)\vec{i} + (y_T - y_N)\vec{j} + (z_T - z_N)\vec{k} \tag{9}$$

$$\vec{N} = \overline{PN} = x_N\vec{i} + y_N\vec{j} + (z_N - z_T)\vec{k} \tag{10}$$

Due to the common normal vector of workpiece and cutting tool, there is:

$$\overline{oN} = \overline{oM} + \overline{MN} = x_N\vec{i} + y_N\vec{j} + z_N\vec{k} = \vec{\rho} + \vec{r} \tag{11}$$

Besides, in terms of the aforesaid equations, N (x_N, y_N, z_N) can be determined as:

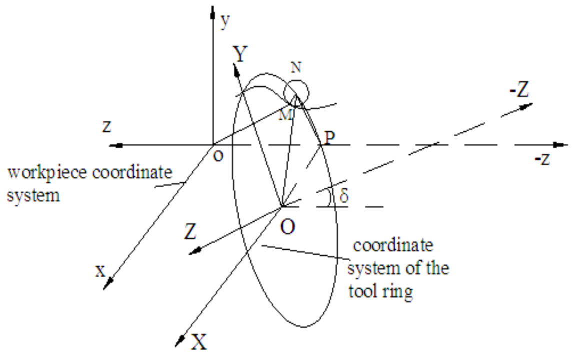

Fig. 3: Schematic program of workpiece and cutting tool

$$\begin{cases} x_N = X + \dfrac{pr(\sin\theta + f'(x)\cos\theta)}{|n|} \\ y_N = Y - \dfrac{pr(\sin\theta - f'(x)\cos\theta)}{|n|} \\ z_N = p\theta + X(\cos\theta - f'(x)\sin\theta) + \\ Y(\sin\theta + f'(x)\cos\theta) \end{cases} \tag{12}$$

where, r is the radius of tool tip and $|n|$ can be expressed as:

$$|n| = \sqrt{p^2(1+f'(x)) + \begin{bmatrix} X(\cos\theta - f'(x)\sin\theta) + \\ Y(\sin\theta + f'(x)\cos\theta) \end{bmatrix}^2} \tag{13}$$

Cutter location data calculation: As shown in Fig. 3, point O (x_T, y_T, z_T) is the central point of tool disk and point N (x_N, y_N, z_N) is the central point of cutting tool, we can have:

$$(x_T - x_N)^2 + (y_T - y_N)^2 + (z_T - z_N)^2 = R^2 \tag{14}$$

where, R is the radius of tool disk.

While point P is on X axis and point N is on XOY plane, so $\vec{N} = \overrightarrow{PN}$ is on XOY plane, moreover, $\vec{R} = \overrightarrow{NO}$ also is on XOY plane, so in this case, the direction of $\vec{R} \times \vec{N}$ is same with the positive direction of Z axis. By assuming that the versor of the positive direction of Z axis is $\overrightarrow{k'}$, there is:

$$\overrightarrow{k'} = \frac{\vec{R} \times \vec{N}}{|\vec{N}||\vec{R}|} \tag{15}$$

According to the process of whirling, there is an installation angle δ between z and Z axes, we have:

$$\vec{k} \cdot \overrightarrow{k'} = \cos\delta \tag{16}$$

Considering the Eq. 9, 10, 15 and 16, there is:

$$(x_T y_N - x_N y_T)^2 = R^2 \cos^2\delta \cdot \begin{bmatrix} x_N^2 + y_N^2 + \\ (z_N - z_T)^2 \end{bmatrix} \tag{17}$$

Figure 3 also shows that the vector $\vec{N} \times \vec{R}$ is perpendicular to vector $\vec{r} \times \vec{R}$ and it can be expressed as:

$$R^2 \vec{r} \cdot \vec{N} - (\vec{r} \cdot \vec{R}) \cdot (\vec{R} \cdot \vec{N}) = 0 \tag{18}$$

where, the coordinates of $\vec{r}$, $\vec{R}$, $\vec{N}$ can be obtained by Eq. 8, 9 and 10.

In this way, the central point of tool disk O (x_T, y_T, z_T) can be calculated by Eq. 14, 17 and 18. Furthermore, the numerical control points of machine tools can be expressed as:

$$\begin{cases} A = \sqrt{x_T^2 + y_T^2} \\ C = \arctan\left(\dfrac{y_T}{x_T}\right) + \dfrac{z_T}{p} \end{cases} \tag{19}$$

where, A is the coordinate of X in machine tools and C is the rotation angle of the work piece.

CASE STUDY

An example part: When whirling the helical groove, the work piece rotates around C axis and the tool disk moves along X axis and Z axis simultaneously. An example is given as follows.

The example part is a double-thread screw shaft. Its lead pz is 88 mm, outer diameter D is 85 mm, root diameter d is 55 mm, tool tip radius r is 1.2 mm and the cross section is determined by sampled points as shown in Table 1.

According to sampled points, cubic spline fitting is used to obtain a complete profile of the cross section and the profile (curve a) is shown in Fig. 4.

Table 1: Sampled points in cross section

x (mm)	y (mm)	x (mm)	y (mm)	x (mm)	y (mm)	x(mm)	y (mm)
27.5	0	25.65	10.84	21.06	22.95	11.97	36.84
27.43	1.96	25.8	12.78	19.93	25.17	7.55	41.82
27.22	3.91	24.45	14.74	18.68	27.32	4.54	42.26
27.03	5.08	23.7	16.71	17.27	29.58	0.91	42.49
26.64	7	22.91	18.74	15.69	31.9	0.3	42.5
26.17	8.92	22.06	20.84	13.91	34.29	0	42.5

Table 2: Part of data points

X	17.7329	12.7796	8.5860	4.8425	1.3875	-1.8741
Y	21.2857	25.8651	29.1286	31.6125	33.5969	35.2460

Table 3: Central point of tool disk

x_T	-65.72963	127.1139	127.31575	124.31384	119.78070	-121.48997
y_T	107.10296	-9.342165	15.25486	34.33287	49.783841	47.52222
z_T	0.469625	-11.43530	-15.56843	-17.77453	-19.65992	-21.55498

Table 4: Cutter location data

A	C	A	C
125.6036	-0.3403	128.9417	-1.8113
127.4563	-0.8898	129.6900	-1.8477
128.2269	-0.9923	130.4537	-1.9119

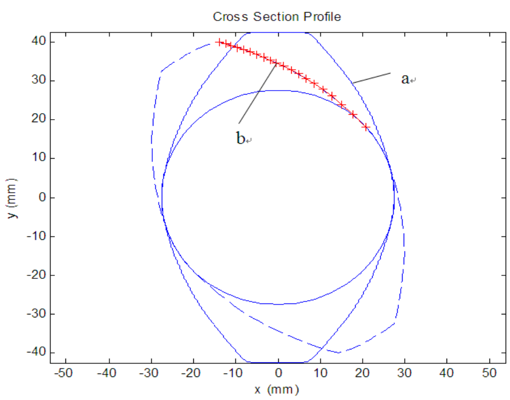

Fig. 4: Cross section profile of example part

In this whirling, the tool disk radius R is 120mm and installation angle δ is 10°, Fig. 4 (curve b) demonstrates part of the theoretical cross section profile when θ is pi/6 and part of the data points are shown in Table 2. In terms of Eq. 14, 17 and 18, just as Table 2 shows part of data points in cross section, Table 3 shows the corresponding central point of tool disk O (x_T, y_T, z_T). Then, in terms of Eq. 19, part of cutter location data are obtained in Table 4.

Error analysis of example: Theoretically, besides the installation error which is determined by operators, another major error source is the interpolation error. Suppose $A_i(x_i, y_i)$ and $A_{i+1}(x_{i+1}, y_{i+1})$ are two adjacent interpolation nodes, p_i, p_{i+1}, φ_i, φ_{i+1} are their polar radius and polar angle, respectively. There is:

$$\rho = \rho_i + \frac{\rho_{i+1} - \rho_i}{\varphi_{i+1} - \varphi_i} \varphi \qquad (20)$$

$A_j(x_j, y_j)$ is one point between A_i and A_{i+1}, r_j and φ_j are its polar radius and polar angle respectively. p_j can be obtained with putting φ_j into Eq. 20. Hence, in order to ensure the interpolation accuracy, there should be $|p_j - r_j| < \Delta$, where, Δ is the allowable error.

CONCLUSION

This study established the mathematical model of whirling helical groove on a 3-axis linkage machine and the approach to calculate the tool path provides a valuable basis for the machining of helical groove. In addition, the case study including error analysis given in this study also showed that the proposed method is effective in calculating tool path, in terms of machine motion along X, Z and C axes.

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