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**Research Article**

# Covariance Intersection Kalman Fuser for Two-Sensor System with Time-Delayed Measurements

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**Abstract:** To handle the estimation fusion problem between local estimation errors for the system with unknown cross-covariances and to avoid a large computed burden and computational complexity of cross-covariances, for a two-sensor linear discrete time-invariant stochastic system with time-delayed measurements, by the measurement transformation method, an equivalent system without measurement delays is obtained and then using the Covariance Intersection (CI) fusion method, the covariance intersection steady-state Kalman fuser is presented. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal Kalman fuser weighted by matrices with known cross-covariances. A Monte-Carlo simulation example shows the above accuracy relation and indicates that its actual accuracy is close to that of the Kalman fuser weighted by matrices, hence it has good performances.

**Keywords:** Consistency, covariance intersection fusion, information fusion kalman fuser, time-delayed measurement, unknown cross-covariance

## INTRODUCTION

Multisensor information has been applied to many fields, such as Guidance, defense, robotics, tracking, signal processing (Hall and Llinas, 1997; Li *et al*., 2003; Bar-Shalom and Li, 2001). There are two kinds of optimal information fusion methods: the centralized and distributed information fusion, where the distributed weighted fusion method applies widely because of its less computational burden. Even if, to compute the optimal distributed weighted fusion Kalman estimators (Sun and Deng, 2004; Bar-Shalom and Campo, 1986), it is required to compute the cross-covariances among local Kalman estimator errors, which needs a large computational burden, because these cross-covariances are generally unknown (Deng *et al*., 2008) or their computation is very complex (Sun and Deng, 2009; Sun *et al*., 2009; Liggins *et al*., 2009). In order to overcome this limitation, a Covariance Intersection (CI) fusion method was presented and developed in Julier and Uhlmann (1997), Chen *et al*. (2002), Julier and Uhlmann (1996, 2009) and Arambel *et al*. (2001), which can solve the fused filtering problems with unknown cross-covariances and has the consistency and robustness (Julier and Uhlmann, 1997). It has wide applications (Julier and Uhlmann, 2007; Guo *et al*., 2010; Bolzani *et al*., 2007). Besides that, the systems with time-delayed measurement exist widely in control and estimation fields, which results that it is more difficult to compute the cross-covariances. Now there are augmented state method (Kailath *et al*., 2000) and non-augmented state method (Sun and Deng, 2009; Zhang and Xie, 2007) to convert the system with time delays into that without time delays.

In this study, first, the fused estimation problem is converted into that for systems without measurement delays (Sun and Deng, 2009; Sun *et al*., 2009; Liggins *et al*., 2009) for the two-sensor system with measurement delays. Second, in order to overcome the difficulty and complexity of computing cross-covariances, by the CI method (Julier and Uhlmann, 1997; Chen *et al*., 2002; Julier and Uhlmann, 1996, 2009; Guo *et al*., 2010; Bolzani *et al*., 2007), a CI fusion steady-state Kalman fuser is presented, whose objective is to avoid to find the cross-covariances and reduce the computational burden. It can handle the fused estimation problem of the system with unknown cross-covariances and it has higher accuracy comparing with local estimators. Third, it is rigorously proved that the accuracy of CI fuser is higher than that of each local filter and is lower than that of optimal fuser weighted by matrices. The CI fuser is consistent, so it has the good performance.

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## PROBLEM FORMULATION

Consider the two-sensor linear discrete time-invariant stochastic system:

$$x(t+1) = \Phi x(t) + \Gamma w(t) \quad (1)$$

$$z_i(t) = H_i x(t-\tau_i) + \xi_i(t), i = 1,2 \quad (2)$$

where,

| | |
|---|---|
| $t$ | = The discrete time |
| $\tau_i$ | = The measurement delay |
| $x(t) \in R^n$ | = The state |
| $z_i(t) \in R^{m_i}$ | = The measurement |
| $w(t) \in R^r$, $\xi_i(t) \in R^{m_i}$ | = Uncorrelated white noises with zero mean and variances $Q_w$ and $Q_{\xi i}$, respectively |

The objectives are to find the local steady-state optimal Kalman estimators $\hat{x}_i^z(t|t+N)$, (N<0, N>), i = 1, 2, the Kalman fusers $\hat{x}_m^z(t|t+N)\,\hat{x}_d^z(t|t+N)$, $\hat{x}_s^z(t|t+N)$ weighted by matrices, diagonal matrices and scalars, the CI fusion Kalman fuser $\hat{x}_{CI}^z(t|t+N)$ and compare their accuracy relations.

## THE STEADY-STATE OPTIMAL WEIGHTED KALMAN FUSER

Introducing the new measurement $y_i(t)$ and measurement noise $v_i(t)$:

$$y_i(t) = z_i(t+\tau_i) \quad (3)$$

$$v_i(t) = \xi_i(t+\tau_i) \quad (4)$$

From (2), we have the new measurement equation without measurement delays:

$$y_i(t) = H_i x(t) + v_i(t), i = 1,2 \quad (5)$$

where, the new measurement noise $v_i(t)$ is white noise with zero mean and variance $Q_{vi} = Q_{\xi i}$.

Denoting the linear space spanned by the stochastic variables ($z_i$(t+N), $z_i$(t+N-1),…) as L($z_i$(t+N), $z_i$(t+N-1),…) and the linear space spanned by the stochastic variables ($y_i$(t+N-$\tau_i$), $y_i$(t+N-$\tau_i$ −1),…) as L($y_i$(t+N-$\tau_i$), $y_i$(t+N-$\tau_i$-1),…), we have the relation between the two linear measurement spaces as:

$$\begin{aligned} &L(z_i(t+N), z_i(t+N-1),\cdots) \\ &= L(y_i(t+N-\tau_i), y_i(t+N-\tau_i-1),\cdots) \end{aligned} \quad (6)$$

So we have the relation as:

$$\hat{x}_i^z(t|t+N) = \hat{x}_i(t|t+N-\tau_i) \quad (7)$$

where, $\hat{x}_i(t|t+N-\tau_i)$, is the linear minimum variance filters of x(t) based on ($y_i$(t+N-$\tau_i$), $y_i$(t+N-$\tau_i$ −1),…) and $\hat{x}_i^z(t|t+N)$ is the linear minimum variance filters based on L($z_i$(t+N), $z_i$(t+N-1),…). So the relation between the estimation errors $\hat{x}_i^z$(t|$t+N$) and $\hat{x}_i$ ($t|t+N$) is:

$$\tilde{x}_i^z(t|t+N) = \tilde{x}_i(t|t+N-\tau_i), i = 1,2 \quad (8)$$

where,

$$\tilde{x}_i^z(t|t+N) = x(t) - \hat{x}_i^z(t|t+N)$$
$$\tilde{x}_i(t|t+N) = x(t) - \hat{x}_i(t|t+N)$$

Defining steady-state estimator error variances and cross-covariances are $P_{ii}^z$ (N) = E[$\hat{x}_i^z$(t|$t+N$) $\hat{x}_i^{zT}$(t|$t+N$)] and $P_{ij}^z$ (N) = E[$\hat{x}_i^z$($t|t+N$) xizT(tt+N)], respectively. So we have obtained:

$$P_{ij}^z(N) = P_{ij}(N-\tau_i, N-\tau_j), i = 1,2 \quad (9)$$

where,

$$P_{ij}(N-\tau_i, N-\tau_j) = \mathrm{E}[\tilde{x}_i(t|t+N-\tau_i)\tilde{x}_i^{\mathrm{T}}(t|t+N-\tau_j)]$$

From (7) we can see that the problem of finding the local steady-state optimal Kalman estimators $\hat{x}_i^z(t|t+N)$ based on the measurement (zi (t+N), zi(t+N-1),…) is converted t that of finding the optimal steady-state Kalman estimators $\hat{x}_i(t|t+N-\tau_i)$ based on the measurement ($y_i$(t+N−$\tau_j$), $y_i$ (t+N-1),…). For N>$\tau_i$, N = $\tau_i$, N<$\tau_i$ the estimators are called the smoothers, filters and predictors, respectively.

**Lemma 1:** (Sun and Deng, 2009) For the two-sensor system (1) and (5), the local steady-state Kalman predictor $\hat{x}_i(t+1|t)$ is given by:

$$\hat{x}_i(t+1|t) = \Psi_{pi}\hat{x}_i(t|t-1) + K_{pi}y_i(t), i = 1,2 \quad (10)$$

where,

$$\Psi_{pi} = \Phi - K_{pi}H_i \quad (11)$$

$$K_{pi} = \Phi\Sigma_i H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1} \quad (12)$$

The prediction error variance matrix $\sum i$ satisfies the steady-state Riccati equation:

$$\Sigma_i = \Phi[\Sigma_i - \Sigma_i H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1}H_i\Sigma_i]\Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} \quad (13)$$

and the cross-covariance matrix $\sum ij$ = E[$\tilde{x}_i$ (t+1|$t$) $\tilde{x}_i^T$(t+1|$t$)] between local prediction errors satisfies the Lyapunov equation:

$$\Sigma_{ij} = \Psi_{pi}\Sigma_{ij}\Psi_{pj}^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}},\ i, j = 1, 2,\ i \neq j \quad (14)$$

The -k$_i$ steps steady-state Kalman predictor $\hat{x}_i$ (t|$t+k_i$) is given by:

$$\hat{x}_i(t \mid t+k_i) = \Phi^{-k_i-1}\hat{x}_i(t+k_i+1 \mid t+k_i),\ k_i \leq -2 \quad (15)$$

The -k$_i$ steps Kalman prediction error variance $P_i(k_i, k_i)$ = E[$\tilde{x}_i$ (t|$t+k_i$) $\tilde{x}_i^T$(t|$t+k_i$)] and cross-covariance $P_{ij}(k_i, k_j)$ = E[$\tilde{x}_i$ (t|$t+k_i$) $\tilde{x}_j^T$($t$|$t+k_j$)] are given by:

$$P_i(k_i, k_i) = \Phi^{-k_i-1}\Sigma_i\Phi^{(-k_i-1)\mathrm{T}} + \sum_{j=2}^{-k_i}\Phi^{-k_i-j}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{(-k_i-j)\mathrm{T}},\ k_i \leq -2 \quad (16)$$

$$P_{ij}(k_i, k_j) = \Phi^{-k_i-1}\Sigma_{ij}\Phi^{(-k_i-1)\mathrm{T}} + \sum_{j=0}^{-\max(k_i,k_j)-2}\Phi^{j}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{j\mathrm{T}},\ k_i, k_j \leq -2 \quad (17)$$

The steady-state Kalman filter $\hat{x}_i$ (t|$t$) is as following:

$$\hat{x}_i(t \mid t) = \Psi_{fi}\hat{x}_i(t-1 \mid t-1) + K_{fi}y_i(t),\ i = 1, 2 \quad (18)$$

where,

$$\Psi_{fi} = (I_n - K_{fi}H_i)\Phi \quad (19)$$

$$K_{fi} = \Sigma_i H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1} \quad (20)$$

The corresponding steady-state Kalman filtering error variance $P_{ii}$ = E[$\tilde{x}_i$(t|$t$) $\tilde{x}_i^T$(t|$t+k_i$)] and cross-covariance $P_{ij}$ = E[$\tilde{x}_i$(t|$t$) $\tilde{x}_j^T$(t|$t+k_i$)] are given by:

$$P_{ii} = (I_n - K_{fi}H_i)\Sigma_i \quad (21)$$

$$P_{ij} = \Psi_{fi}P_{ij}\Psi_{fj}^{\mathrm{T}} + (I_n - K_{fi}H_i)\Gamma Q_w\Gamma^{\mathrm{T}}(I_n - K_{fj}H_j)^{\mathrm{T}}$$
$$i, j = 1, 2,\ i \neq j \quad (22)$$

The steady-state Kalman smoother $\hat{x}_i$ (t|$t+k_i$), ( $k_i > 0$) is given by:

$$\hat{x}_i(t \mid t+k_i) = \hat{x}_i(t \mid t-1) + \sum_{r=0}^{k_i}K_i(r)\varepsilon_i(t+r)\ i = 1, 2 \quad (23)$$

where,

$$K_i(r) = \Sigma_i\Psi_{pi}^{\mathrm{T}r}H_i^{\mathrm{T}}(H_i\Sigma_i H_i^{\mathrm{T}} + Q_{vi})^{-1} \quad (24)$$

$$\varepsilon_i(t) = y_i(t) - H_i\hat{x}_i(t \mid t-1) \quad (25)$$

The Kalman smoothing error variance $P_i(k_i, k_i)$ = E[$\tilde{x}_i$ ($t$|$t+k_i$) $\tilde{x}_i^T$(t|$t+k_i$)] and cross-covariance $P_{ij}(k_i, k_j)$ = E[$\tilde{x}_i$ (t|$t+k_i$) $\tilde{x}_j^T$(t|$t+k_j$)] are given by:

$$P_i(k_i, k_i) = \Sigma_i - \sum_{r=0}^{k_i}K_i(r)(H\Sigma_i H^{\mathrm{T}} + Q_{vi})K_i^{\mathrm{T}}(r),\ k_i > 0 \quad (26)$$

$$P_{ij}(k_i, k_j) = \Sigma_{ij} - \sum_{r=0}^{\min(k_i,k_j)}K_i(r)H\Sigma_{ij}H^{\mathrm{T}}K_j^{\mathrm{T}}(r),\ k_i, k_j > 0 \quad (27)$$

**Lemma 2:** Sun and Deng (2009) For the two-sensor system (1) and (5), the steady-state Kalman filtering error variance $P_i(k_i, k_i)$ = E[$\hat{x}_i$ ($t$|$t+k_i$) $\tilde{x}_i^T$($t$|$t+k_i$)] and cross- covariance $P_{ij}(k_i, k_i)$= E[$\hat{x}_i$ ($t$|$t+ki$) $xjT$($tt+kj$)], ($ki$ = N-$\tau i$, $kj$ = N$-\tau j$) are given by

**Case 1:** When $k_i = 0$, $k_j = 0$:

$$P_i(k_i, k_i) = P_{ii} \quad (28)$$

$$P_{ij}(k_i, k_j) = P_{ij} \quad (29)$$

**Case 2:** When $k_i < 0$, $k_j < 0$:

$$P_i(k_i, k_i) = \Sigma_i, (k_i = -1) \quad (30)$$

$$P_{ij}(k_i, k_j) = \Sigma_{ij}, (k_i = -1, k_j = -1) \quad (31)$$

$$P_i(k_i, k_i) = \Phi^{-k_i-1}\Sigma_i\Phi^{(-k_i-1)\mathrm{T}} + \sum_{j=0}^{-k_i-2}\Phi^{j}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{j\mathrm{T}},\ (k_i \leq -2) \quad (32)$$

$$P_{ij}(k_i, k_j) = \Phi^{-k_i-1}\Psi_{pi}^{k_i-k_j}\Sigma_{ij}\Phi^{(-k_i-1)\mathrm{T}} + \sum_{r=-k_i-1}^{-k_j-2}\Phi^{-k_i-1}\Psi_{pi}^{k_i+r+1}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}} + \sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},\ (k_j \leq k_i \leq -2) \quad (33)$$

$$P_{ij}(k_i, k_j) = \Phi^{-k_i-1}\Sigma_{ij}\Psi_{pi}^{(k_j-k_i)\mathrm{T}}\Phi^{(-k_j-1)\mathrm{T}} + \sum_{r=-k_j-1}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{k_j+r+1}\Phi^{(-1-k_j)\mathrm{T}} + \sum_{r=-k_j-1}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{k_j+r+1}\Phi^{(-1-k_j)\mathrm{T}} + \sum_{r=0}^{-k_j-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},\ (k_i \leq k_j \leq -2) \quad (34)$$

$$P_{ij}(k_i,k_j)=\Psi_{pi}^{-k_j-1}\Sigma_{ij}\Phi^{(-k_j-1)\mathrm{T}} + \sum_{r=0}^{-k_j-2}\Psi_{pi}^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}},(k_i=-1,k_j\le -2) \tag{35}$$

$$P_{ij}(k_i,k_j)=\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pi}^{(-k_i-1)\mathrm{T}} + \sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{r\mathrm{T}},(k_i\le -2,k_j=-1) \tag{36}$$

**Case 3:** When $k_i > 0$, $k_j > 0$:

$$P_{ii}(k_i,k_i)=\Sigma_i-\sum_{r=0}^{k_i}K_i(r)[H_i\Sigma_iH_i^{\mathrm{T}}+Q_{vi}]K_i^{\mathrm{T}}(r),(k_i>0) \tag{37}$$

$$P_{ij}(k_i,k_j)=\sum_{r=0}^{k_i}[I_n\delta_{r0}-K_i(r)H_i\Psi_{pi}^{r}]\Sigma_{ij}\sum_{s=0}^{k_j}[I_n\delta_{s0}-K_j(s)Hj\Psi_{pj}^{s}]^{\mathrm{T}} + \sum_{r=0}^{\min(k_i,k_j)}\sum_{u=0}^{r-1}K_i(r)H_i\Psi_{pi}^{u\mathrm{T}}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{u\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(r),(i\ne j) \tag{38}$$

**Case 4:** When $k_i < 0$, $k_j \ge 0$:

$$P_{ij}(k_i,k_j)=\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pj}^{(-1-k_i)\mathrm{T}} - \sum_{s=0}^{k_j}\Phi^{-k_i-1}\Sigma_{ij}\Psi_{pj}^{(s-k_i-1)\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(s) + \sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{r\mathrm{T}} + \sum_{s=0}^{k_j}\sum_{r=0}^{-k_i-2}\Phi^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Psi_{pj}^{(s+r)\mathrm{T}}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(s) \tag{39}$$

**Case 5:** When $k_i \ge 0$, $k_j < 0$:

$$P_{ij}(k_i,k_j)=\Psi_{pj}^{-k_j-1}\Sigma_{ij}\Phi^{(-k_i-1)\mathrm{T}} - \sum_{s=0}^{k_i}K_i(s)H_i\Psi_{pi}^{(s-k_j-1)}\Sigma_{ij}\Phi^{(-k_j-1)\mathrm{T}} + \sum_{r=0}^{-k_j-2}\Psi_{pi}^{r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}} + \sum_{s=0}^{k_i}\sum_{r=0}^{-k_j-2}K_i(s)H_j\Psi_{pi}^{s+r}\Gamma Q_w\Gamma^{\mathrm{T}}\Phi^{r\mathrm{T}} \tag{40}$$

**Lemma 3:** Sun and Deng (2004) for the two-sensor system (1) and (5), the optimal Kalman fuser weighted by matrices is given by:

$$\begin{aligned}\hat{x}_m^z(t\mid t+N)&=\Omega_1\hat{x}_1^z(t\mid t+N)+\Omega_2\hat{x}_2^z(t\mid t+N)\\&=\Omega_1\hat{x}_1(t\mid t+N-\tau_1)+\Omega_2\hat{x}_2(t\mid t+N-\tau_2)\end{aligned} \tag{41}$$

where, the weighted matrix is given by:

$$[\Omega_1,\Omega_2]=(e^{\mathrm{T}}P^{-1}(k_1,k_2)e)^{-1}e^{\mathrm{T}}P^{-1}(k_1,k_2) \tag{42}$$

$$P(k_i,k_j)=(P_{ij}(k_i,k_j))_{NL\times NL},(i,j=1,2) \tag{43}$$

where, $e^T=[I_n\ I_n]$ and the fused error variance matrix $P_m$ is given as:

$$P_m=[e^{\mathrm{T}}P^{-1}(k_i,k_j)e]^{-1} \tag{44}$$

The optimal Kalman fuser $\tilde{x}_s^z(t|t+N)$ weighted by scalars is given by:

$$\begin{aligned}\hat{x}_s^z(t\mid t+N)&=a_1\hat{x}_1^z(t\mid t+N)+a_2\hat{x}_2^z(t\mid t+N)\\&=a_1\hat{x}_1(t\mid t+N-\tau_1)+a_2\hat{x}_2(t\mid t+N-\tau_2)\end{aligned} \tag{45}$$

where the optimal weighted coefficients $a=[a_1\ a_1]$ is given as:

$$a=\frac{e_s^{\mathrm{T}}P_{tr}^{-1}}{e_s^{\mathrm{T}}P_{tr}^{-1}e_s} \tag{46}$$

where,

$$e_s^{\mathrm{T}}=\begin{bmatrix}1 & 1\end{bmatrix} \tag{47}$$

$$P_{tr}=\begin{bmatrix}\mathrm{tr}P_1(k_1,k_1) & \mathrm{tr}P_{12}(k_1,k_2)\\ \mathrm{tr}P_{21}(k_2,k_1) & \mathrm{tr}P_2(k_2,k_2)\end{bmatrix} \tag{48}$$

The corresponding optimal fused estimation error variance $P_s$ is given by:

$$P_s=\sum_{i=1}^{2}\sum_{j=1}^{2}a_ia_jP_{ij}(k_i,k_j) \tag{49}$$

The optimal Kalman fuser $\hat{x}_d^z(t|t+N)$ weighted by diagonal matrices is given by:

$$\begin{aligned}\hat{x}_d^z(t\mid t+N)&=A_1\hat{x}_1^z(t\mid t+N)+A_2\hat{x}_2^z(t\mid t+N)\\&=A_1\hat{x}_1(t\mid t+N-\tau_1)+A_2\hat{x}_2(t\mid t+N-\tau_2)\end{aligned} \tag{50}$$

where, the optimal weighted diagonal matrices $A_j$ is given as:

$$A_j=\begin{bmatrix}a_{j1} & & & \\ & a_{j2} & & \\ & & \ddots & \\ & & & a_{jn}\end{bmatrix},\ j=1,2 \tag{51}$$

The component expression of $\hat{x}_j^z$ and $\hat{x}_d^z$ are given by:

$$\hat{x}_j^z = \begin{bmatrix} \hat{x}_{j1}^z \\ \vdots \\ \hat{x}_{jn}^z \end{bmatrix}, \hat{x}_d^z = \begin{bmatrix} \hat{x}_{d1}^z \\ \vdots \\ \hat{x}_{dn}^z \end{bmatrix} \quad (52)$$

So the Eq. (50) is equivalent with:

$$\hat{x}_{dl}^z(t \mid t+N) = \sum_{j=1}^{2} a_{jl} \hat{x}_{jl}^z(t \mid t+N), l = 1, \cdots, n \quad (53)$$

where, the optimal weighted coefficient vector is:

$$a_l = \begin{bmatrix} a_{1l} & a_{2l} \end{bmatrix} = \frac{e_d^{\mathrm{T}}(P^{ll})^{-1}}{e_d^{\mathrm{T}}(P^{ll})^{-1} e_d} \quad (54)$$

where, $e_d^T = [1 \ \dots \ 1]$ and defining:

$$P^{ll} = \begin{bmatrix} P_1^{(ll)} & P_{12}^{(ll)} \\ P_{21}^{(ll)} & P_2^{(ll)} \end{bmatrix} \quad (55)$$

where, $P_{ij}^{ll}$ is the (*l*,*l*) th diagonal component of $P_{ij}(k_i, k_j)$ and the optimal component fused error variance $P_{dl}$ and the optimal fused error variance $P_d$ are given by:

$$P_{dl} = [e_d^{\mathrm{T}}(P^{ll})^{-1} e_d]^{-1}, l = 1, \cdots, n$$
$$P_d = \sum_{i=1}^{2} \sum_{j=1}^{2} A_i P_{ij} A_j^{\mathrm{T}} \quad (56)$$

## THE STEADY-STATE CI KALMAN FUSER

From the proposed interpretation, it is clear that the computation of getting the cross-covariances between local filtering is very complex and the computed burden is very large. So a CI fusion method was presented (Julier and Uhlmann, 1997; Chen *et al*., 2002).

Consider a two-sensor system (1) (2) and (5), when $P_1$ and $P_2$ are known, but the cross-covariance $P_{12}$ is unknown, the CI Kalman fuser without $P_{12}$ is given by:

$$\hat{x}_{CI}^z(t \mid t+N) = P_{CI}[\omega P_1^{-1} \hat{x}_1^z(t \mid t+N) + (1-\omega) P_2^{-1} \hat{x}_2^z(t \mid t+N)] \quad (57)$$

where, the CI fusion error variance matrix $P_{CI}$ is given by:

$$P_{CI} = [\omega P_1^{-1} + (1-\omega) P_2^{-1}]^{-1} \quad (58)$$

where, $P_i \triangleq P_i\ (k_i, k_i)$, I = 1, 2, $P_{12} \triangleq P_{12}\ (k_i, k_2)$, $P_{21} \triangleq P_{21}\ (k_2, k_1)$ and the weighted coefficient $\omega \in [0,1]$ and minimizes the performance index:

$$\mathrm{J} = \min_{\omega} \mathrm{tr} P_{CI} = \min_{\omega \in [0,1]} \mathrm{tr}\{[\omega P_1^{-1} + (1-\omega) P_2^{-1}]^{-1}\} \quad (59)$$

where, the notation tr denotes the trace of matrix. The optimal weighting coefficient $\omega$ can be solved by the gold section method or Fibonacci method (Julier and Uhlmann, 1996).

## THE CONSISTENCY AND ACCURACY OF CI KALMAN FUSER

**Theorem 1:** The actual CI fusion error variance matrix $\bar{P}_{CI}$ is given by:

$$\bar{P}_{CI} = P_{CI}[\omega^2 P_1^{-1} + \omega(1-\omega) P_1^{-1} P_{12} P_2^{-1} + \omega(1-\omega) P_2^{-1} P_{21} P_1^{-1} + (1-\omega)^2 P_2^{-1}] P_{CI} \quad (60)$$

**Proof:** From (58), we can obtain $\bar{P}_{CI}\,[\omega P_1^{-1} + (1-\omega) P_1^{-1}] = \mathrm{I}$, which yields:

$$x(t) = P_{CI}[\omega P_1^{-1} x(t) + (1-\omega) P_2^{-1} x(t)] \quad (61)$$

Subtracting (57) from (61), we get the actual CI fuser error:

$$\tilde{x}_{CI}(t \mid t+N) = P_{CI}[\omega P_1^{-1} \tilde{x}_1(t \mid t+N) + (1-\omega) P_2^{-1} \tilde{x}_2(t \mid t+N)] \quad (62)$$

The actual CI fusion error variance $\bar{P}_{CI} = \mathrm{E}[\tilde{x}_{CI}(t|t+N)\, \tilde{x}_{CI}^T(t|t+N)]$, where E denotes the mathematical expectation, T denotes the transpose. Substituting (62) into $\bar{P}_{CI}$ yields that (60) holds. The proof is completed.

**Theorem 2**: For local and fused Kalman filters, we have the accuracy relations:

$$P_m \le P_d, P_m \le P_s, P_m \le \bar{P}_{CI}, P_m \le P_i, i = 1, 2 \quad (63)$$

$$\bar{P}_{CI} \le P_{CI} \quad (64)$$

$$\mathrm{tr} P_m \le \mathrm{tr} P_d \le \mathrm{tr} P_s \le \mathrm{tr} P_i, i = 1, 2 \quad (65)$$

$$\mathrm{tr} P_m \le \mathrm{tr} \bar{P}_{CI} \le \mathrm{tr} P_{CI} \le \mathrm{tr} P_i, i = 1, 2 \quad (66)$$

**Proof:** From (41), $\hat{x}_m^z(t|t+N)$ is the Unbiased Linear Minimum Variance (ULMV) estimate of x(t) based on the linear space $L_m = L_m\ (\hat{x}_1^z(t|t+N), \hat{x}_2^z(t|t+N))$

spanned by ($\hat{x}_1^z$(t|$t+N$)$,$ $\hat{x}_2^z$(t|$t+N$)). From (45), (50), (57), we have ($\hat{x}_0^z$(t|$t+N$) ∈ $L_m$, θs. D, CI, i, which yields (63). It has been proven $\bar{P}_{CI} \le P_{CI}$ and tr$P_m\le$ tr$P_d\le$ tr$P_s\le$ tr$P_i$, I = 1, 2 in the reference (Sun and Deng, 2004; Julier and Uhlmann, 1997; Chen *et al*., 2002), so we have (64) and (65) hold. In the performance index (59), taking ω = 0, we have J = tr$P_2$ and taking ω = 1, we have J = tr$P_1$. Hence the optimal weighting coefficient $\omega \in [0, 1]$ yields tr$P_{CI}$ ≤ tr$P_1$, tr$P_{CI}$ tr$P_2$, i.e. tr$P_{CI}$ ≤ tr$P_i$, I = 1, 2. Taking trace operation for (63) and (64), we have (66) holds. The proof is completed.

## SIMULATION EXAMPLE

Consider the tracking system with two-sensor and with time-delayed measurements:

$$x(t+1)=\begin{bmatrix}1 & T_0\\ 0 & 1\end{bmatrix}x(t)+\begin{bmatrix}0.5T_0^2\\ T_0\end{bmatrix}$$

$$z_i(t)=H_i x(t-\tau_i)+\xi_i(t),\ i=1,2$$

$$H_1=\begin{bmatrix}1 & 0\end{bmatrix}, H_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tau_1=1, \tau_2=2$$

where, x(t)∈ $R^2$ is the state, $z_i$(t) is the measurement with delays for sensor i, w(t) and $\xi_i$(t) are white noise with zero mean and variances $Q_w$ and $Q_{\xi i}$, respectively. According to the corresponding transformation, we obtain the measurement $y_i$(t) without time delays:

$$y_i(t)=z_i(t+\tau_i)=H_i x(t)+v_i(t)$$

where, $v_i$(t) = $\xi_i$(t+$\tau_i$) is the transformed measurement white noise with zero mean and variance $Q_{vi}$ = $Q_{\xi i}$. Thus the problem of finding the local steady-state optimal Kalman smoother $\hat{x}_i^z$ ($t|t+1$), (i = 1, 2) based on the measurement $z_i$(t) is converted into the problem of finding the local steady-state optimal Kalman filter $\hat{x}_1$ $(t|t)$ and Kalman predictor $\hat{x}_2$ $(t|t-1)$

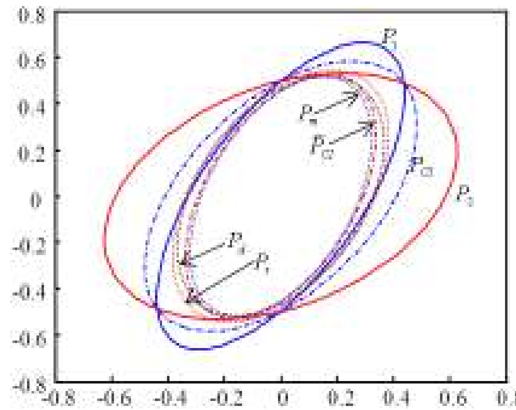


Fig. 1: The accuracy comparison of $P_1$, $P_2$, $P_0$, $P_s$, $P_d$, $P_{CI}$, and $\bar{P}_{CI}$

based on the new measurement $y_i$(t), respectively. Further, applying the Kalman filtering method yields the steady-state optimal fusion Kalman smoothers $\hat{x}_m^z$(t|$t+1$), $\hat{x}_d^z$ ($t|t+1$), $\hat{x}_s^z$(t|$t+1$) weighted by matrices, diagonal matrices and scalars and CI fused Kalman smoother $\hat{x}_{CI}^z(t|t+1)$. In simulation, we take $T_0$ = 0.4, $Q_w$ = 1, $Q_{\xi 1}$ = 0.36, $Q_{\xi 2}$ = $\begin{bmatrix}4 & 0\\ 0 & 0.16\end{bmatrix}$.

In order to give a powerful geometric interpretation with respect to accuracy relations of local and fusers, the covariance ellipse for a covariance matrix $P$ is defined as the locus of points {x:$x^T P^{-1}$x = c} where, c is a constant. In the sequel, c = 1 will be assumed without loss of generality. It was proved (Julier and Uhlmann, 1996) that $P_a\le P_b$ is equivalent to that the ellipse for $P_a$ is enclosed in the ellipse for $P_b$.The simulation results are shown in Fig. 1.

From Fig. 1, it is clearly shown that the covariance ellipse for $P_m$ lies within the intersection of ellipses for $P_1$ and $P_2$, the ellipse for CI fused theoretical covariance $P_{CI}$ encloses the intersection region of ellipses for $P_1$ an

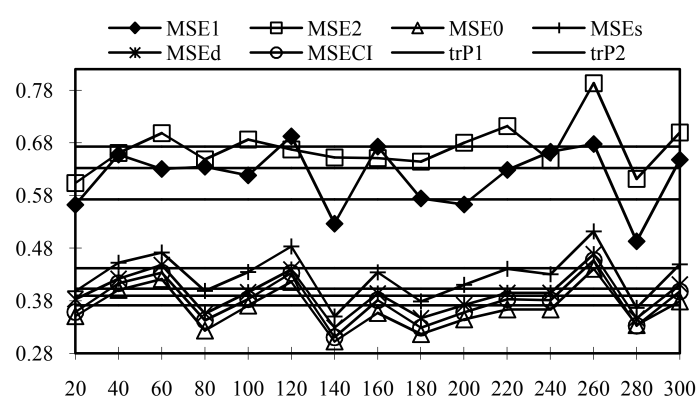


Fig. 2: The curves of $MSE_i$(t)

$P_2$ (Chen *et al*., 2002) and passes through the four points of intersection of ellipses for $P_1$ and $P_2$. The ellipse for CI fused actual covariance $\bar{P}_{CI}$ lies within the ellipse for theoretical covariance $P_{CI}$.

In order to verify the above theoretical results for accuracy relation, 200 Monte-Carlo runs are performed for t = 1,…, 300, the results are shown in Fig. 2.

where the straight lines denote tr$P_i$, I = 0, 1, 2, CI and tr$\bar{P}_{CI}$, the curves denote the corresponding mean square errors $MSE_i(t)$. We see the CI fuser has good performance, whose accuracy is approximates to the accuracy of the optimal smoother, because $MSE_{CI}(t)$ is approximates to $MSE_m(t)$.

## CONCLUSION

For the two-sensor system with time-delayed measurements and with unknown cross-covariance matrices, a CI fusion steady-state Kalman fuser has been presented, which has the advantage that the computation of cross-covariances can be avoided. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal fuser weighted by matrices. A two-sensor system Monte-Carlo simulation example shows that its accuracy is approximates to the accuracy of the optimal fuser and is higher than those of the fusers weighted by diagonal matrices and scalars.

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