Research Journal of Applied Sciences, Engineering and Technology 6(20): 3767-3771, 2013
DOI:10.19026/rjaset.6.3589
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: January 10, 2013 Accepted: March 02, 2013 Published: November 10, 2013

**Research Article**

# Study on a New Character of Chaotic States to a Nonlinear Measure Model: Pseudo Temperature

Huiz-Hong Cheng and Xie-Feng Cheng
Atmospheric Observation Technology Center of Yunnan Protection, Yunnan, Kunming, China

**Abstract:** Chaos is ubiquitous and extremely complicated phenomenon in the decision of nonlinear systems. Bifurcations of periodic, quasi-periodic and chaotic oscillations are frequently observed in forced nonlinear circuits described by no autonomous differential equations with a periodic external forcing term. Qualitative analysis using the Poincare mapping is commonly applied in the study of bifurcation phenomena as well as in numerical analysis of periodic and non-periodic solutions. In the measurement of electronics, electrical engineering, when the measurement object is nonlinear, the results of measurement may occur as chaotic state. The study used the nonlinear circuit as the measure model, studies probability density of "Pseudo-Temperature", which can be used to represent the degree of chaotic states and provides a new way of quantitative analysis to chaotic states of the nonlinear measure system. The experiment results show Pseudo-Temperature is new characteristic parameters, it can measure wide-area information from the normal to the randomness in the chaotic state.

**Keywords:** Chaotic, nonlinear circuit, probability density, pseudo temperature

## INTRODUCTION

Chaos is ubiquitous and extremely complicated phenomenon in the decision of nonlinear systems. Chaotic state is a steady-state response of nonlinear system which is always restricted to limited area and its track never overlaps with complicated behavior and random movements. In the measurement of electronics, electrical engineering, when the measurement object is nonlinear, the results of measurement may occur as chaotic state.

In order to give the background of chaotic states and to support the new concept of pseudo temperature, There are several algorithms had proposed for describing the chaotic phenomenon in the nonlinear circuit, such as Louodop *et al*. (2012), Chen and Wu (2012), Trzaska (2011), Cheng *et al*. (2008), Ning *et al*. (2006) and Kawakami (1992).

In this study, the purpose is to present some elementary mechanisms of the bifurcations and to discuss numerical methods for analyzing these nonlinear phenomena. As illustrated examples, numerical results of forced oscillatory circuits containing storable inductors are presented. We used the nonlinear circuit as the measure model. At the same time, we propose a new concept "Pseudo-Temperature" which can be used to represent when it comes into chaotic states and provides a new way of quantitative analysis to chaotic states of the nonlinear measure system. The experiment results show Pseudo-Temperature is new characteristic parameters and it's a new way to measure wide-area information from the normal to the randomness in the chaotic state.

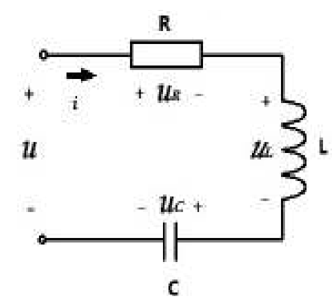



Fig. 1: The RLC nonlinear circuit

## LITRATURE REVIEW

In Fig. 1, the RLC of nonlinear circuit connection in series can be regarded as a simplified model for some certain nonlinear systems, the circuit equation:

$$L\frac{di}{dt} + Ri + Vc = V_o \sin \omega t \tag{1}$$

where,

C = The voltage-controlled variable capacitance diodes

$V_c$ = The terminal voltage of the nonlinear capacitance

**Corresponding Author:** Huiz-Hong Cheng, Atmospheric Observation Technology Center of Yunnan Protection, Yunnan, Kunming, China, Tel.: 15850781687

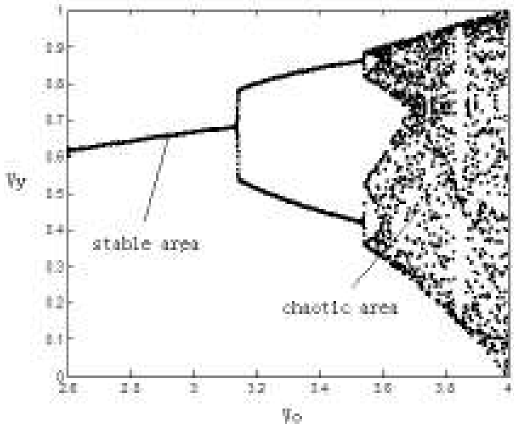



Fig. 2: Phenomenon of bifurcation to chaotic state in the logistic system

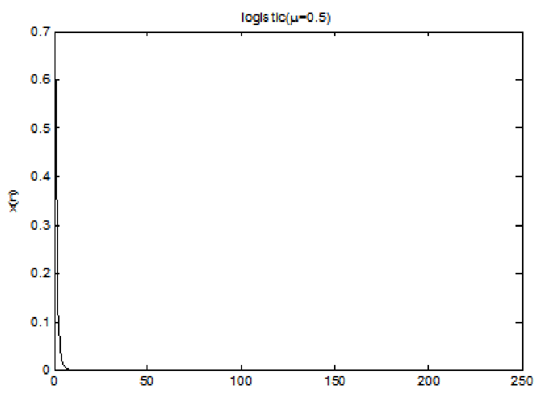
$$\because C = C_o / (1+\beta V_C)^{\frac{1}{2}}$$
$$\therefore V_c = (C_0{}^2 - C^2)/\beta C^2 \qquad (2)$$

Thus, Eq. (1) can be written as：

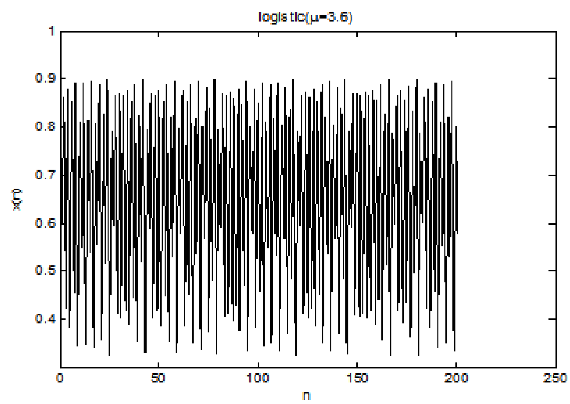
$$q + \frac{R}{L} q + \frac{1}{L}\left(\frac{C_0{}^2 - C^2}{\beta C^2}\right) = \frac{V_0}{L}\sin \omega t \qquad (3)$$

Acting as a measurement of electric current of the excitation signal function V (t), also as other control parameters, ω, R, L, C0 and β function to study. For this type of circuit model, Lindsay and Testa, had observed the capacitor terminal voltage Vc (shown as the ordinate $V_y$) early in 1981 in the experiment. and the amplitude of the excitation signal V (t), $V_0$ ($V_0$ shown in the abscissa) in a relationship that the phenomenon of turning from the bifurcation of the two-cycle state to the chaotic state (Fig. 2). Besides, we give the Logistic phase diagram of different values of μ with shown in Fig. 3. Using nonlinear circuit as the measure model, the paper studies the probability density and gives a new concept "Pseudo Temperature".

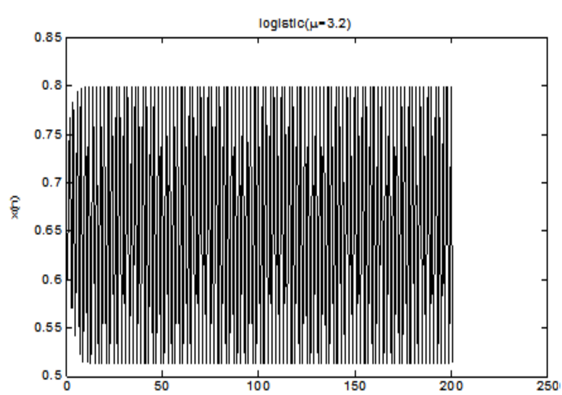
Further study into the impact on the Logistic system when the value is changing: Suppose the initial value is 0.6 and m changes from 2.6 to 4, then the Logistic iteration limit morphology diagram is shown in Fig. 2 (Thompson AND Stewart, 1986), with the value changes, the system dynamics form changes



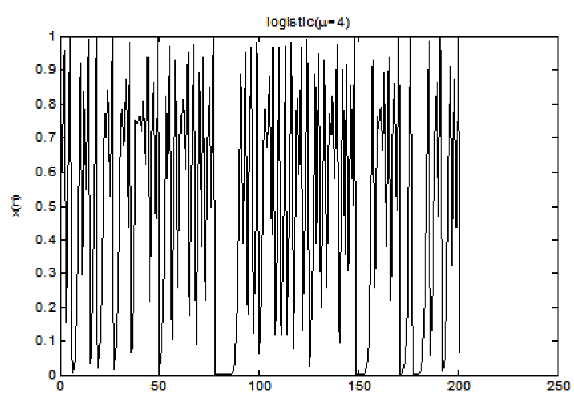

(a)




(b)




(c)




(d)

Fig. 3: Logistic phase diagram with different values of μ

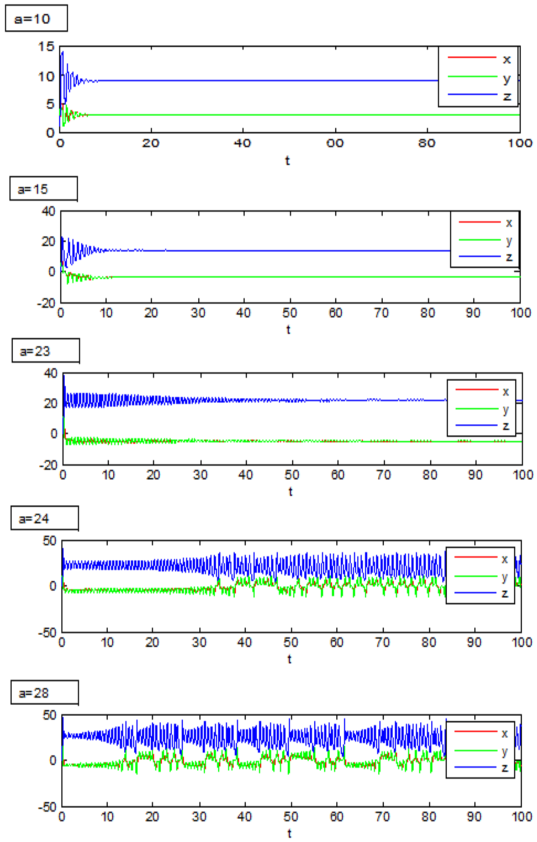



Fig. 4: Waveforms of time domain response in the Lorenz system with different values

constantly and finally we will see chaotic state (Gonzalez and Woods, 1992; Jain, 1989).

Another common chaotic model is Lorenz system. American meteorologist Lorenz posed a strange attractor dynamics system in the performance of atmospheric science magazine in 1963. The chaotic system model can be described by the following differential equations:

$$\frac{dx}{dt} = -c(x-y) \qquad \frac{dy}{dt} = ax - y - xz \qquad \frac{dz}{dt} = b(xy - z) \tag{4}$$

We use MATLAB mathematical software to solve the above differential equations and begin numerical simulations. At first, we need to establish the M-file Lorenz to define the script function and then we start the programming calls.

Fixed other parameters and set the initial values f0 and computation time t0 we can find the phenomenon by changing the parameters a so that the system gradually enters into the chaotic state of the process. The Fig. 4 gives us the circumstance of waveforms in the Lorenz model with different values.

We believe the chaotic state probability density and its Pseudo temperature are symbols that are measured objects in a chaotic state "chaos in the orderly", with big significance signing important regularity. And knowing the probability density, we will obtain the random process of the object, so that determines the chaotic state of the pseudo random process. It provides new characteristic parameters so that intelligent instruments can measure wide-area information from the normal to the randomness in the chaotic state

## MATERIALS AND METHODS

Because the chaotic state is a pseudo-random state, we will take each pseudorandom state of chaotic state as spatial representation points, which we called "pseudo-particle" (Li and Yuan, 2008; Trzaska, 2011). Changes of the Representation Points in the phase space are equivalent to the pseudo-random variation of the chaotic state. Therefore, in order to describe the circuit model chaotic signals---pseudorandom changes of the chaotic state, we define state variables**:**

$$q_1(t) = q(t), q_2(t) = q(t), \cdots q_n(t) = q^{(n-1)}(t) \tag{5}$$

Here we assume that the phase space is flat (q1, q2).Pseudorandom changes in the chaotic state are corresponding to the pseudo random motions of the representation point in the phase space and the dominated differential equations describe the point motions in the phase space has been proposed by Ning *et al*. (2006):

$$q_i = f_i(q_1, q_2, \cdots, q_n) + f_i(t) \tag{6}$$

According to the Fokker-Planck Eq. (3), (4):

$$\frac{\partial p}{\partial t} + \sum_i \frac{\partial p f_i}{\partial q_i} = -\sum_i \lambda \frac{\partial p}{\partial q_i} + \frac{1}{2}\sum_i S_{ii}\frac{\partial^2}{\partial q_i^2} + \sum_{i<j} S_{ij}\frac{\partial^2 p}{\partial q_i \partial q_j} \tag{7}$$

$\lambda_i$, $S_{ii}$, $S_{ij}$ are the coefficients of this equation, defined as:

$$\lambda_i = \lim_{\Delta t \to 0} \frac{1}{\Delta t} < \int_t^{t+\Delta t} F_i(\tau)d\tau > \tag{8}$$

$$S_{ii} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} < \{\int_t^{t+\Delta t} F_i(\tau)d\tau\}^2 > \tag{9}$$

$$S_{ij} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} < \int_t^{t+\Delta t} F_i(\tau)dt \int_t^{t+\Delta t} F_j(\tau)dt > \tag{10}$$

Following a series of calculations below, we can get the probability density of the chaotic state. First, from the formula (4), (5):

$$\begin{cases} q_1 = q_2 \\ q_2 = -\frac{R}{L}q - \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2}) + \frac{V_0}{L}\sin\omega t \end{cases} \quad (11)$$

Then from the formula (4), (5), (6) we can know:

$$\begin{cases} f_1 = q_1(t) = q_2(t), F_1 \equiv 0 \\ f_2 = -\frac{R}{L}q - \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2}), F_2 \equiv \frac{V_0}{L}\sin\omega t \end{cases} \quad (12)$$

Base on $F_1 = 0$, $F_2 = V_0/L \sin\omega t$ According to the definition of formula (8), (9) and (10) we get this by calculating:

$$\begin{Bmatrix} \lambda_1 = 0 \\ \lambda_2 = \lim_{\Delta t \to 0} \frac{1}{\Delta t}(\frac{V_0}{L}) < \int_t^{t+\Delta t} \sin\omega\tau d\tau \end{Bmatrix}^2_{\geq 0} \quad (13)$$

Obviously, $S_{11} = 0$, $S_{12} = 0$ and we can get this from formula (8) by calculating:

$$S = S_{22} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} < \left\{ \int_t^{t+\Delta t} (\frac{V_0}{L}) \sin\omega\tau d\tau \right\}^2 >= \frac{V_0^{\ 2}}{2\omega L^2} \quad (14)$$

Take formula (12), (13) and (14) into formula (7), obtaining the nonlinear RLC Eq. (4), corresponding to the Fokker-Planck equation:

$$\frac{\partial p}{\partial t} + q_2 \frac{\partial p}{\partial q_1} + \frac{\partial p}{\partial q_2}\left\{ -\frac{R}{L}q - \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2}) \right\} = \frac{S}{2}\frac{\partial^2 P}{\partial q_2^{\ 2}} \quad (15)$$

For the steady state that $\partial p/\partial t = 0$, so the formula (15) turns into:

$$\frac{S}{2}\frac{\partial^2 p}{\partial q_2^{\ 2}} - q_2\frac{\partial p}{\partial q_1} + \frac{\partial}{\partial q_2}\left\{ [\frac{R}{L}q + \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2})]p \right\} = 0 \quad (16)$$

Turn the formula (16) into variant as follows:

$$\frac{\partial}{\partial q_2}\left[ \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2})p \right] + \frac{LS}{2R}\frac{\partial}{\partial q_2}\frac{\partial p}{\partial q_1} + \frac{R}{L}\frac{\partial}{\partial q_2}(qp) + \frac{R}{L}\frac{\partial}{\partial q_2}(\frac{LS}{2R}\frac{\partial}{\partial q_2}) - \frac{\partial}{\partial q_1}(q_2 p) - \frac{\partial}{\partial q_1}(\frac{LS}{2R}\frac{\partial}{\partial q_2}) = 0 \quad (17)$$

Then the formula (18) becomes:

$$\frac{\partial}{\partial q_1}\left[ \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2})p + \frac{LS}{2R}\frac{\partial p}{\partial q_1} \right] + (\frac{R}{L}\frac{\partial}{\partial q_2} - \frac{\partial}{\partial q_1})\left[ q_2 p + \frac{LS}{2R}\frac{\partial p}{\partial q_2} \right] = 0 \quad (18)$$

Eq. (18) and the following equations can be solved:

$$\begin{cases} \sum_{j=1}^{n} L_j(\vec{x})\left[ \beta_j(x_j)p + r_j(x_j)\frac{\partial p}{\partial x_j} \right] = 0 \\ \beta_j(x_j)p + r_j(x_j)\frac{\partial p}{\partial x_j} = 0 \end{cases} \quad (19)$$

Similarly, comparing with formula (16):

$$\begin{cases} L_1 = \frac{\partial}{\partial q_2}, L_2 = \frac{R}{L}\frac{\partial}{\partial q_2} - \frac{\partial}{\partial q_1} \\ \beta_1(x_1) = \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2}), \beta_2 = q_2 \\ r_1(x_1) = r_2(x_2) = \frac{LS}{2R} \end{cases} \quad (20)$$

The solution of equation: (Zhang *et al*., 1998):

$$p(\vec{x}) = C\prod_{j=1}^{n} \exp\left[ -\int_0^{x_j} \frac{\beta_j(x_j)}{r_j(x_j)} dx_j \right] \quad (21)$$

Take Eq. (20) into (21), then we can get the solution of Eq. (18):

$$\begin{aligned} p(q_1, q_2) &= C\exp\left\{ -\frac{2R}{LS}\left[ \int_0^{q_1} \frac{1}{L}(\frac{C_0^{\ 2} - C^2}{\beta C^2})dq_1 + \frac{1}{2}q_2^{\ 2} \right] \right\} \\ &= C\exp\left\{ -\frac{2R}{L^2 S}\left[ \frac{1}{2}Lq_2^{\ 2} + \int_0^{q_1} V_c dq_1 \right] \right\} \end{aligned} \quad (22)$$

Integration of the index above $\int_0^{q_1} V_c q_1$ is the integration to the nonlinear potential energy Vc, it should be the electric potential energy stored in the nonlinear capacitor, however $\frac{1}{2}Lq_2^{\ 2} = \frac{1}{2}Li^2$ is the energy of motion, so the sum of the two is the total energy in the circuit, therefore we can get:

$$p(q, i) = C\exp\left\{ -\frac{2R}{L^2 S}W \right\} \quad (23)$$

## RESULTS

Eq. (23) is how we use the calculation method to obtain the description of Fig. 1, the probability density of chaotic state in the nonlinear circuit model. Obviously, it is similar in the shape with the distribution law of Boltzmann:

$$p(x, x) = k\exp(-\frac{W}{kT}) \quad (24)$$

So we can make a new definition:

$$\frac{SL^2}{2Rk} = T^* \quad (25)$$

We call this $T^*$ as "Pseudo Temperature". Obviously, the dimension remains true temperature. In this way, the formula (24) can be written as:

$$p(q,i) = C\exp\left\{-\frac{W}{kT^*}\right\} \quad (26)$$

## CONCLUSION

The study used the nonlinear circuit as the measure model, studies probability density of "Pseudo-Temperature", which can be used to represent the degree of chaotic states and provides a new way of quantitative analysis to chaotic states of the nonlinear measure system. Some typical illustrative example above using chaotic circuit system has shown satisfactory performances to support the theory.

So, "Pseudo Temperature" is a new concept in the study. Pseudo temperature $T^*$ characterizes the random changes of the phase space- A kind of intensity describes the "Pseudo thermal motion''. At the same time, indirectly characterizes the nonlinear circuit model appears degree of pseudo-random chaotic state. Indirectly characterizes the degree of pseudo-random of chaotic state appears in the nonlinear circuit model. Similar research for other chaotic phenomena can be studied likewise.

## ACKNOWLEDGMENT

This study is supported by the National Natural Science Foundation of China (Grant Nos. 61271334) and Science Project of Atmospheric Observation Technology Center of Yunnan Protection.

## REFERENCES

Chen, H. and L. Wu, 2012. Design and Implementation of a Simple Nonlinear Transformation Circuit for Generating More Chaotic Signals. pp: 3131-3136.

Cheng, X.F., *et al*., 2008. A new single-channel mix signal separation technique in noise. J. Comput. Inform. Syst., 1(4): 321-328.

Gonzalez, R.C. and R.E. Woods, 1992. Digital Image Processing. Addison-Wesley Publ., Reading, MA.

Jain, A.K., 1989. Fundamentals of Digital Image Processing. Prentice Hall, ISBN: 0-13-336165-9.

Kawakami, H., 1992. Bifurcation and chaotic state in forced oscillatory circuits containing saturable inductors. Int. J. Appl. Electromagn. Mater., 3(3): 215-220.

Li, B.B. and Z.F. Yuan, 2008. Non-linear and chaos characteristics of heart sound time series. P. I. Mech. Eng. H, 222(3): 265-272.

Louodop, P., H. Fotsin1 and S. Bowong, 2012. A strategy for adaptive synchronization of an electrical chaotic circuit based on nonlinear control. Physica Scripta, 85(2).

Ning, X.B., C.H. Bian and J. Wang, 2006. Research progress in nonlinear analysis of heart electric activities. Chinese Sci. Bull., 51(4): 385-393.

Thompson, J.T.M. and H.B. Stewart, 1986. Nonlinear Dynamics and Chaos. John Wiley, New York.

Trzaska, M., 2011. Chaotic oscillations in fractional-order nonlinear circuit models of bipolar pulsed electroplatings. Proceeding of the European Conference on Circuit Theory and Design, ECCTD, pp: 166-169.

Zhang, J.H., X.F. Cheng, *et al*., 1998. Celeration limit of BH. Int. J. Theor. Phys., pp: 3031-3040.