

## Research Article

### Higher Order Cumulants based Digital Modulation Recognition Scheme

<sup>1</sup>Sajjad Ahmed Ghauri, <sup>2,3</sup>Ijaz Mansoor Qureshi, <sup>1,3</sup>Aqdas Naveed Malik and

<sup>1,3</sup>Tanveer Ahmed Cheema

<sup>1</sup>NUML, Islamabad, Pakistan

<sup>2</sup>Air University, Islamabad, Pakistan

<sup>3</sup>Institute of Signals, Systems and Soft Computing (ISSS), Pakistan

**Abstract:** In this study, we have presented that Higher Order Cumulants (HOC) based modulation recognition scheme for Pulse Amplitude Modulation (PAM), Quadrature Amplitude Modulation (QAM) and Phase Shift Keying (PSK) modulated signals having orders of 2 to 64. Modulation recognition is a process to recognize the signal modulation type which is received by the receiver in the presence of channel noise. The HOC based MR is accomplished in two modules. First is feature extraction using higher order cumulants. These features are distinct for different modulated signals. Second is recognition process which gives decision based upon the features extracted from higher order cumulants. The Probability of Correctness (POC) curves shows the recognition accuracy for sample size and number of iterations. The Additive White Gaussian Noise (AWGN) is considered throughout the simulations.

**Keywords:** Additive White Gaussian Noise (AWGN), Higher Order Cumulants (HOC), Modulation Recognition (MR), Probability of Correctness (POC)

## INTRODUCTION

Modulation Recognition (MR) is the intermediate step between detection of the information contained in the signal and demodulation of signal. Automatic Modulation Recognition (AMR) has various applications in Cognitive Radio (cooperative and non-cooperative communication), civilian and military communication, electronic warfare and surveillance. AMR is to recognize the received signal modulation type, which has undergone through channel effects like fading, noise and interference etc., during transmission of the signal. AMR is basically a non-cooperative communication, (Panagiotou *et al.*, 2000) which also includes some aspects of cooperative communication, such as tracking and identification of channel and estimation and detection of signal parameters.

The major techniques of automatic modulation recognition are decision theoretic methods and pattern recognition. The decision theoretic approach is based on likelihood function of the received signal. The modulation recognition in decision theoretic approach can be viewed as multiple hypothesis tests, or may be considered sequence of pair-wise multiple hypothesis test. Once the likelihood function of the received signal is constituted, Average Likelihood Ratio Test (ALRT) and Generalized Likelihood Ratio Test (GLRT) can be pragmatic to determine the modulation type. Likelihood based approach is theoretically optimal but

computationally complex (Wang and Wang, 2010). Due to phase errors, frequency offset, channel effects and timing jitter, the decision theoretic methods are not robust to model mismatch (Wei and Mendel, 2000; Yucek and Arslan, 2004; Zhao and Tao, 2004).

The Feature Based (FB) pattern recognition method is the suboptimal solution (Swami and Sadler, 2000). In FB approach modulation recognition is carried out in two modules; the first module is feature extraction subsystem, in which features are extracted from the received signal with channel effects; the second module is pattern recognizer subsystem, in which features extracted from the received signal are compared with the theoretical values of the reference features and determines the modulation type of transmitted signal. Due to robustness with respect to model mismatches and low computational complexity, FB approach is used for modulation recognition. Few research contributions in this are Marchand *et al.* (1997), Dobre *et al.* (2003), Kadambe and Jiang, (2004), Guan *et al.* (2004) and Dobre *et al.* (2007).

In this study we have used the Higher Order Cumulants (HOC) to recognize the modulation type of received signal corrupted by additive white Gaussian noise. The features extracted based on moments and cumulants for recognition purpose considering PAM and QAM modulations for order 2 to 64. The theoretical values of higher order moments and higher order cumulants for considered modulations are

included. The following modulated signals PAM 2 to PAM 64, QAM 2 to QAM 64 and PSK 2 to PSK 64 are used for recognition purpose. The Probability of Correctness (POC) curves are simulated, based on Signal to Noise Ratio (SNR), number of iterations and sample size. The simulation results using HOC for the considered modulated signals show that high recognition rate is achieved at low SNR. The cummulants based tree structure for recognition of PSK signals is briefly presented.

**SYSTEM MODEL AND FEATURES USED**

**System model:** Figure 1 shows the system model. The generalized expression for signal received is given by:

$$r(n) = s(n) + y(n) \tag{1}$$

where,  
 $r(n)$  : Complex baseband envelop of received signal  
 $y(n)$  : The additive white gaussian noise  
 $s(n)$  : Given by:

$$s(n) = Ke^{i(2\pi f_0 nT + \theta_n)} \sum_{j=-\infty}^{j=\infty} s(l)h(nT - jT + \epsilon_T T) \tag{2}$$

where,  
 $s(l)$  : Input symbol sequence which is drawn from set of M constellations of known symbols and it is not necessary that symbols are equi-probable  
 K : Amplitude of signal  
 $f_0$  : Frequency offset constant  
 T : Symbol spacing  
 $\theta_n$  : The phase jitter which varies from symbol to symbol  
 $h(\dots)$ : Channel effects  
 $\epsilon_T$  : The timing jitter

**Features used:** As Cummulants are made up of moments, so various moments have been used as features. For the complex valued stationary random process  $r(n)$ , Cummulants of 2<sup>nd</sup>, 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> order have the following definitions:

$$C_{20} = E[y^2(n)] = cumm\{y(n), y(n)\} \tag{3}$$

$$C_{21} = E[|y(n)|^2] = cumm\{y(n), y^*(n)\} \tag{4}$$

$$C_{40} = M_{40} - 3M_{20}^2 = cumm\{y(n), y(n), y(n), y(n)\} \tag{5}$$

$$C_{41} = M_{40} - 3M_{20}M_{21} = cumm\{y(n), y(n), y(n), y^*(n)\} \tag{6}$$

$$C_{42} = M_{42} - |M_{20}|^2 - 2M_{21} = cumm\{y(n), y(n), y^*(n), y^*(n)\} \tag{7}$$

$$C_{60} = M_{60} - 15M_{20}M_{40} + 30M_{20}^3 = cumm\{y(n), y(n), y(n), y(n), y(n), y(n)\} \tag{8}$$

$$C_{61} = M_{61} - 5M_{21}M_{40} - 10M_{20}M_{41} + 30M_{20}^2M_{21} = cumm\{y(n), y(n), y(n), y(n), y(n), y^*(n)\} \tag{9}$$

$$C_{62} = M_{62} - 6M_{20}M_{42} - 8M_{21}M_{41} - M_{22}M_{40} + 6M_{20}^2M_{22} + 24M_{21}^2M_{22} = cumm\{y(n), y(n), y(n), y(n), y^*(n), y^*(n)\} \tag{10}$$

$$C_{63} = M_{63} - 9M_{21}M_{42} + 12M_{21}^3 - 3M_{20}M_{43} - 3M_{22}M_{41} + 18M_{20}M_{21}M_{22} = cumm\{y(n), y(n), y(n), y^*(n), y^*(n), y^*(n)\} \tag{11}$$

$$C_{80} = M_{80} - 35M_{40}^2 - 28M_{60}M_{20} + 420M_{40}M_{20}^2 - 630M_{20}^4 = cumm\{y(n), y(n), y(n), y(n), y(n), y(n), y(n), y(n)\} \tag{12}$$

$$C_{84} = M_{84} - 16C_{63}C_{21} + |C_{40}|^2 - 18C_{42}^2 - 72C_{42}C_{21}^2 - 24C_{21}^4 = cumm\left\{\begin{matrix} y(n), y(n), y(n), \\ y(n), y^*(n), y^*(n), y^*(n), y^*(n) \end{matrix}\right\} \tag{13}$$

$M_{pq}$  stands for moments of received signal and it is given:

$$M_{pq} = E[y(k)^p y^*(k)^q] \tag{14}$$

**Feature values based on moments and cummulants:** The theoretical values of Moments and Cummulants which were used for various signal constellations of

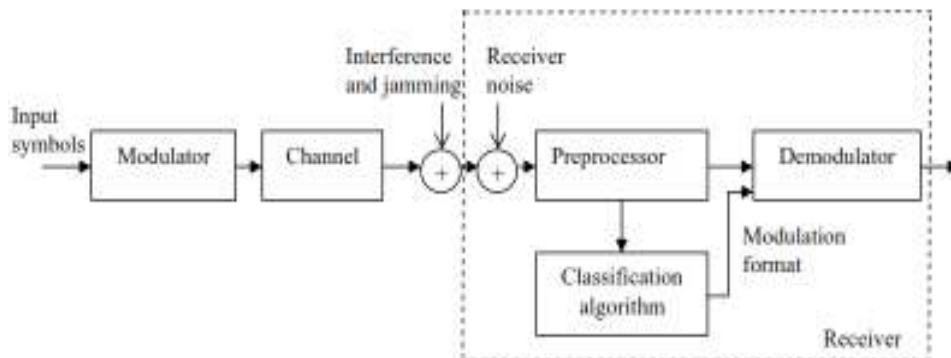


Fig. 1: The system model

Table 1: Theoretical values of moments of different modulation types

PSK	PSK 2	PSK 4	PSK 8	PSK 16	PSK 32	PSK 64
M20	1	0	0	0	0	0
M21	1	1	1	1	1	1
M40	1	1	0	0	0	0
M41	1	0	0	0	0	0
M42	1	1	1	1	1	1
M60	1	0	0	0	0	0
M61	1	1	0	0	0	0
M63	1	1	1	1	1	1
M80	1	1	1	0	0	0
M84	1	1	0	1	0	0
QAM	QAM 2	QAM 4	QAM 8	QAM 16	QAM 32	QAM 64
M20	1	0	4	0	0	1
M21	2	0	1	1	0	0
M40	2	1	1	1	0	1
M41	1	0	0	0	0	0
M42	0	7	1.20	1.29	1.32	1.38
M60	0	0	0	0	0	0
M61	1	8	1.18	1.32	1.30	1.29
M63	1	3	1.80	1.96	2	2.22
M80	0.8	0.9	1.50	2.20	2	1.92
M84	1	2.5	2.90	3.12	3.50	3.96
PAM	PAM2	PAM 4	PAM 8	PAM 16	PAM 32	PAM 64
M20	1	5	21	85	344	1356
M21	2	1	1	1	1	1
M40	2	1	1	1	1	1
M41	1	40	769	12835	211795	3327731
M42	0	47	861	14444	237102	3678478
M60	0	0	0	0	0	0
M61	1	358	33203	2294370	154364560	9741354452
M63	1	14	123	1012	8274	65013
M80	0	0	0	0	0	0
M84	1	40	769	12835	211795	3327731

Table 2: Theoretical values of cummulants of different modulation types

PSK	PSK2	PSK4	PSK8	PSK16	PSK32	PSK64
C20	1	0	0	0	0	0
C21	1	1	1	1	1	1
C40	2	1	0	0	0	0
C41	2	0	0	0	0	0
C42	2	1	1	1	1	1
C60	31	4	0	0	0	0
C63	13	4	4	4	4	4
C80	350	35	0	0	0	0
C84	163	34	34	34	34	34
QAM	QAM 2	QAM 4	QAM 8	QAM 16	QAM 32	QAM 64
C20	1	0	4	0	0	0
C21	1	1	1	1	1	1
C40	2	1	1	1	0	1
C41	2	0	1	0	0	0
C42	2	1	1	1	1	1
C60	31	0	0	0	0	0
C63	13	1.96	2.2	2	1.9	1.8
C80	350	13.60	13.5	13	12	11
C84	163	13.60	13.5	13	12	11
PAM	PAM2	PAM 4	PAM 8	PAM 16	PAM 32	PAM 64
C20	1	5	21	85	337	1368
C21	1	5	21	85	337	1368
C40	2	1	1	1	1	1
C41	2	1	1	1	1	1
C42	2	1	1	1	1	1
C60	31	2210	139244	9574215	605695009	40748052374
C63	13	707	52226	3636415	230197704	15363424446
C80	350	402993	113587175	32331650963	8152670391483	2206949690426600
C84	163	44060	12929717	3693480387	931687158728	252221335666583

interest are given in Table 1 and 2. The values are obtained by calculating the ensemble averages under noise free channel conditions.

In Table 1, the computed values of moments of PSK, QAM and PAM with orders 2 to 64 are listed. The moments (row wise) M 21, M 42 and M 63 are same for all orders of PSK modulated signals. The moments M 20, M 41 and M 60 are also same for all orders of PSK modulated signals. The moment M 60 has same value for all orders of QAM modulated signals? Also all moments for PSK 2 have same constant value i.e., 1.

In Table 2, the computed values of cummulants of PSK, QAM and PAM with orders 2 to 64 are listed. The cummulants C 21 and C 42 are same value for all orders of PSK and QAM modulated signals.

### SIMULATION RESULTS

The recognition of PAM and QAM modulated signals in the presence of additive white Gaussian noise is evaluated here. The modulated signals considered here is PAM 2, PAM 4, PAM 8, PAM 16, PAM 32, PAM64 and QAM 2, QAM 4, QAM8, QAM 16, QAM

Table 3: The correct rate of recognition of PAM and QAM under different SNR

	SNR in dB					
	-5	0	5	10	15	20
PAM						
PAM2	80	95	100	100	100	100
PAM4	90	100	100	100	100	100
PAM8	89	100	100	100	100	100
PAM16	91	100	100	100	100	100
PAM32	90	100	100	100	100	100
PAM64	90	100	100	100	100	100
QAM						
QAM2	90	95	100	100	100	100
QAM4	80	100	100	100	100	100
QAM8	90	100	100	100	100	100
QAM16	100	100	100	100	100	100
QAM32	80	100	100	100	100	100
QAM64	90	100	100	100	100	100

32, QAM 64. Table 3 shows the simulation results of correct recognition of modulated signals using the higher order Cummulants features under different SNR. The recognition of PAM and QAM modulated signals are acceptable above -5 dB. As SNR increases from -5 to 20 dB, the correct rate of recognition also increases, while at SNR = 0 dB the correct rate of recognition reaches 100%.

For example considering the modulated signal PAM2; the graphical representation of the probability of correctness curve for varying SNR is shown in Fig. 2. The probability of correctness gradually increases with the increase in SNR. The probability of correctness approaches 1 at SNR = -2 dB.

For example considering the modulated signal PAM4; the graphical representation of the probability

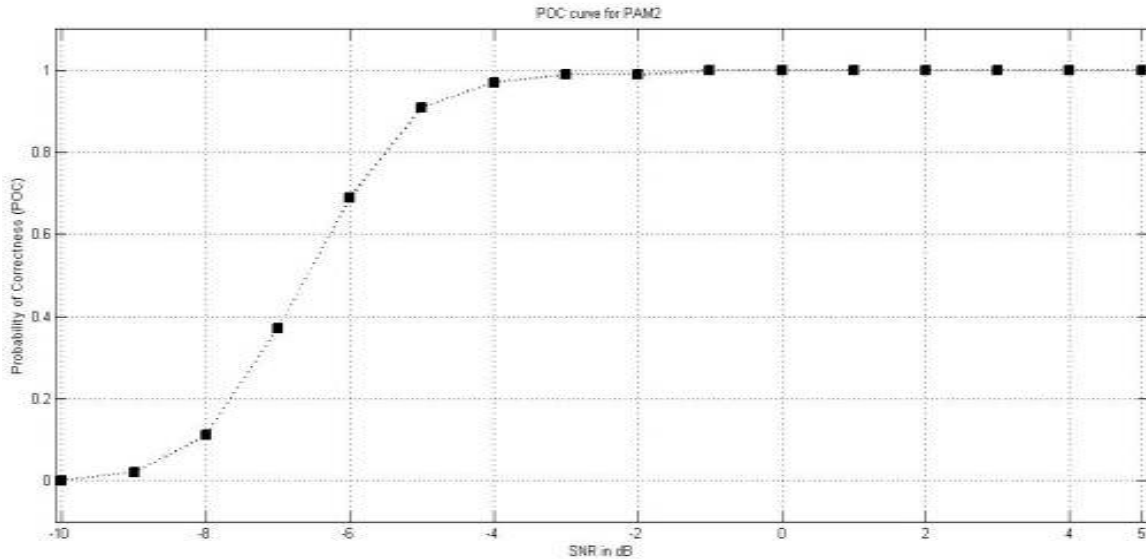


Fig. 2: POC curves for the varying SNR

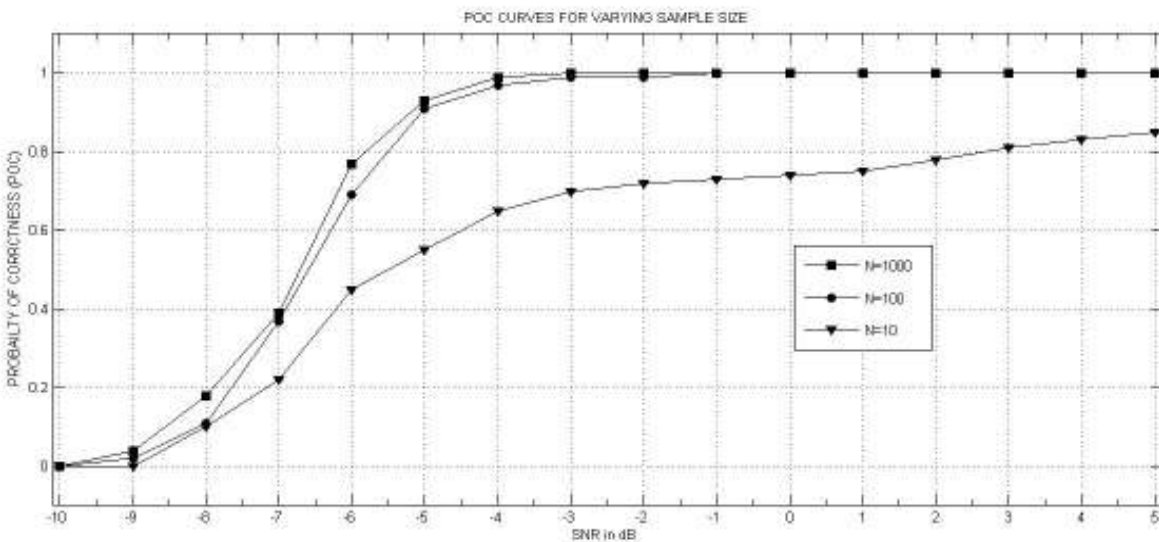


Fig. 3: POC curves for the varying sample size

of correctness curves for varying sample size are shown in Fig. 3. When the sample size is small ( $N = 10$ ), the probability of correctness is approximately 0.8 for SNR = 5 dB. When sample size is increased ( $N = 100$ ), the probability of correctness is approximately 1 for SNR = -1 dB. When the sample size is further increased ( $N = 1000$ ), the probability of correctness is approximately 1 for SNR = -3 dB.

For example considering the modulated signal QAM8; the graphical representation of the probability of correctness curves for different numbers of iterations are shown in Fig. 4. When the number of iterations is small ( $K = 100$ ), the probability of correctness is approximately 1 for SNR = -1 dB. When number of iterations is increased ( $K = 1000$ ), the probability of correctness is approximately 1 for SNR = -3 dB. When the number of iterations is further increased ( $K = 2000$ ),

the probability of correctness is approximately 1 for SNR = -4 dB.

The recognition of PSK modulated signals are shown in tree diagram in which only two Cummulants are used for classifications of PSK modulated signals. If  $|C60| < 31$ , the PSK 4, PSK 8, PSK 16, PSK 32, PSK 64 are in one class and second class is PSK2. If  $|C80| < 35$ , the PSK 8, PSK 16, PSK 32, PSK 64 are in one class and PSK 4 is in one class. The higher order Cummulants for PSK 8, PSK 16, PSK 32 and PSK 64 are same so the Cummulants are not used for the recognition of this subclass of PSK modulated signals PSK 8, PSK 16, PSK 32 and PSK 64. The Fig. 5 shows the tree structure for PSK modulated sequence recognition using higher order Cummulants and moments.

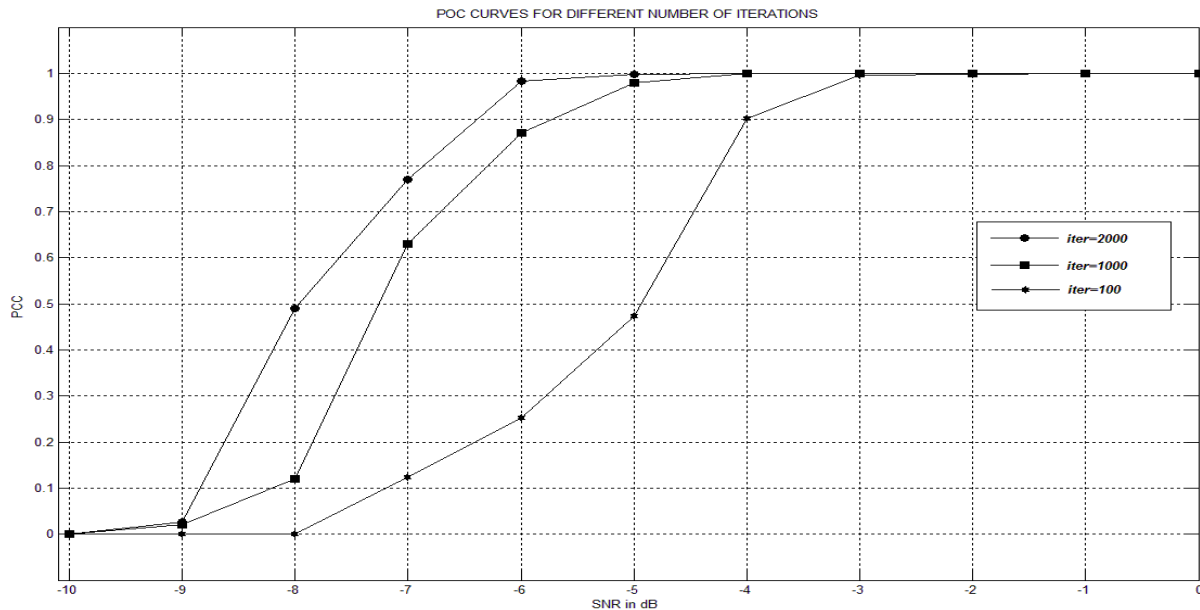


Fig. 4: POC curves for the different number of iterations

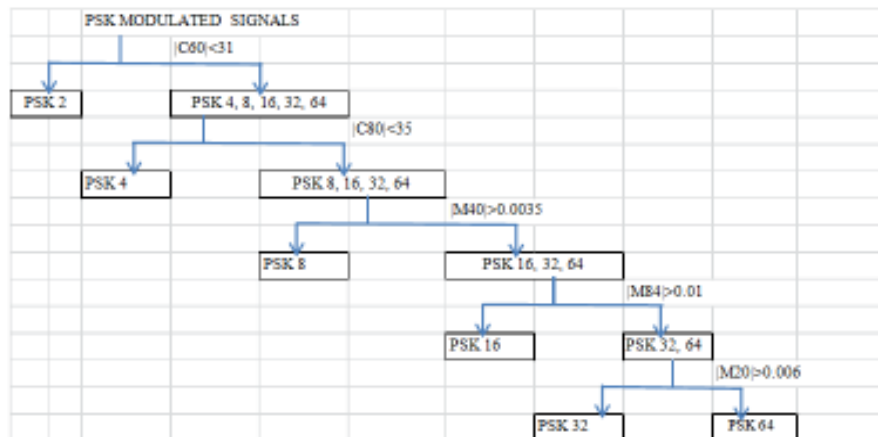


Fig. 5: Tree structure for PSK signals recognition under AWGN channel

## CONCLUSION

In this study, the characteristics of the cumulative amount of the modulated signals were analyzed. The paper chooses the HOS of the signal in which higher order moments and higher order cummulants are used for the recognition purpose. The recognition of PAM and QAM signals are done using decision rule, in which probability of correctness is based on a Likelihood ratio test which gives high recognition performance. The PSK signals recognition is done using tree diagram. Through simulation it is showed that the recognition process is correct. The channel is chosen to be Gaussian white noise channel.

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