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# Research Article Modeling and Control of a Quadrotor Helicopter System under Impact of Wind Field

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**Abstract:** Aiming at the hovering problem of a quadrotor helicopter system under impact of wind field, in this study, a nonlinear integral backtepping controller was designed. The quadrotor helicopter is a nonlinear system which is underactuated and strongly coupled. The wind field would lead to the nonlinear change of aerodynamic force and moment and make the flight condition worse. For the highly nonlinear characteristic of the system, first we establish the dynamic model that considers the effect of wind field via Newton-Euler formalism; and then we develop a controller based on integral backstepping algorithm and validate the stability of the system by Lyapunov theory. Simulation results demonstrate that the model can accurately reflects dynamic performance of the system and the controller presents good robustness in the effect of wind field.

Keywords: Attitude control, integral backstepping, lyapunov theory, position control, quadrotor helicopter, wind field

## INTRODUCTION

Recently, as a member of Vertical Take-Off and Landing (VTOL) Unmanned Aerial Vehicles (UAVs), the quadrotor helicopter has been more widely used in both military and civilian fields. Compared to fixedwing aircrafts, quadrotors can fly at low altitude and hovering at set point. Compared to traditional helicopters, quadrotors have several advantages including: simple mechanical structure, good maneuverability and small size, low cost and strong concealment. These excellent features make quadrotors able to perform in constrained area with more effectiveness and reliability.

The quadrotor system is highly nonlinear because the aerodynamic of the four rotors. Like traditional aircraft, the control of quadrotor involves attitude control and position control. The main difference is that, due to unique body structure as well as rotor aerodynamic, the attitude dynamics and position dynamics are strongly coupled (Abhijit *et al.*, 2009). Moreover, because the motion of the quadrotor is six degrees of freedom (6 DOF) but with only four driving forces, so the system is underactuated.

In the relevant literatures, a lot of work has been done to deal with the problem of modeling and control of the quadrotor system. Hoffmann *et al.* (2009, 2011) developed the STARMAC || research platform to validate multiple algorithms such as reactive collision avoidance, path planning, cooperative search and aggressive maneuvering. In the early study, PID control scheme is widely used. Bouabdallah *et al.* (2004) used PID control and LQ regulation to control the system. Salih et al. (2010) introduced a PID controller to the set point flight of a quadrotor. Compared to linear control method, nonlinear control method can substantially enhance the capability of the controller. As a kind of nonlinear control method, backstepping control was implemented by many researches (Bouabdallah and Siegwart, 2007; Ashfaq and Wang, 2008; Bouchoucha et al., 2011; Madani and Benallegue, 2006a). Ashfaq and Wang (2008) proposed a backstepping-based PID controller for a quadrotor under the condition of hovering and near hovering. Bouchoucha et al. (2011) developed an integral backstepping controller for attitude tracking. Madani and Benallegue (2006b) presented a full-state backstepping technique based on Lyapunov stability theory. There are also other nonlinear control methods used for the control of quadrotor system. Raffo et al. (2010) presented an integral predictive and nonlinear robust control strategy to solve the path following problem. Lee et al. (2009) discussed the effect of feedback linearization controller and sliding mode controller for trajectory tracking control. Carrillo et al. (2011) proposed a vision-based position control method; this method can measure the position variables that are difficult to compute when using conventional navigation systems.

However, few researches considered the impact of wind field on modeling and control of the quadrotor system. In the actual flight, for quadrotor flying at low altitude, it is more susceptible to wind field that could

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significantly affect the aerodynamic performance and stability (Su *et al.*, 2007). Therefore, it is necessary to take the impact of wind field into account in the study of quadrotor system's modeling and control.

To overcome this problem, in this study, a dynamic model of the quadrotor considering the influence of wind field is established. The quadrotor system is divided into two interconnected subsystem: rotor subsystem and body subsystem. The rotor subsystem's aerodynamic model which takes the impact of the wind field into consideration is built through blade element theory and momentum theory. The dynamic model of the body subsystem is established by Newton-Euler formalism. In order to control the position and attitude of the nonlinear system, an integral backstepping controller is designed and the system stability is conducted through the Lyapunov theory. Three numerical simulation experiments are demonstrated and the conclusions are drawn finally.

## DYNAMIC MODEL OF QUADROTOR HELICOPTER SYSTEM

The quadrotor system is composed by body and four rotors, as presented in Fig. 1. Set up two reference frames: the earth-fixed reference frame  $E = \{E_x, E_y, E_z\}$ and the body-fixed reference frame  $B = \{B_x, B_y, B_z\}$ . The absolute position  $X = [x, y, z]^T$  and attitude angle  $\Theta$ =  $[\varphi, \theta, \psi]^T$  of the system are defined in the reference frame *E*. These three Euler Angels are called roll angle  $(-\pi/2 < \varphi < -\pi/2)$ , pitch angle  $(-\pi/2 < \theta < \pi/2)$  and yaw angle  $(-\pi < \psi < \pi)$ .

The rotation transformation matrix from **B** to **E** is:

$$\boldsymbol{R}(\boldsymbol{\Omega}) = \begin{bmatrix} C_{\psi}C_{\theta} & C_{\psi}S_{\theta}S_{\varphi} - S_{\psi}C_{\varphi} & C_{\psi}S_{\theta}C_{\varphi} + S_{\psi}S_{\varphi} \\ S_{\psi}C_{\theta} & S_{\psi}S_{\theta}S_{\varphi} + C_{\psi}C_{\varphi} & S_{\psi}S_{\theta}C_{\varphi} - C_{\psi}S_{\varphi} \\ -S_{\theta} & C_{\theta}S_{\varphi} & C_{\theta}C_{\varphi} \end{bmatrix}$$
(1)

where,  $S(.) = \sin(.)$  and  $C(.) = \cos(.)$ .

The thrust forces  $F_{Ti}$  (i =1, 2, 3, 4) is generated by the four rotors. The motion of the quadrotor is controlled by varying the rotation speed of the four rotors to change the thrust and the torque produced by each one. Four rotors are divided into two pairs-pair (1, 3) and pair (2, 4). The rotate direction of the two pairs is contrary in order to counteract the aerodynamic torque generated by the rotors' rotation. Increase or decrease the rotation speed of the four rotors simultaneously will generate vertical motion. Independently varying the speed of the rotor pair (1, 3)can control the pitch angle  $(\theta)$  about the y-axis and the translational motion along the x-axis. Accordingly, independently varying the speed of the rotor pair (2, 4)can control the roll angle ( $\varphi$ ) about the x-axis and the



Fig. 1: Sketch of quadrotor helicopter

translational motion along the y-axis. The yaw angle  $(\psi)$  about the z-axis is determined by the yaw torque which is the sum of the reaction torques generated by each rotor.

**Rotor aerodynamics:** The two speed coefficients advance ratio ( $\mu$ ) and inflow ratio ( $\lambda$ ) of the rotor is:

$$\mu = \frac{U_x^B}{\Omega r} = \frac{V_x^B - R^T(\Omega) \cdot W_x^E}{\Omega r}$$

$$\lambda = \frac{v - U_z^B}{\Omega r} = \frac{v - (V_z^B - R^T(\Omega) \cdot W_z^E)}{\Omega r}$$
(2)

where, U is the air speed, V is the ground speed and W is the wind speed. In reference frame G,  $W^E = [W^E_x, W^F_y, W^E_z]^T$  is a known quantity.  $\Omega$  is the angular velocity of the rotor, r is the radius of the rotor, v is the induced velocity. The aerodynamic coefficients: thrust coefficient  $C_T$ , drag coefficient  $C_H$ , torque coefficient  $C_Q$  and roll coefficient  $C_R$  can be derived according to  $\mu$ and  $\lambda$ , thereby the thrust force  $F_T$ , drag force  $F_H$ , torque  $M_Q$  and rolling moment  $M_R$  can be obtained:

$$\begin{bmatrix} \boldsymbol{F}_{T} \\ \boldsymbol{F}_{H} \\ \boldsymbol{M}_{\boldsymbol{\varrho}} \\ \boldsymbol{M}_{R} \end{bmatrix} = \begin{bmatrix} C_{T} \rho A r^{2} \boldsymbol{\varrho}^{2} \\ C_{H} \rho A r^{2} \boldsymbol{\varrho}^{2} \\ C_{\varrho} \rho A r^{2} \boldsymbol{\varrho}^{2} \\ C_{R} \rho A r^{2} \boldsymbol{\varrho}^{2} r \end{bmatrix}$$
(3)

where,  $\rho = 1.293 \ kg/m^3$  is the air density and A is the area of propeller disk.

**System General Forces and Moments:** Assume that the quadrotor is a rigid-body structure and is completely symmetrical. Establish the translational dynamic equation and the rotational dynamic equation according to Newton-Euler formalism.

Step 1: Establish the translational dynamic equation:

$$\boldsymbol{F}_{total} = \boldsymbol{m} \boldsymbol{\ddot{X}} \tag{4}$$

where,  $F_{total}$  is the external resultant force, such as:

$$\boldsymbol{F}_{total} = \boldsymbol{F}_{rotor} - \boldsymbol{F}_{aero} - \boldsymbol{F}_{G}$$
(5)

where  $F_G = mG$  is the gravity,  $G = [0,0,g]^T$ .  $F_{rotor}$  represents the aerodynamic forces of the rotor and  $F_{aero}$  is the air resistance of the body:

$$F_{rotor} = R(\Omega) \left( \sum_{i=1}^{4} F_{Ti} - \sum_{i=1}^{4} F_{Hi} \right)$$

$$F_{aero} = \frac{1}{2} \rho A C (U^B)^2$$
(6)

where,  $C = diag[C_x, C_y, C_z].$ 

From Eq. (4-6), the translational dynamic equations are obtained:

$$\ddot{x} = \frac{1}{m} [(S_{\varphi}S_{\psi} + C_{\psi}S_{\theta}C_{\varphi})\sum_{i=1}^{4}F_{Ti} - \sum_{i=1}^{4}F_{Hxi} - \frac{1}{2}\rho AC_{x}(U_{x}^{B})^{2}]$$
(7)

$$\ddot{y} = \frac{1}{m} [(-S_{\varphi}C_{\psi} + S_{\psi}S_{\theta}C_{\varphi})\sum_{i=1}^{4}F_{T_{i}} - \sum_{i=1}^{4}F_{Hyi} - \frac{1}{2}\rho AC_{y}(U_{y}^{B})^{2}]$$
(8)

$$\ddot{z} = \frac{1}{m} (C_{\theta} C_{\varphi} \sum_{i=1}^{4} F_{Ti} - \frac{1}{2} \rho A C_{z} (U_{z}^{B})^{2}) - g \qquad (9)$$

Step 2: Establish the rotational dynamic equation:

$$\boldsymbol{M}_{total} = \boldsymbol{I}\boldsymbol{\Theta} + \boldsymbol{\Theta} \times (\boldsymbol{I}\boldsymbol{\Theta}) \tag{10}$$

where  $M_{total}$  is the external resultant moment, such as:

$$M_{total} = M_c + M_g + M_R \tag{11}$$

where,  $M_g = J_r \cdot (-1)^{i+1} \sum_{i=1}^{4} \dot{\boldsymbol{\Theta}} \times \boldsymbol{\Omega}_i$  is the gyroscopic torque of the rotor,  $J_r$  is the rotor's moment of inertia,  $M_R = (-1)^{i+1} \sum_{i=1}^{4} M_{Ri}$  is the rolling moment and  $M_c$  is the control moment produced by the rotors:

$$M_{c} = \begin{bmatrix} l(-F_{T2} + F_{T4}) \\ l(F_{T1} - F_{T3}) \\ (-1)^{i+1} \sum_{i=1}^{4} M_{Qi} \end{bmatrix}$$
(12)

where, l is the arm length of the quadrotor. From Eq. (10-12), the rotational dynamic equations are obtained:

$$\ddot{\rho} = \frac{I_{yy} - I_{zz}}{I_{xx}} \dot{\theta} \dot{\psi} - \frac{J_r}{I_{xx}} \dot{\theta} \Omega_r + \frac{1}{I_{xx}} (-1)^{i+1} \sum_{i=1}^4 M_{Rxi} + \frac{l}{I_{xx}} (F_{T4} - F_{T2})$$
(13)

$$\ddot{\theta} = \frac{I_{zz} - I_{xx}}{I_{yy}} \dot{\phi} \dot{\psi} + \frac{J_r}{I_{yy}} \dot{\phi} \Omega_r + \frac{1}{I_{yy}} (-1)^{i+1} \sum_{i=1}^4 M_{Ryi} + \frac{l}{I_{yy}} (F_{T1} - F_{T3})$$
(14)

$$\ddot{\psi} = \frac{I_{xx} - I_{yy}}{I_{zz}} \dot{\phi} \dot{\theta} - \frac{J_r}{I_{zz}} \dot{\Omega}_r + \frac{1}{I_{zz}} (-1)^{i+1} \sum_{i=1}^4 M_{Qi}$$
(15)

where,  $\Omega_r = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$ .

## INTEGRAL BACKSTEPPING CONTROL SCH EME

Integral backstepping control scheme is used for this system because:

- The backstepping technique is applicable to nonlinear system and has strong robustness to disturbances (Kanellakopoulos and Krein, 1993)
- The integral term of states parameters error is introduced to the backstepping technique in order to eliminate the static error of the system (Skjetne and Fossen, 2004)

Assume that the angular velocity of the system in reference frame *E* is equal to it in reference frame *B*; ignore the drag force, rolling moment of rotor and air resistance of body. Consider the thrust coefficient *b* and drag coefficient *d* as constants. Based on the above assumptions, the equation of the system model in state-space is obtained:  $\dot{\mathbf{S}} = f(\mathbf{S}, \mathbf{U})$ , where  $\mathbf{S} = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \varphi, \dot{\varphi}, \dot{\theta}, \dot{\varphi}, \psi, \dot{\psi}]^T$  is the state vector and  $U = [U_1, U_2, U_3, U_4]^T$  is the input vector. The inputs  $U_1, U_2, U_3$  and  $U_4$  are defined as:

$$\begin{bmatrix} U_{1} \\ U_{2} \\ U_{3} \\ U_{4} \end{bmatrix} = \begin{bmatrix} b(\Omega_{1}^{2} + \Omega_{2}^{2} + \Omega_{3}^{2} + \Omega_{4}^{2}) \\ b(-\Omega_{2}^{2} + \Omega_{4}^{2}) \\ b(\Omega_{1}^{2} - \Omega_{3}^{2}) \\ d(\Omega_{1}^{2} - \Omega_{2}^{2} + \Omega_{3}^{2} - \Omega_{4}^{2}) \end{bmatrix}$$
(16)

where,  $U_1$  is designed for altitude control and  $U_2$ ,  $U_3$ ,  $U_4$  is used for the control of the three attitude angles respectively. The control equation is written as:



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Fig. 2: Structure of control system

$$f(\mathbf{S}, \mathbf{U}) = \begin{bmatrix} \dot{x} \\ \frac{C_{\varphi}S_{\theta}C_{\psi} + S_{\varphi}S_{\psi}}{m}U_{1} \\ \dot{y} \\ \frac{C_{\varphi}S_{\theta}S_{\psi} - S_{\varphi}C_{\psi}}{m}U_{1} \\ \frac{\dot{z}}{m} \\ \frac{C_{\varphi}C_{\theta}}{m}U_{1} - g \\ \dot{\phi} \\ \dot{\phi} \\ \dot{\phi} \\ \dot{\psi} \\ \frac{I_{yy} - I_{zz}}{I_{xx}} + \dot{\theta} \\ \frac{J_{r}}{I_{xx}}\Omega_{r} + \frac{l}{I_{xx}}U_{2} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\phi} \\ \dot{\psi} \\ \frac{I_{zz} - I_{xx}}{I_{yy}} - \dot{\phi} \\ \frac{J_{r}}{I_{yy}}\Omega_{r} + \frac{l}{I_{yy}}U_{3} \\ \dot{\psi} \\ \dot{\phi} \\ \dot{\psi} \\ \frac{\dot{\phi}}{I_{zz}} \\ - \frac{I_{yy}}{I_{zz}} + \frac{l}{I_{zz}}U_{4} \end{bmatrix}$$
(17)

The structure of the control system is shown in Fig. 2. The controller consists of three parts: attitude controller, altitude controller and position controller. Altitude controller outputs  $U_1$  according to the desired altitude  $(z_d)$ , current altitude (z) and attitude angle  $(\varphi, \theta, \psi)$ . Position controller receives  $U_1$ , combines the desired position  $(x_d, y_d)$  and current position (x, y), outputs desired roll angle  $(\varphi_d)$  and desired pitch angle  $(\varphi_d, \theta_d, \psi_d)$  and current attitude angle  $(\varphi, \theta, \psi)$ , outputs  $U_2$ ,  $U_3$ ,  $U_4$ . The dynamic model receives input vector from controller, integrates with wind field, outputs the state of next time step and feeds back to the controller.

Here we deduce the control of roll angle as an example to explain the design of controller.

**Step 1:** Set the tracking-error of roll angle ( $\varphi$ ) as:

$$e_{\varphi} = \varphi_d - \varphi \tag{18}$$

Set the first Lyapunov function as:

$$V_{1} = \frac{1}{2} (e_{\phi}^{2} + \lambda_{1} \chi_{1}^{2})$$
(19)

where,  $\chi_1 = \int_0^t e_{\varphi}(\tau) d\tau$  is the integral of tracking-error of roll angle  $(\varphi)$ ,  $\lambda_1 > 0$ . The derivation of Eq. (19) is:

$$\dot{V}_{1} = e_{\phi}\dot{e}_{\phi} + \lambda_{1}\chi_{1}e_{\phi}$$

$$= e_{\phi}(\dot{e}_{\phi} + \lambda_{1}\chi_{1})$$

$$= e_{\phi}(\dot{\phi}_{d} - \dot{\phi} + \lambda_{1}\chi_{1})$$
(20)

If we set the virtual control  $(\dot{\phi})_d$  of  $\dot{\phi}$  as:

$$(\dot{\phi})_{d} = \dot{\phi}_{d} + \lambda_{1}\chi_{1} + c_{1}e_{\sigma}, c_{1} > 0$$
(21)

then:

$$V_1 = -c_1 e_{\varphi}^2 \tag{22}$$

Hence when  $e_{\varphi} \neq 0$ ,  $\dot{V}_1 < 0$ .

Set the tracking-error of  $\dot{\phi}$  as:

$$\boldsymbol{e}_{\dot{\boldsymbol{\phi}}} = (\dot{\boldsymbol{\phi}})_d - \dot{\boldsymbol{\phi}} \tag{23}$$

Set the second Lyapunov function as:

$$V_{2} = \frac{1}{2} (e_{\varphi}^{2} + \lambda_{1} \chi_{1}^{2} + e_{\dot{\varphi}}^{2} + \lambda_{2} \chi_{2}^{2})$$
(24)

where,  $\chi_2 = \int_0^t e_{\phi}(\tau) d\tau$  is the integral of tracking-error of roll angular velocity ( $\dot{\phi}$ ),  $\lambda_2 > 0$ . The derivation of Eq. (24) is:

$$\dot{V}_{2} = e_{\varphi} ((1 + \lambda_{1} - c_{1}^{2})e_{\varphi} + \ddot{\varphi}_{d} + c_{1}e_{\varphi} - c_{1}\lambda_{1}\chi_{1} - \ddot{\varphi} + \lambda_{2}\chi_{2}) - c_{1}e_{\varphi}^{2}$$
(25)

If we set the virtual control  $(\ddot{\varphi})_d$  of  $\ddot{\varphi}$  as:

$$\begin{aligned} (\ddot{\varphi})_{d} &= (1 + \lambda_{1} - c_{1}^{2})e_{\varphi} + \ddot{\varphi}_{d} + \\ (c_{1} + c_{2})e_{\dot{\varphi}} - c_{1}\lambda_{1}\chi_{1} + \lambda_{2}\chi_{2} \end{aligned}$$
 (26)

where,  $c_1 > 0$ ,  $c_2 > 0$ , then:

$$\dot{V}_2 = -(c_1 e_{\phi}^2 + c_2 e_{\phi}^2) \tag{27}$$

Hence when  $e_{\varphi} \neq 0$  and  $e_{\varphi} \neq 0$ ,  $\dot{V}_2 < 0$ . We can know that the closed-loop system is asymptotically stable according to Lyapunov theorem.

**Step 3:** According to the control equation of roll angle  $(\varphi)$  in Eq. (17):

$$\ddot{\varphi} = \dot{\theta} \dot{\psi} (I_{yy} - I_{zz} / I_{xx}) + \dot{\theta} \Omega_r J_r / I_{xx} + U_2 l / I_{xx}$$

 $U_2$  is obtained:

$$U_{2} = \frac{I_{xx}}{l} [(1 + \lambda_{1} - c_{1}^{2})e_{\phi} + \ddot{\phi}_{d} + (c_{1} + c_{2})e_{\phi} - c_{1}\lambda_{1}\chi_{1} + \lambda_{2}\chi_{2} - \dot{\theta}\dot{\psi}(I_{yy} - I_{zz})/I_{xx} - \dot{\theta}\Omega_{r}J_{r}/I_{xx}]$$
(28)

Similarly, pitch angle ( $\theta$ ) control  $U_3$  can be obtained:

$$U_{3} = \frac{I_{yy}}{l} [(1 + \lambda_{3} - c_{3}^{2})e_{\theta} + \ddot{\theta}_{d} + (c_{3} + c_{4})e_{\dot{\theta}} - (29)$$

$$c_{3}\lambda_{3}\chi_{3} + \lambda_{4}\chi_{4} - \dot{\phi}\dot{\psi}(I_{zz} - I_{xx})/I_{yy} + \dot{\phi}\Omega_{r}J_{r}/I_{yy}]$$

where,  $c_3, c_4, \lambda_3, \lambda_4$  are positive constants and  $\chi_3$ ,  $\chi_4$  are the integral of tracking-error of pitch angle ( $\theta$ ) and pitch angular velocity ( $\dot{\theta}$ ).

Table	1:	Structural	parameters

Parameter	Definition	Value	Unit
т	Mass	0.723	kg
l	Arm length	0.314	m
$J_r$	Rotor inertia	7.321×10 <sup>-5</sup>	kg·m <sup>2</sup>
$I_{xx}$	X Inertia	8.678×10 <sup>-3</sup>	kg·m <sup>2</sup>
$I_{yy}$	Y Inertia	8.678×10 <sup>-3</sup>	kg·m <sup>2</sup>
Izz	Z Inertia	3.217×10 <sup>-2</sup>	kg·m <sup>2</sup>
b	Trust factor	5.324×10 <sup>-5</sup>	$N \cdot s^2$
d	Drag factor	8.721×10 <sup>-7</sup>	Nm·s <sup>2</sup>

Table 2: Control parameters									
Item	Value	Item	Value	Item	Value				
<i>c</i> <sub>1</sub>	10	$C_2$	3	C3	10.5				
C4	3.5	C5	4	$C_6$	3				
C7	4	$c_8$	2.5	C9	3				
C10	1	$c_{11}$	2	$c_{12}$	1				
$\lambda_1 \sim \lambda_{12}$	0.05								

Yaw angle ( $\psi$ ) control  $U_4$  can be obtained:

$$U_{4} = \frac{I_{zz}}{l} [(1 + \lambda_{5} - c_{5}^{2})e_{\psi} + \ddot{\psi}_{d} + (c_{5} + c_{6})e_{\psi} - c_{5}\lambda_{5}\chi_{5} + \lambda_{6}\chi_{6} - \dot{\phi}\dot{\theta}(I_{xx} - I_{yy})/I_{zz}]$$
(30)

where,  $c_5, c_6, \lambda_5, \lambda_6$  are positive constants and  $\chi_5$ ,  $\chi_6$  are the integral of tracking-error of yaw angle ( $\psi$ ) and yaw angular velocity ( $\dot{\psi}$ ).

Altitude control  $U_l$  can be obtained:

$$U_{1} = \frac{m}{C_{\varphi}C_{\theta}} [g + (1 - c_{\gamma}^{2} + \lambda_{\gamma})e_{z} + (c_{\gamma} + c_{8})e_{z} - c_{\gamma}\lambda_{\gamma}\chi_{\gamma}]$$
(31)

where,  $c_7, c_8, \lambda_7$  are positive constants and  $\chi_7$  is the integral of tracking-error of altitude *z*.

#### SIMULATION RESULTS

This section validates the effective of proposed model and control scheme by three numerical simulation experiments. Firstly, we test the performance of the system in the absence of wind field. Then, wind field with velocity of  $W^E = [1, 2, 0]^T$  m/s and  $W^E = [3, 3, 0]^T$  m/s is introduced separately. Table 1 summarizes the structural parameters of the model. Table 2 lists the control parameters of the controller.

In the absence of wind field: Expect system hovering at  $X = 1_{3\times 1}$  m. The initial conditions is  $X = 1_{3\times 1}$  m,  $\Theta = 0_{3\times 1}$  rad, the desired conditions is  $X_d = 1_{3\times 1}$  m,  $\psi_d = 0$ rad. The translational and rotational velocity in both initial and desired conditions is 0. The position and attitude angle response of the system in the absence of wind field are shown in Fig. 3 and 4. We can observe



Fig. 3: Position response (in the absence of wind field)



Fig. 4: Attitude angle response (in the absence of wind field)



Fig. 5: Position response (In the presence of a  $W_E = [1, 2, 0]^T$  m/s wind field)

that the position reach to desired value rapidly; the attitude angle have slight oscillation at beginning, but the controller stabilized it at 0 rad in a short period of time. In this situation, the controller presents good performance.

In the presence of a  $W^{\mathcal{E}} = [1, 2, 0]^{T}$  m/s wind field: Maintain the same initial and desired conditions, the system's position and attitude angle response in the



Fig. 6: Attitude angle response (In the presence of a  $W_E = [1, 2, 0]^T$  m/s wind field)



Fig. 7: Position response (in the presence of a  $W_E = [3, 3, 0]^T$  m/s wind field)

presence of a  $W^{\mathcal{E}} = [1, 2, 0]^T$  m/s wind field are shown in Fig. 5 and 6. We can see that the position can be stabilized at the desired value in about 15 seconds. Pitch angle ( $\theta$ ) is stabilized at 0.13 rad, roll angle ( $\varphi$ ) is stabilized at -0.27 rad, This is due to the influence of lateral wind field, the aircraft nose of the two directions need to be placed into a certain angle in order to achieve hovering. For the wind speed in y direction is greater than it in x direction, the roll angle ( $\varphi$ ) is larger than pitch angle ( $\theta$ ). In this situation, the oscillation of



Fig. 8: Attitude angle response (In the presence of a  $W_E = [3, 3, 0]^T$  m/s wind field)

attitude angle and the time achieve stable are greater than them in experiment 1.

In the presence of a  $W^{E} = [3, 3, 0]^{T}$  m/s wind field: The wind speed is increased to  $W^E = [3,3,0]^T$  m/s, still maintaining the same initial and desired conditions, the system's position and attitude angle response in this experiment are shown in Fig. 7 and 8. We can observe that position can be stabilized at the desired value in about 20 seconds. Pitch angle ( $\theta$ ) is stabilized at 0.33 rad, roll angle ( $\phi$ ) is stabilized at -0.33 rad and both of them are larger than that in experiment 2. This is due to the increase of wind speed the aircraft nose needs to be placed into a lager angle in order to resists the reinforced wind. We can note that because the wind speed is equal in x direction and y direction in this experiment, pitch angle ( $\theta$ ) and roll angle ( $\varphi$ ) are of the same. Compared with experiment 2, we can see that with the increase of wind speed, the oscillation of attitude angle and the time achieving stable are become greater.

### CONCLUSION

The dynamic model of a quadrotor helicopter system under impact of wind field is established and the nonlinear controller based on integral backstepping algorithm is designed. The performance of the system under effect of wind field is studied by numerical simulation experiment. The conclusions from the results can be summarized as follows:

 Position and attitude angle response of the system is accurate to wind field and that is line with principles of flight dynamics; stable time and oscillation are greater when the speed of wind field is increased, this indicates the impact wind field reduce the stability of the system. Therefore, this model can accurately reflect the dynamic performance of the system in the effect of wind field

• The controller can make the system achieve hovering in the desired location even under the interference of wind field. Therefore, the controller has good performance.

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