

## Research Article

### Non-Local Fractional Differential-Based Approach for Image Enhancement

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**Abstract:** This study proposed an image enhancing method which is based on the non-local fractional order differential operator. In this method, a matrix form representation of discrete fractional order differentiation is introduced to enhance the digital image, which is effective to reduce the computation error caused by the traditional local approximate method of the fractional order differentiation. The proposed enhancing method is able to make effective use of the whole image information and improve the enhancing performance of the image enhancing algorithm based on the local mask. The color image enhancing strategy based on the non-local fractional differential also is given. A lot of experiments demonstrate that the proposed method is capable of enhancing gray and color image effectively.

**Keywords:** Digital image, fractional differential, image enhancement, non-local

#### INTRODUCTION

Fractional calculus is an old and mysterious mathematical discipline dealing with the non-integer order differentiation and integration which is proposed in the seventeenth century and developed mainly in the nineteenth century (Podlubny, 1999). Recently, the concepts of fractional calculus have been widely used in various areas of image processing because of some of its characteristics superior to the integer order, which includes image restoration (Jian and Feng, 2007), enhancement (Pu *et al.*, 2010; Chen *et al.*, 2011, 2012), edge detection (Mathieu *et al.*, 2003), motion estimation (Chen *et al.*, 2010) and so on.

In this study, we aim our interests at the image enhancement. The integer-order differential mask is an important tool in edge detection, but it damages the texture detail information in image. The texture detail feature of image has fractal-like structure and fractional differentiation is an effective tool to deal with this fractal problem. Based on this fact, six fractional differentiation-based image enhancement masks were proposed for texture image enhancement which was proved to have a good effect to deal with the complex texture details in smooth area (Pu *et al.*, 2010). In addition, an adaptive image enhancing method based on the 2-D fractional order Savitzky-Golay mask was present to deal with the problem of parameter optimization (Chen *et al.*, 2012). Although the performances of these methods have shown impressive results, the limitation of the use of local fractional differential numerical approximate have recently become of great concern (Podlubny, 2000).

#### METHODOLOGY

**Riemann-liouville definition:** Let  $f \in C^1([a, b])$  and  $\alpha \in (n-1, n)$ . The left Riemann-Liouville fractional order integral is defined by the following formula:

$${}_a I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau, t \in [a, b], \quad (1)$$

where the gamma function  $\Gamma(z)$  is obtained by the integral:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt. \quad (2)$$

The right Riemann-Liouville fractional integral of order  $\alpha$  is defined as:

$${}_t I_b^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_t^b (t-\tau)^{\alpha-1} f(\tau) d\tau, t \in [a, b], \quad (3)$$

Suppose  $f(t)$  is a continuous function in  $[a, b]$ , the left Riemann-Liouville fractional order derivative is defined by the following formula:

$${}_a D_t^\alpha f(t) = \frac{d^n}{dt^n} {}_a I_t^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau, t \in [a, b], \quad (4)$$

and the right Riemann-Liouville fractional derivative is given by the following formula:

$${}_t D_b^\alpha f(t) = \left(-\frac{d}{dt}\right)^n {}_t I_b^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(-\frac{d}{dt}\right)^n \int_t^b (t-\tau)^{n-\alpha-1} f(\tau) d\tau, t \in [a, b]. \quad (5)$$

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**Matrix approximate method:** An efficient matrix approximate method for Eq. (4) and (5) was proposed by Podlubny (2000), which leads to significant simplification of the numerical solution. For easy reading, we reproduce the relevant results.

Let us assume that the given signal  $f$  is sampled from its continuous version at an uniformly spaced grid size of  $\Delta h$ . Thereby, the discrete form  $f_i = f(i\Delta h)$ , for  $i = 0, 1, \dots, m$ . In image processing applications, the grid size  $\Delta h$  is one, so we set  $\Delta h = 1$  for easy description. From Grünwald-Letnikov fractional derivative definition, the discrete formula of the left fractional order derivative of the digital signal can be defined by the following formula:

$${}^L D_x^\alpha f_i \approx \sum_{k=0}^m w_k^{(\alpha)} f_{i-k}, \quad (6)$$

where,  $w_k^{(\alpha)} = (-1)^k \binom{\alpha}{k}$  represent the coefficients of the polynomial  $(1-z)^\alpha$ .

The coefficients can also be obtained recursively from:

$$w_0^{(\alpha)} = 1, w_k^{(\alpha)} = \left(1 - \frac{\alpha+1}{k}\right) w_{k-1}^{(\alpha)}, k = 1, 2 \quad (7)$$

Using the matrix approximate method, we can rewrite Eq. (6) by the following form:

$${}^L D_x^\alpha \vec{f} \approx M_L \vec{f} \quad (8)$$

where,  $\vec{f} = [f_0 \ f_1 \ \dots \ f_m]^T$  and  $M_L$  is a matrix defined by the following formula:

$$M_L = \begin{bmatrix} w_0^{(\alpha)} & 0 & \dots & 0 \\ w_1^{(\alpha)} & w_0^{(\alpha)} & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ w_m^{(\alpha)} & w_{m-1}^{(\alpha)} & \dots & w_0^{(\alpha)} \end{bmatrix} \quad (9)$$

From Eq. (4), (5) and (8), the discrete formula of the right fractional order derivative of the digital signal can be computed by the following formula:

$${}^R D_x^\alpha \vec{f} \approx M_R \vec{f} = M_L^T \vec{f} \quad (10)$$

In the numerical method of the fractional order derivative, the computing error caused by the cut of the computing coefficients is inevitable due to the long memory character of fractional derivative. An effective method to reduce this error is to increase the number of computing coefficient, so the number of computing coefficient is an important criterion for evaluating the performance of numerical method. In matrix approximate method, the length of coefficient is related to the location of the estimated signal, so we use the average of the length of the coefficient to access the accuracy of numerical algorithm, which is obtained by the following formula:

$$L_1^{ave} = \sum_{k=1}^{m+1} k / (m+1) = 0.5m + 1. \quad (11)$$

From Eq. (11), we can see that the bigger  $m$  is, the better the accuracy of matrix algorithm is.

**Proposed enhancing method:** In practical image enhancing applications, assume that the digital image  $u$  is of  $m \times n$  pixels, which is sampled from its continuous version at an uniformly spaced grid size of  $\Delta h$ . Thereby, the digital image  $u(i, j) = u(i\Delta h, j\Delta h)$  for  $i = 0, 1, \dots, m$  and  $j = 0, 1, \dots, n$ . In image processing applications, the grid size  $\Delta h$  is chosen as  $\Delta h = 1$ . Based on the Grünwald-Letnikov fractional derivative definition, when  $\alpha \in (0, 1)$ , the discrete formula of the left fractional order derivative of the digital image in horizontal direction can be defined by the following formula:

$${}^L D_x^\alpha u(i, j) \approx \sum_{k=0}^m w_k^{(\alpha)} u(i-k, j) \quad (12)$$

Using the matrix approximate method, the equation (12) can be rewritten by the following formula:

$${}^L D_x^\alpha U \approx M_L U \quad (13)$$

here  $U$  is the image matrix defined by:

$$U = \begin{bmatrix} u(0,0) & u(0,1) & \dots & u(0,n) \\ u(1,0) & u(1,1) & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ u(m,0) & u(m,1) & \dots & u(m,n) \end{bmatrix} \quad (14)$$

In the same way, the right fractional order derivative of the digital image can be calculated by:

$${}^R D_x^\alpha U \approx M_L^T U. \quad (15)$$

In order to increase the average of the length of the coefficient used in image enhancement, we combine  $M_L$  and  $M_R$  into one matrix,  $M_{L-R}$ , which is obtained by the following equation:

$$M_{L-R} = \begin{bmatrix} w_0^{(\alpha)} & w_1^{(\alpha)} & \dots & \dots & w_m^{(\alpha)} \\ 0 & w_0^{(\alpha)} & \dots & \dots & w_{m-1}^{(\alpha)} \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & w_0^{(\alpha)} & \dots & w_r^{(\alpha)} \\ w_c^{(\alpha)} & \dots & w_1^{(\alpha)} & w_0^{(\alpha)} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ w_{m-1}^{(\alpha)} & \dots & \dots & w_0^{(\alpha)} & 0 & \dots & \dots \\ w_m^{(\alpha)} & \dots & \dots & w_1^{(\alpha)} & w_0^{(\alpha)} \end{bmatrix} \quad (16)$$

where,

$$\begin{cases} r = c - 1 = \frac{m}{2}, & \text{if } m \text{ is even} \\ r = c = \frac{m+1}{2}, & \text{if } m \text{ is odd} \end{cases} \quad (17)$$

Based on the matrix  $M_{L-R}$ , we can calculus the fractional order derivative of the digital image in horizontal direction using the following formula:

$$D_x^\alpha U \approx M_{L-R} U, \quad (18)$$

and the average of the length of the coefficient used in Eq. (18) is calculated by:

$$L_2^{ave} = \begin{cases} \frac{3m^2+8m+4}{4(m+1)}, & \text{if } m \text{ is even} \\ \frac{3m+5}{4}, & \text{if } m \text{ is odd} \end{cases} \quad (19)$$

From Eq. (11) and (19), it can be seen that  $L_2^{ave} \gg L_1^{ave}$  when  $m$  is the positive integer, so the accuracy of matrix approximate method defined by Eq. (19) is much better than that of the method defined by Eq. (11).

In the same way, the fractional order derivative of the digital image in vertical direction can be calculated by the following formula:

$$D_y^\alpha U \approx U M_{L-R}^T \quad (20)$$

Based on the Eq. (18) and (20), the enhanced image is obtained by the following equation:

$$U_{Enhanced} = \min\{\max\{\max\{D_x^\alpha U, D_y^\alpha U\}, 0\}, LE\} \quad (21)$$

here  $U_{Enhanced}$  is the enhanced image and  $LE$  is the gray level of image.

Moreover, the proposed non-local fractional order differential-based image enhancing algorithm also is available for the color image. We all know that the RGB color model is the most popular color model. However, the three components, red (R), Green (G) and Blue (B), have strong coupling with each other. If we use Eq. (21) in these three channels directly, it will result in the color distortion. LAB color model is the one of the typical color models. In LAB color model, the three components have weak coupling with each other. Based on this fact, we use LAB color model instead of RGB model to prevent the color distorted. For easy reading, the main process is described as follows. Firstly, we transform RGB color model into Lab color model. Then, we process the 'L' component using Eq. (21) because the 'L' component closely matches human perception of lightness. Finally, we transform the processed Lab color model back to RGB color model.

## EXPERIMENTS AND ANALYSIS

**Relationship between the fractional order and performance of proposed enhancing method:** The fractional order  $\alpha$  is the only parameter in our proposed enhancing method whose choice directly affects the result of enhanced image. In this section, our purpose is to reveal the relationship between the fractional order and performance of our proposed enhancing algorithm.

For this purpose, the typical  $512 \times 512$  texture-rich gray image is used as the test image. We enhance

this image by our proposed non-local fractional differential method with the different fractional order,  $\alpha = 0, 0.2, 0.4, 0.6, 0.8$  and  $1$ . The original image and its enhanced results are shown in Fig. 1.

From Eq. (16), the matrix  $M_{L-R}$  is the identity matrix, so the enhanced image  $U_{Enhanced}$  is equal to original image  $U$  when  $\alpha = 0$ . When  $\alpha = 1$ , the enhanced image is the edge image obtained by the difference operator, which is shown in Fig. 1f. From this figure, it can be seen that the edge in original image is enhanced greatly but many detailed textures are removed. The enhanced images ( $\alpha \in (0, 1)$ ) can be seen as the natural interpolation between original image the ( $\alpha = 0$ ) and the edge image ( $\alpha = 1$ ) which are shown in Fig. 1b-e. From these figures, we can see that, when  $\alpha$  increases from 0 to 1, the texture information is firstly enhanced and then removed and the edge information is enhanced more and more. Therefore, we are able to find a optimal enhanced result between  $\alpha = 0$  and  $\alpha = 1$ .

**Enhancing results of gray image:** In this section, some experiments are given to assess the performance of our proposed enhancing algorithm for gray image. For this purpose, two typical  $512 \times 512$  gray images, shown in Fig. 2a and e, are used as the test image. These two images are enhanced by our proposed enhancing method with fractional order  $\alpha = 0.2$  and their enhanced results are shown in Fig. 2b and f respectively.

From Fig. 2b and f, we can see that the contrast of enhanced result is much better than that of original image and the detailed texture is more clear. In order to show the clear comparisons, the partial enlarged views of the original images and their enhanced images are shown in Fig. 2c, d, g and h. Comparing with the Fig. 2c, the detail information in Fig. 2d is clearer, for example, the hair looks more sheen, the hat looks more tridimensional and the gap of feather is easier to distinguish. From Fig. 2g and h, we can get the same conclusion. It is obvious that the beard in Fig. 2h is easier to distinguish than that of Fig. 2g.

Therefore, from these experiment results, we can conclude that the proposed fractional differential enhancing method is able to achieve a good trade-off between the detail texture preservation and the image enhancement.

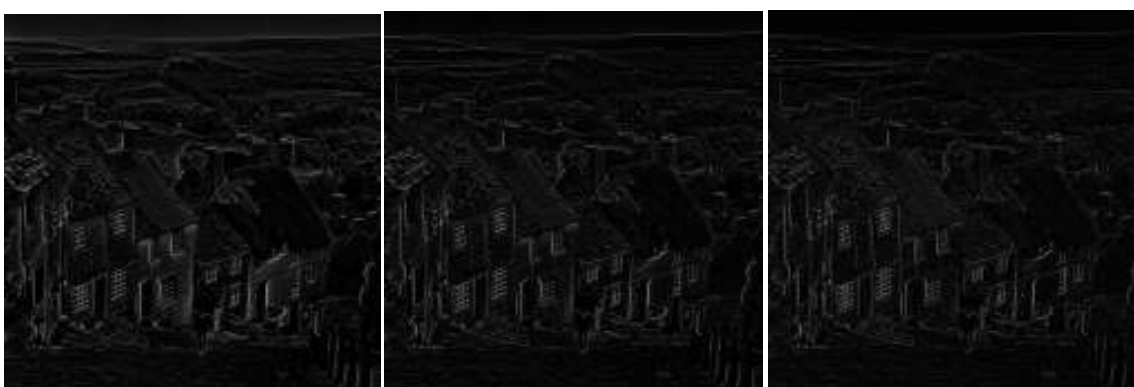
**Enhancing results of color image:** In this section, some experiments are shown to verify the validity of our proposed enhancing method for the color image. For this purpose, we choose four typical  $384 \times 512$  color texture images as test images which are respectively shown in Fig. 3a, c, e and g. Our proposed enhancing strategy is used to process these four images and their enhanced results are shown in Fig. 3b, d, f and h respectively. In these experiments, we set fractional



(a) Enhanced image with  $\alpha = 0$

(b) Enhanced image with  $\alpha = 0.2$

(c) Enhanced image with  $\alpha = 0.4$



(d) Enhanced image with  $\alpha = 0.6$

(e) Enhanced image with  $\alpha = 0.8$

(f) Enhanced image with  $\alpha = 1$

Fig. 1: The comparison among enhanced images with different fractional order

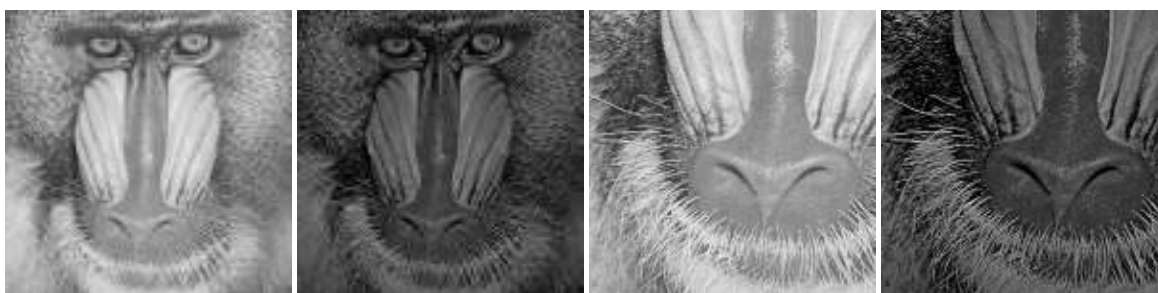


(a) Original image

(b) Enhanced image

(c) Partial view of (a)

(d) Partial view of (b)



(e) Original image

(f) Enhanced image

(g) Partial view of (e)

(h) Partial view of (f)

Fig. 2: Comparison between original images and enhanced results



Fig. 3: Comparison between original images and enhanced results

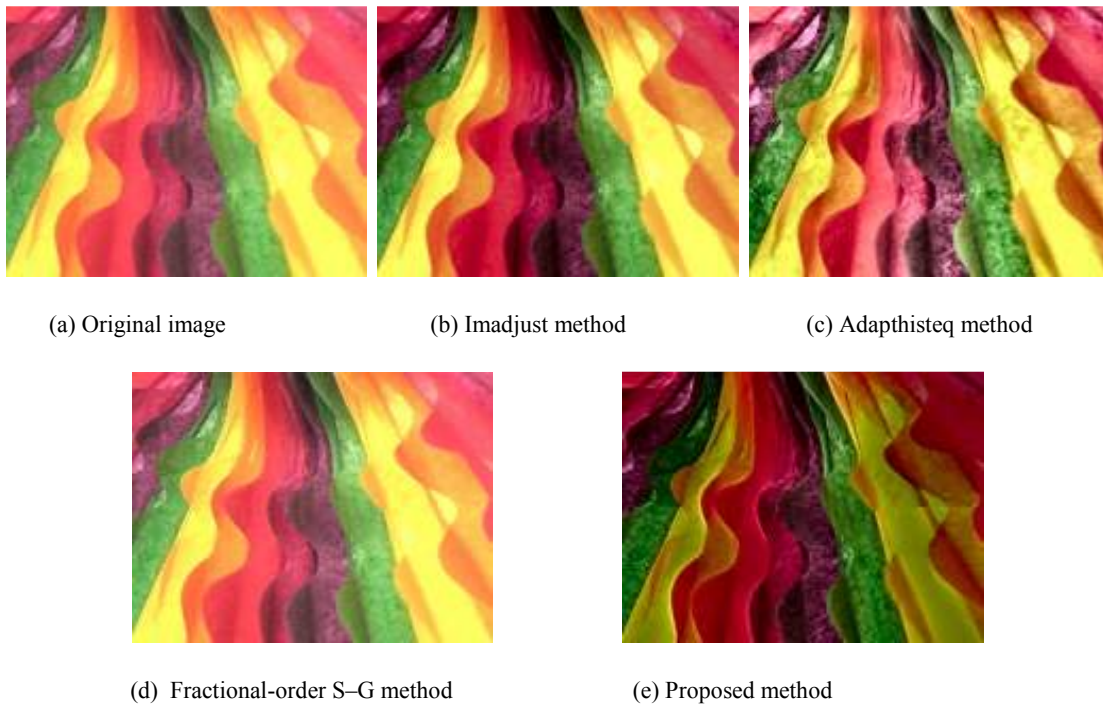


Fig. 4: Comparison among the proposed image enhancement method and three typical image enhancement methods

order  $\alpha = 0.2$ . Comparing with the original images, we can see that the enhanced results have the more abundant color and the better observability. Specially, the detail texture information is enhanced greatly.

Therefore, we have conclusion that our proposed enhancing method is effective for the color image enhancement.

In order to further assess the performance of our proposed fractional differential enhancing method, we

compare the proposed fractional order enhancing method with three typical enhancing methods, including the imadjust method (Gonzalez and Woods, 1992), adapthisteq method (Gonzalez and Woods, 1992) and fractional-order Savitzky-Golay method (Chen *et al.*, 2012). For this purpose, we choose a typical  $384 \times 512$  color texture images as the test image which is shown in Fig. 4a. The enhanced images, obtained by imadjust method, adapthisteq method,



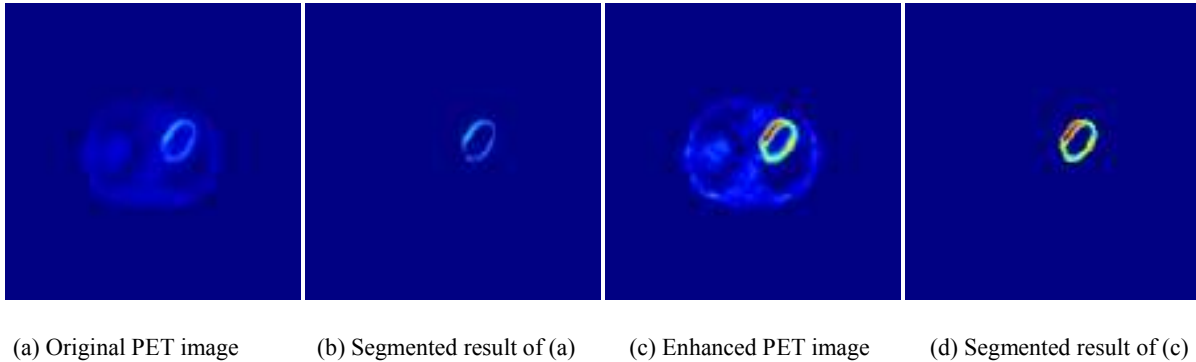


Fig. 5: Comparison between the segmented results of original PET image and enhanced PET image

fractional-order Savitzky-Golay method and our proposed method, are respectively shown in Fig. 4b, c, d and e. In the experiment, the parameters of these methods are set to ensure the optimal performance. Comparing with the enhanced results shown in Fig. 2b, c and d, we can see that, in Fig. 2e, not only is the image contrast enhanced, but also the brightness distribution in image is protected. The texture is clear and the gradation is distinct.

Therefore, we can conclude that the proposed enhancing method has the better performance than the other three enhancing methods.

**Enhancing results of PET image:** Image enhancement technology not only can improve the observability of the image, but also can increase the accuracy of some postprocessing technologies. In order to assess this criterion, the PET image, downloaded from the DICOM7 medical imaging database, is used as the test image. We use the same segment method to process the original image and the enhanced images respectively. For easy observation, the pseudo-color technology is used to process the test image and experiment results and the processed images are shown in Fig. 5. The Fig. 5a shows the original PET image. In the figure, the luminous annular region is the cardiac muscular tissue which is the important standard for evaluating myocardial vitality. In the medical image processing, the segment of the cardiac muscular tissue is an important step. The segmented result of the original PET image is shown in Fig. 5b. Our proposed enhancing method is used to enhance the original PET image and the result is shown in Fig. 5c. For comparison, the segmented result of enhanced image is shown in Fig. 5d. From the comparison between Fig. 5b and d, we can see that a gap exists in the annular region of Fig. 5b, while the annular region in Fig. 5d is clearer and more helpful for the doctor to estimate the condition of the cardiac muscular tissue.

Therefore, we can conclude that the proposed enhancing method is helpful for improving the accuracy of the postprocessing technologies in the PET image segment application.

## CONCLUSION

A non-local fractional differential-based approach was proposed for image enhancement in this study, which includes the following contributions. Firstly, the matrix approximate method is extended to new version which is suitable for the image enhancement. Secondly, the effective non-local fractional order differential image enhancing method is proposed based on the improved matrix approximate method, which solves the problem of use of local fractional differential numerical approximate. Finally, a lot of experiments verify the validity of proposed method. Future works involve constructing the adaptive strategy for this model and applying it in the medical image enhancement.

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