The Public Opinion Control Model Based on the Connecting Multi-Small-World-Network

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Abstract: Based on the propagation mechanism of the rumor control, this study proposes a mode of propagation found on the information content to describe the dissemination of two opposite rumors on the same subject among crowds and sets up public opinion control model on the basis of this mode. Two opposite rumors on the same subject in our mode of propagation can respectively represent rumor and truth, so we investigate their interactions during the dissemination among crowd and simulate it in the connecting multi-small-world-network. Finally, by adjusting the factors which can affect the control effect of the model, we propose a corresponding rumor immunization strategy. Based on that, we conduct the analogy analysis of interactions of many opposite rumors on the same subject when they spread among crowds.

Keywords: Connecting multi-small-world-network, organizational communication, rumor control, SIRS model

INTRODUCTION

In recent years, with the popularizing of computers and the Internet as well as the development of network communication technology, it has been more convenient and efficient for people to communicate with each other, which also makes the environment more convenient for the propagation of public opinion on the Internet. For example, in order to achieve some specific goal, certain individuals and organizations tend to disseminate fake news to slander other people or organizations and sometimes even be engaged in separatist activities. If no effective measure is taken, individuals, enterprises and the nation will suffer huge losses.

The propagation of public opinion in groups plays an important role in the public information management. Once the rumor occurs, relevant information will fast disseminate among different groups through the network and receivers will react with stress reaction. In a typical rumor spreading case, there is abundant and complex information and the changes in rumor are irregular and unpredictable. Considering the burstiness, uncertainty and urgency of rumor, the information asymmetry is ubiquity. The propagation of public opinion in different cases can exacerbate public fears about crises, which might further leads to the spread of crises makes them out of control, resulting in new social crises. Thus, how to reduce negative effects of crises and improve public capacity to deal with crises has become a key issue in public information management.

The propagation of public opinion, which was traditionally the subjects of psychological field (Anthony, 1973), has become complicated on the Internet. In reality, many technological networks (Zou et al., 2002; Myers, 2003; Zhao et al., 2006) often revealed the scale-free behavior. Meanwhile, as Barabasi once said, “Our world is small because our society is a dense network. Barabasi and Albert (1999). The network in the real world is usually a network between regular network and random network, with the”small-world effect”. Watts and Strogatz first promoted Small-world network in 1998, (Watts and Strogatz, 1998) which randomly rewires links and is called WS-SWN. One obvious characteristic of the small-world network is its shorter average path length and higher clustering coefficient. In addition, its degree distribution conforms to the exponential distribution. Later on, Newman and Watts proposed a modified WS-SWN named NW-SWN (Newman and Watts, 1999) which randomly rewires links and is called WS-SWN. One obvious characteristic of the small-world network is its shorter average path length and higher clustering coefficient. In addition, its degree distribution conforms to the exponential distribution. Later on, Newman and Watts proposed a modified WS-SWN named NW-SWN (Newman and Watts, 1999) which randomly rewires links and is called WS-SWN. One obvious characteristic of the small-world network is its shorter average path length and higher clustering coefficient. In addition, its degree distribution conforms to the exponential distribution.
interconnection model which depicted the propagation process. Taking the group structure into consideration, they used the single small-world network to express the propagation of public opinion in groups and interconnection process to express the propagation of public opinion among communities.

In this study, in order to control the propagation of public opinion, we firstly set up the rules of self-evolution for rumor control. The general principle in rumor propagation employing the Susceptible-Infective-Refractory-Susceptible (SIRS) model, an epidemiology model introduced by Daley and Kendall for information spreading (Liu et al., 2003; Zhou et al., 2007), is as follows: at the beginning, only one individual is infected and all the remnant population are susceptible. Later, the infected individual i will randomly contact one of its neighbors j. If j is in the susceptible state, i transmits the rumor and j becomes infected. If, on the other hand, j is already infected or refractory, then i loses her interest in the rumor and becomes refractory. Zhou et al. (2007) However, since this model simplifies the propagation mode, it can only depict a rumor at a time. The key to depict two rumors or more is to establish the credibility of information. Cholvy (2011) and Warhen and Jacquelyn (2002) conducted relevant researches on this issue and drew the conclusion that: for individuals, the credibility of information greatly depends on the authority of the information source and receiver’s perceptions of information. Thus, this study proposes a propagation mode based on information content to describe the spreading of two opposite rumors on the same subject among crowds. Among these two opposite opinions, one can represent rumor and the other can represent the truth. So we can investigate their interactions in the spreading among crowds and further extend to the discussion about interactions of different opinions, which are also opposite, in their spreading among crowds. Then, we make a simulation under the small-world network structure that xing and his partners (Xing et al., 2011) has set up. Finally, by adjusting factors which can affect the control effect of rumor control model on rumor, we propose a better strategy to avoid rumors.

RUMOR CONTROL MODEL

As mentioned in the first part, the propagation of simplified propagation behaviors on the Internet has its limitations. Although Xing and partners (Xing et al., 2011) have modified part of the rules when they built up the CM-SWN, these are still not enough for research on interactions of different opinions, which are also opposite, in their spreading among crowds. Therefore, in this part, we will set up a rumor control model to adapt its propagation rule to the spreading of two or more rumors which are opposite to each other among crowds.

Connecting Multi-Small-World-Network model (CM-SWN): In current society, many communities have a strict structure but a great gap exists in their communication with the outside world. Within this group structure, the great gap provides favorable environment for the propagation of public opinion in such group environment. Xing and his partners (Xing et al., 2011) have well reflected this social structure in the form of a pyramid network structure when they were building the CM-SWN. Thus, the social network structure in this study will adopt CM-SWN, the simplified structure of which is shown in Fig. 1.

Building the rumor control model: Once the crisis occurs, relevant information will spread within the network. At the moment t, node k is a node belonging to the node set V(G), which forms the network in Fig. 1. Parameters $R_k$ and $T_k$ respectively represent the level of information about the rumor and the truth when node k has received at the moment t, so the whole information level of node k at moment t is $I_k$. We define $I_k$ as follows: $I_k = R_k + T_k$, among which $I_k = 1$, $0 \leq R_k$, $T_k \leq 1$. The difference of their information levels can be described by $D_k$ with $D_k = R_k - T_k$. So if $D_k > 0$, then node k represents higher information level of the rumor. Thus, the node k is more inclined to the rumor in both the propagation of the information and its credibility. So if $D_k < 0$, then node k represents higher information level of the truth. Thus, the node k is more inclined to the truth in both the propagation of the information and its credibility. When $D_k$ reaches certain level in which node k in the infective state, there is a great gap between the information levels of the rumor and the truth that the node k represents at moment t, so there is a desire to spread the information. Therefore, the desire to spread the information for node k at moment t can be expressed by $|D_k|$-the absolute value of $D_k$.

In correspondence to the SIRS model, we define that the state determines the threshold value: Threshold1 and Threshold2. Then if $|D_k| < \text{Threshold1}$, node k is in the susceptible state; if $\text{Threshold1} < |D_k| < \text{Threshold2}$, node k is in the infective state; if $|D_k| > \text{Threshold2}$, node k is in the refractory state. And Threshold1 and Threshold2 are determined by the group’s sensitivity to the rumor.
In the initial period, the information level of rumor and truth of all nodes are the same, which means ∀k ∈ V, R^k_0 = 0.5, T^k_0 = 0.5, D^k_0 = 0. When it comes to moment α, node m begins to spread rumor (or truth), which usually occurs later than rumor (Steven Fink.1986.Crisis Management: Planning for the inevitable [M]. New York:American Management Association.) in the network, then R^m_α > 0.5, T^m_α < 0.5, D^m_α > 0 (or R^m_α < 0.5, T^m_α > 0.5, D^m_α < 0). Besides, the desire to spread the information reaches limit critical value, which means |D^m_α| = Threshold^d = σ, among which σ is the sensitive actor, α<<1. And the value of σ is also determined by the group’s sensitivity to the rumor. When it comes to moment b, since node n has received enough information about the rumor(truth) from its neighbors in the network, then Threshold < |D^n_b|. Therefore, the node n varies from the infective state to the refractory state and R^n_α = 1, T^n_α = 0, D^n_α = 1 ( or R^n_α = 0, T^n_α = 1, D^n_α = −1).

The rule of information spreading in the CM-SWN is:

i. Assume that node i at moment t is in the infective state and its neighbor j is in the susceptible state. According to the state analysis we have referred in the previous part, we can draw a conclusion that: |D^i_t| < |D^j_t|, which also means node i have a stronger desire to spread the information than node j. Thus, node j is the receiver and it is able to receive relevant information (rumor or truth). In addition, when the time changes as t → t+1, it will change its information level as the following formula, based on the information they have received:

\[
R^{i+1}_t = \begin{cases} 
R^i_t + |D^j_t| & |R^i_t - R^j_t|, D^j_t > 0 \\
R^i_t - |D^j_t| & |T^i_t - T^j_t|, D^j_t < 0 
\end{cases} \\
T^{i+1}_t = \begin{cases} 
T^j_t - |D^i_t| & |R^i_t - R^j_t|, D^i_t > 0 \\
T^j_t + |D^i_t| & |T^i_t - T^j_t|, D^i_t < 0 
\end{cases}
\]

This rule can correspond to the situation in which a person who is crazy about information dissemination has communication with the one who is sensitive to the information in the reality. The former is crazy about spreading some information to the latter basing on his confidence in the information. When the person who is sensitive to the information receives the information, he will add more information on the viewpoint and deny part of the information on the opposite point of view. Then the deviation will exist in his information level, leading him to believe in the rumor or the truth.

ii. Assume that node i at moment t is in the infective state and its neighbor j is in the refractory state. According to the state analysis we have referred in the previous part, we can draw a conclusion that: |D^j_t| < |D^i_t|. But since node j is in the refractory state, so the node j will lose the desire to spread the information and it will react on i. If node i and node j believe in the same opinion (rumor or truth), then node i will get more information from its occasional communication with node j and become more confirmed on its opinion. In addition, when the time changes as t → t+1, node i will change its information level as the following formula, based on the information they have received:

\[
R^{i+1}_t = \begin{cases} 
R^i_t + |D^j_t| & |R^i_t - R^j_t|, D^j_t = 1, D^i_t > 0 \\
R^i_t - |D^j_t| & |T^i_t - T^j_t|, D^j_t = -1, D^i_t < 0 
\end{cases} \\
T^{i+1}_t = \begin{cases} 
T^i_t - |D^j_t| & |R^i_t - R^j_t|, D^i_t = 1, D^j_t > 0 \\
T^i_t + |D^j_t| & |T^i_t - T^j_t|, D^i_t = -1, D^j_t < 0 
\end{cases}
\]

But, when node j at moment t believes in the rumor, then the truth spread by the node i will wavers the opinion of j and node j will return to the initial susceptible state, among which R^j_t = 0.5, T^j_t = 0.5, D^j_t = 0.

This rule can also correspond to the situation in which a person who is crazy about information dissemination has communication with the one who is sensitive to but bored with the information in the reality; then the information level of the latter will remain the same. What’s more, he will receive more information and become more confident in it, which will react on the former. The person who is crazy about information dissemination will receive unexpected information, leading him to be inclined to believe in the original information. But when two persons inclined to opposite opinions talk with each other, one of their states will change only under the circumstance that a person who is crazy about information dissemination on truth has communication with the one who is sensitive to but bored with the information on rumor. As rumor usually does not stand up to scrutiny, when the person who is sensitive to but bored with the information on rumor receives the truth, he will doubt the previous information and overthrow it and be interested in this topic again.

iii. Assume that node i at moment t is in the infective state and its neighbor j is in the infective state. If node j and node i believe in the same opinion (rumor or truth), their desires to spread the information reach the limit critical value, which means |D^j_t| = Threshold^d = σ. Then one of them will transfer into the refractory state.

This rule can also correspond to the situation in which person who is crazy about information dissemination has communication with the same kind of person in the reality, they hold the same view and both of their desires to spread the information reach the limit critical value. Thus, there will be no quarrel and persuasion between them, which will lead them to be bored with this topic and one of them will believe in but be bored with the information.
In conclusion, the rule of information spreading among the network will be showed as follows:

\[ \frac{dS(t)}{dt} = -\varepsilon \mu_s(t)K(t)S(t)I_r(t) + \varepsilon \mu_I(t)K(t)I_r(t)R_f(t) \]

\[ \frac{dI_r(t)}{dt} = \varepsilon \mu_s(t)K(t)S(t)I_r(t) - \varepsilon \mu_I(t)K(t)I_r(t)I_f(t) + \varepsilon \mu_I(t)K(t)I_r(t)R_f(t) \]

\[ \frac{dI_f(t)}{dt} = \varepsilon \mu_I(t)K(t)I_r(t)I_f(t) - \varepsilon \mu_I(t)K(t)I_r(t)I_f(t) + \varepsilon \mu_I(t)K(t)I_r(t)R_f(t) \]

\[ \frac{dR_f(t)}{dt} = \varepsilon \mu_I(t)K(t)I_r(t)I_f(t) + \varepsilon \mu_I(t)K(t)I_f(t)R_f(t) \]

where, \( \varepsilon \) is a positive deviation factor and \( \varepsilon \geq 1 \) because of the hub node in CM-SWN improves the propagation efficiency. \(<k>\) denotes the average nodes, to which each node are connected. Meanwhile, we provide the following equations to quantify the transition probabilities:

\[
\lambda_s(t) = \frac{N_s(t)}{N} \\
\lambda_I(t) = \frac{N_I(t)}{N} \\
\lambda_f(t) = \lambda_s(t) + \lambda_I(t) = \frac{N_s(t) + N_I(t)}{N} \\
\mu_s(t) = \frac{N_s(t)}{N} \\
\mu_I(t) = \frac{N_I(t) + N_f(t)}{N} \\
\mu_f(t) = \frac{N_f(t)}{N}
\]

where, \( N \) represents the amount of points in the point set \( V(G) \) in the network and \( N = |V(G)|; N_{I_R}(t) \) the amount of points in the \( I_R; N_{I_f}(t) \) the amount of points in the \( R_f; N_{R_f}(t) \) the amount of points in the \( R_f \).

**Evaluation system:** In order to examine the operation process of the model based on the crisis information spreading in the small-world network, this study sets four indexes to measure the control level of the rumor control model. The average warmly degree of the rumor in the group and the number of population believing in the rumor in the group are indexes to measure the information level of the rumor in the group. Since information management organizations do not expect the spreading of the rumor which is public harmful, the lower these two indexes are the better. On the other hand, the average warmly degree of the truth in the group and the number population believing in the truth in the group are indexes to measure the information level of the truth in the group. And the higher these two indexes are the better.

**The average warmly degree of the rumor in the group:** Among the whole group \( N \) at moment \( t \), all individuals who receive more information about rumor than truth make up a set \( G_{R} \) and the average warmly degree of the rumor in the group represents the ratio of the total desires to spread information in individuals of \( G_{R} \) compared to the total number of population. It can be expressed as follows:

\[ WD_R = \frac{\sum |G_{R}|}{N} \]

**The population believing in the rumor in the group:** Among the whole group \( N \) at moment \( t \), all individuals
who receive more information about rumor than truth make up a set $\text{Group}_R^t$ and the population believing in the rumor in the group represents the total number of individuals in the $\text{Group}_R^t$, that is:

$$P_x^t = |\text{Group}_x^t|$$

The average warmly degree of the truth in the group: Among the whole group $N$ at moment $t$, all individuals who receive more information about truth than rumor make up a set $\text{Group}_R^t$ and the average warmly degree of the truth in the group represents the ratio of the total desires to spread information in individuals of $\text{Group}_R^t$ compared to the total number of population.

It can be expressed as follows:

$$W_D_t = \frac{\sum_{i \in \text{Group}_R^t} |P|}{N}$$

The population believing in the truth in the group: Among the whole group $N$ at moment $t$, all individuals who receive more information about truth than rumor make up a set $\text{Group}_R^t$ and the population believing in the truth in the group represents the total number of individuals of $\text{Group}_R^t$, that is:

$$P_x^t = |\text{Group}_x^t|$$

RESULTS AND DISCUSSION

In this part, we will firstly study the interesting phenomenon of two opposite opinions about the same topic spreading among crowds and use the public opinion control model to describe it. In order to simulate this phenomenon, we adopt MATLAB to simulate the public opinion control model in CM-SWN. Later, in order to reflect the control effect of this model, we adjust the factors which can affect the control effect of the model and offer a better rumor immunization strategy.

Here, we simulate the phenomenon of the rumor spreading in the youngster-aged group and adopt the simplest structure in CM-SWN with three sub-networks (as we have mentioned in the second part), each of which has 500 points. And we set other parameters in the CM-SWN: $K = 2$, $p = 0.1$.

Description of the phenomenon: In the following part, we will connect the result we get from the simulation with reality and further explain whether this model can describe the phenomenon of two opposite opinions about the same topic spreading among crowds. Here, we uniformly regulate that the truth occurs five steps later than the rumor and the truth randomly occurs in the whole group.

As can be seen from Fig. 2 and 3, the information exchange of the truth occurs approximately in the 5th to 16th steps and the information exchange of the rumor occurs approximately in the 1st to 22nd steps. However, since the truth occurs later than the rumor, there has been rumor existing in the circle when the truth comes up, the truth whose core point has infected by the rumor cannot spread to other circles. After that, when information about truth has accumulated to some extent, the circle that truth occurs will change into the period of state mutation; meanwhile, since the rumor has spread to new circles, then special circumstance occurs: one circle is in the period of information exchange and the other is in the period of state mutation. Consequently, truth can only spread in the circle that truth has occurred and other circles are cheated by the rumor. This state is the same as the circumstance in our real life where truth propagation does not go well and it can spread in a few circles, which can well demonstrate that the public opinion control model can exactly describe the phenomenon of two opposite opinions about the same topic spreading among crowds.

Fig. 2: The average warmly degree of the rumor and the average warmly degree of the truth
The optimal control effect of the rumor: There are three main factors that can affect the control effect of the public opinion control model on the rumor: the circle where truth occurs, whether the truth occurs at the core point of the circle and the time when truth occurs. Hence, we should figure out the optimal control effect of the public opinion control model on the rumor.

Whether truth will occur in the circle where rumor firstly occurs: We firstly have a discussion over the
circle where truth occurs. Usually, truth occurs later than rumor. Here, we uniformly regulates the truth occurs five steps later than the rumor and both of them occur in the whole group-in the form of random points. In our real life, people commonly have their social communication circles. Similar with rumor, truth spreading also starts from one circle and then expands to other circles. Thus, when the truth occurs in the circle where rumor has firstly occurred, the control effect of the public opinion control model on the rumor is different from the circumstance where truth occurs in other circles.

As can be seen from Fig. 4 and 5, when truth and rumor do not occur in the same circle, the increase of the group’s average warmly degree of the rumor and the group’s average warmly degree of the truth approximately remain the same. They both increase as soon as they occur with short period of information exchange. But due to the hysteresis characteristic of the occurrence of truth, when it spreads to other circles where rumor has been in the favorable position, average warmly degree of the truth will go down and truth will fail to spread to other circles. When truth and rumor occur in the same circle, they will experience dramatic confrontation and the period of information exchange will be quite long. But due to the hysteresis characteristic of the occurrence of the truth, the average warmly degree of the truth will be limited until both truth and rumor change into the period of state mutation in the new circle. Consequently, truth and rumor almost enter the new circle at the same time. For the new circle, although it has received more information about rumor than the truth, the hysteresis characteristic of truth almost disappears. Furthermore, truth continues to spread in the original circle, so the average warmly degree of the truth will rapidly go up, which can build a solid information basis for the latter period of state mutation. Therefore, the population believing in the truth will increase and the rumor will be limited.

**Assumption on the basic simulation rule:** Then we will discuss over whether the truth will occur at the core point of the circle. Here, we uniformly regulates the truth occurs five steps later than the rumor and both of them occur in the same circle. From the CM-SWN set up by Xing and these partners, we can know

![Fig. 6: The group’s average warmly degree of the rumor and the group’s average warmly degree of the truth when truth occurs/does not occur in the core point of the circle](image)

![Fig. 7: The group’s population believing in the rumor and the group does population believe in the truth when truth occurs/does not occur in the core point of the circle](image)
that every circle has a core point and the degree of the core point is generally higher than the others. It has better connectivity. Thus, when the truth occurs in the core point of the circle, the control effect of the public opinion control model on the rumor will be different from the circumstance where the truth does not occur in the core point of the circle.

As shown in Fig. 6 and 7, since the truth occurs in the core point of the circle, it has better connectivity. As a result, truth will rapidly spread out of the circle and the spreading of the rumor will be slower than that of the truth, so that rumor cannot spread out of the circle. But in the original circle, due to the hysteresis characteristic of the occurrence of truth and the fact that better connectivity cannot make up this disadvantage, rumor will break out in this circle. Only after that, it will slowly disappear. Compared with the circumstance where truth does not occur in the core point of the circle, taking either the group’s average warmly degree of the rumor or the group’s population believing in the rumor into consideration, the circumstance where truth occurs in the core point of the circle will get better results. The control effect of the public opinion control model in the original circle is not optimal, which is caused by the great gap between the degree of the core point $K$ and the number of nodes in a single circle. If we narrow the gap, the control effect of the public opinion control model may be better.

**Local simulation rule:** At last, we will have a discussion about the initial time to control public opinion, namely the time when truth occurs. Here, we uniformly regulate truth and rumor occurs in the same circle and truth occurs at the core point. By comparing the difference of the time of public opinion control, we get the effect of the time of public opinion control on the control effect of the public opinion control model. From the former simulation experiment, we can know that the first period of information exchange for rumor mainly occurs in the $1^{st}$ to $10^{th}$ step. Thus, if we compare the results after the $10^{th}$ step, where rumor has changed into the period of state mutation, it will make no sense. Therefore, we make comparisons among the different effects of $2^{nd}$, $5^{th}$ and $8^{th}$ steps.

As shown in the Fig. 8 and 9, an interesting phenomenon occurs in the simulation result: the earlier the truth occurs; the better is not the truth. This phenomenon seems to be unreasonable. However, through analysis we can conclude that: the reason why people believing in rumor do not turn into those sensitive to information and finally change to believe

Fig. 8: The group’s average warmly degree of the rumor and the group’s population believing in the rumor when the initial time to control public opinion is respectively the $2^{nd}$, $5^{th}$, $8^{th}$ step.
Fig. 9: The group’s average warmly degree of the truth and the group’s population believing in the truth when the initial time to control public opinion is respectively the 2\textsuperscript{nd}, 5\textsuperscript{th}, 8\textsuperscript{th} step.

in the truth is because truth occurs too early and no one will spread truth in the later period. And at that time, nobody will disseminate the truth to those who believe in rumor. Early initial time to control public opinion will hold down the group’s average warmly degree of the rumor to a comparatively low level. But this always leads to high controlling cost (constantly observe whether rumor occurs or not) and the control effect is not quite satisfactory.

Compared with the control effect of rumor in the 5\textsuperscript{th} and 8\textsuperscript{th} steps, there is no obvious difference. Although the group’s highest warmly degree in the 8\textsuperscript{th} step is higher than that of the 5\textsuperscript{th} step, it disappears faster than the 5\textsuperscript{th} step. The group’s population believing in the rumor, the group’s average warmly degree of the truth and the group’s population believing in the truth are similar in these two steps, with a few fluctuations and time differences. Therefore, if we want to acquire the optimal control effect, we can choose to control rumor at the middle of the period of information exchange; if we want the rumor to disappear as soon as possible, we can choose to control the rumor in the second half which is also close to the middle of the period of information exchange.

CONCLUSION

Rumor spreading is one of the basic mechanisms for information dissemination in networks. This study uses results of the computer simulation to well verify the analysis of the features of information propagation in different periods from the quantitative perspective. It can not only embody the information level of the rumor and truth at different moments, but also dynamically reflect the propagation law and mode of evolution of information spreading in the network. By adjusting factors which can affect the control effect of rumor control model on rumor, we discover that public information management organizations should strengthen their regulation and monitoring on the non-mainstream media and the Internet to reduce rumor. Meanwhile, if public information management organizations release truth in the circle that rumor occurs, they can control the propagation of public opinion in a more targeted way. Besides, the broader truth covers, the higher level of information will be released, which will be more favorable to the rumor control. Meanwhile, in consideration of the specificity of the public opinion control model and CM-SWN, which simulates the social communication circle, this
model can be generally applied in multi-information exchange in the area of group’s propagation of public opinion. In the future, we expect to be able to further improve public opinion control model to achieve more intensive process of information exchange, more obvious interactions between information and various approaches to controlling public opinion.

REFERENCES