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Research Article

An Approach for Monitoring a Two-stage Process with Profile Qualitative Characteristic in Phase II

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Abstract: Aim of this study is introduction one Approach for Monitoring a Two-Stage Process by Profile Quality Characteristic in the Second Stage. Nowadays, many processes are multistage and such processes often depend on each other. The implication is; the specification features for the product that are used to monitor the quality of that product and are usually assessed in one stage of the process, not only take form on the same stage but also take shape in the different phases of the process. This topic in statistical quality control is known as cascade property in multistage processes. In such a case, care must be taken that the lack of attention to this detail will cause an error in the analysis of control charts. Thus, in reviewing the literature, some methods are presented to reduce the error. In many situations, the quality of process or product is described by using the relationship between a response variable and an independent variable. Thus at each stage of sampling, a set of data is collected which can be shown by using a profile. Our goal in this study is to assess the cascade property for evaluating linear profiles that are in various stages of the processes. We have named this project as profile monitoring and evaluation of multistage processes. Hence, in this study, results have been studied by simulation of the average run length in Phase II.

Keywords: Average Run Length (ARL), cascade property, multistage processes, profile monitoring

INTRODUCTION

In recent years, various research activities in the use of control charts have been done. Most of the researches have emphasized so much on the proper use of control charts in correct position. Several studies have been conducted on the errors resulting of improper use. Two of these studies are the major source of this article, which is trying to consider the both together. The first group had tried to describe the quality of the product and the process performance by monitoring the relationship between a response variable and one or more independent variables. They have named this equation (relationship) as profile (2004), The second group believed that because many of the manufacturing processes are complex systems and this process is often not a single step, hence, the output quality should be evaluated by monitoring several interdependent processes that take place. This type of control is called multistage processes monitoring (Zhang, 1980). Multistage processes have cascade properties. This means that at each stage of the process, quality is dependent on two parameters. One is particular quality, which is the quality of operations in the current period. And the other is the overall quality, which is defined as the quality of pre-and current stages.

The studies undertaken which are based on profile monitoring, can be found in Gupta *et al*. (2006), Zou *et al*. (2006) and Saghaei *et al*. (2009) which has been carried out in the second phase and Mahmoud and Woodall (2004) and Mahmoud *et al*. (2007) in the first phase and Kang and Albin (2000) and Kim *et al*. (2003) in both phases. Monitoring polynomial profiles by Kazemzadeh *et al*. (2008, 2009) are examined in the first and second phases. Zou *et al*. (2007) and Amiri *et al*. (2012) studied the multiple linear profile monitoring. In monitoring linear profiles with multiple multivariate Noorossana *et al*. (2009, 2010a) offered simple solutions. About monitoring nonlinear profiles (Jin and Shi, 1999) can be valuable. Also activities of Walker and Wright (2002), Ding *et al*. (2006), Williams *et al*. (2007), Moguerza *et al*. (2007), Vaghefi *et al*. (2009), Qiu and Zou (2010a) and Qiu *et al*. (2010b) can be noted. The effects of non-normality residual on simple linear profile monitoring by Noorossana *et al*. (2010b, 2004) and the effects of non-independent data on profiles monitoring by Jensen *et al*. (2008), Noorossana *et al*. (2008) and Soleimani *et al*. (2009) are examined. Niaki *et al*. (2007) used generalized linear model for the monitoring of simple linear profiles. Zhu and Lin (2010) focused on monitoring the slope of the linear profile. Chen and Nembhard (2010) was with high-dimensional control chart for monitoring the linear profiles. Noghondarian and Ghobadi (2012) fuzzy profile monitoring approach for phaseI.

Zhang (1982, 1984, 1985, 1989a, 1989b, 1992) first carried out monitoring a multistage processes. The foundation of these efforts were based on the cascade

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property, then Hawkins (1991, 1993) provided similar charts regardless of the cascade property. This new control chart created new horizons in the analysis and improvement of a multistage processes and then Wade and Woodall (1993) and Yang and Yang (2006a) began to develop, expand and emphasize the use of the charts. Several examples of multistage processes in the semiconductor industry by Skinner *et al*. (2004) and Jearkpaporn *et al*. (2003, 2005, 2007) have been raised, assuming that the data is not normalized. Yang (1999) and Sulek *et al*. (2006) studied a multistage processes model in the banking system and supermarket. Loredo *et al*. (2002), Shu and Tsung (2003) and Yang and Yang (2005) conducted their research with premise of data correlation. Yang and Su (2006b, 2007a, b) began the application of adaptive control charts in monitoring a multistage processes. In economic design of control charts for monitoring multistage processes (Yang, 1997, 1998, 2003a; Yang and Chen, 2003b; Yang and Yang, 2006c) provided valuable research. Also using neural network by Niaki and Davoodi (2009) was studied.

In the research that has been cited, few studies have been carried on these two topics; profile monitoring and controlling multistage processes, or together. Our focus in this study is the simultaneous analysis of the impact that profile monitoring and control of multistage processes will have on the control charts. One of the researches in this field can be Niaki *et al*. (2012) study. In this study a two-step process has been considered that in each step, rather than quantitative characteristics, a profile exist and the impact of cascade effect on profiles monitoring have been measured in the second phase. In this study there has been an attempt to measure the effect of the coefficients in a two-step process, in a way that in the first phase, there exists a qualitative characterization and in the second phase there is a profile. And qualitative characteristics of the first stage act as the independent variables of the second stage. So for monitoring the qualitative characteristics of the first step graph $\bar{x}$ - R, for monitoring profile parameters of the second step graph T^2 and for monitoring the residuals graph x^2 are used. This study was conducted in the second phase and aims to monitor the impact of coefficients changes on a multistage processes profile monitoring.

METODOLOGY

Defining the problem and model assumptions: In many situations, the quality of a process or a product is characterized by the relationship between a response variable and one independent variable. Thus at each stage of sampling, a set of data is collected which can be shown by using a profile. But sometimes it is necessary that monitoring take place at different stages of processes. This type of monitoring is named multistage processes monitoring. In fact, in this case the steps are not independent of each other. And based on the cascade property, former stages have their impact on the latter stages. Such as Fig. 1 a two-stage process that in the first phase requires a qualitative characteristic x_1 and in the second phase a profile Y_2 to be monitored simultaneously. According to Eq. (1) the qualitative characteristics of the first stage would affect the response variable on the second stage:

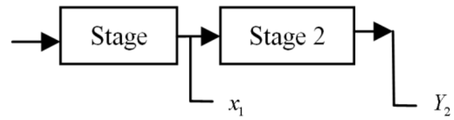

Fig. 1: A typical profile of a two-stage process

$$\begin{cases} x_i \sim (\mu_{xi}, \sigma_{xi}^2) \\ y_2 = \beta_0 + \gamma x_1 + \beta_1 x_2 + \varepsilon \end{cases} \quad (1)$$

In Eq. (1) β_1 's and the profile coefficients γ in second stage are the kinds that show the expected change in y_2 as per one unit change in x_1 itself or x_2, with all the other variables being constant. x_2 is the qualitative characteristics affecting the profile of the second stage and x_1 is the qualitative characteristic in the first step and ε is the error characteristics. Model assumptions are:

- ε has a normal distribution

$$\varepsilon \sim \mathrm{N}(0, \sigma_\varepsilon^2) \quad (2)$$

- Due to regression, the values of x_2 are constant (not random variables)
- There is no autocorrelation within the profiles
- The profiles are intended to be linear

The estimation equations for intercept and slope of the profile in a two-stage process: According to Eq. (1) and considering that it is a two-stage process, to obtain the coefficients of the profile, we have: y_{ijk}, x_{ijl} and x_{kl} that:

$$\begin{aligned} &(y_{111}, x_{111}, x_{12}), (y_{112}, x_{111}, x_{22}), \ldots, (y_{11n_2}, x_{111}, x_{n_2 2}) \\ &, (y_{121}, x_{121}, x_{12}), (y_{122}, x_{121}, x_{22}), \ldots, (y_{12n_2}, x_{121}, x_{n_2 2}) \\ &, \ldots, (y_{1n_1 1}, x_{1n_1 1}, x_{12}), (y_{1n_1 2}, x_{1n_1 1}, x_{22}), \ldots, (y_{1n_1 n_2}, x_{1n_1 1}, x_{n_2 2}) \\ &, \ldots, (y_{m11}, x_{m11}, x_{12}), (y_{m12}, x_{m11}, x_{22}), \ldots, (y_{m1n_2}, x_{m11}, x_{n_2 2}) \\ &, \ldots, (y_{mn_1 1}, x_{mn_1 1}, x_{12}), (y_{mn_1 2}, x_{mn_1 1}, x_{22}), \ldots, (y_{mn_1 n_2}, x_{mn_1 1}, x_{n_2 2}) \end{aligned} \quad (3)$$

where, l is the counter of the steps and by premises l = 1, 2 and j is the counter of the sample size of the qualitative characteristic of the first stage, j = 1, …, n_1 and k is counter of the sample size of the effective qualitative characteristic on profile in the second stage k = 1, …, n_2 and i is the counters of repeating sample i = 1, …, m so, we would have.

$$\begin{aligned} &If\ l = 1 \Rightarrow x_{ij1} \\ &If\ l = 2 \Rightarrow x_{k2} \end{aligned} \tag{4}$$

According to Eq. (3 and 4), x_{ij1} is the j[th] amount in each of n_1 sample of qualitative characteristics of the first stage in i[th] repeat sampling, x_{k2} is the k[th] amount in each of n_2 sample of qualitative characteristics of the second stage and y_{ijk} is the amount of profile for the j[th] amount in each of n_1 sample of qualitative characteristics of the first stage and the k[th] amount of n_2 sample of qualitative characteristics of the second stage in i[th] repeating samples.

However, with respect to the above equations, the matrix designs are the following:

$$\begin{bmatrix} 1 & x_{111} & x_{12} \\ 1 & x_{111} & x_{22} \\ 1 & x_{111} & x_{32} \\ . & . & . \\ 1 & x_{111} & x_{n_2 2} \\ 1 & x_{121} & x_{12} \\ 1 & x_{121} & x_{22} \\ . & . & . \\ 1 & x_{121} & x_{n_2 2} \\ . & . & . \\ 1 & x_{1n_1 1} & x_{12} \\ . & . & . \\ 1 & x_{1n_1 1} & x_{n_2 2} \\ 1 & x_{211} & x_{12} \\ . & . & . \\ 1 & x_{2n_1 1} & x_{n_2 2} \\ . & . & . \\ 1 & x_{mn_1 1} & x_{n_2 2} \end{bmatrix}_{(m\times n_1\times n_2)\times 3} \times \begin{bmatrix} \beta_0 \\ \gamma \\ \beta_1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} y_{111} \\ y_{112} \\ y_{113} \\ . \\ y_{11n_2} \\ y_{121} \\ y_{122} \\ . \\ y_{12n_2} \\ . \\ y_{1n_1 1} \\ . \\ y_{1n_1 n_2} \\ y_{211} \\ . \\ y_{2n_1 n_2} \\ . \\ y_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} \tag{5}$$

$$\hat{Y} = \hat{\beta}_0 + \hat{\gamma} x_1 + \hat{\beta}_1 X_2 \tag{6}$$

Considering the method of least squared errors from Eq. (6)we have:

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 \tag{7}$$

$$Y = X\beta + e \tag{8}$$

$$\begin{bmatrix} \hat{y}_{111} \\ \hat{y}_{112} \\ \hat{y}_{113} \\ . \\ \hat{y}_{11n_2} \\ \hat{y}_{121} \\ . \\ . \\ \hat{y}_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} = \begin{bmatrix} 1 & x_{111} & x_{12} \\ 1 & x_{111} & x_{22} \\ 1 & x_{111} & x_{32} \\ . & . & . \\ 1 & x_{111} & x_{n_2 2} \\ 1 & x_{121} & x_{12} \\ . & . & . \\ . & . & . \\ 1 & x_{mn_1 1} & x_{n_2 2} \end{bmatrix}_{(m\times n_1\times n_2)\times 3} \times \begin{bmatrix} \hat{\beta}_0 \\ \hat{\gamma} \\ \hat{\beta}_1 \end{bmatrix}_{3\times 1} \tag{9}$$

$$\begin{bmatrix} e_{111} \\ e_{112} \\ e_{113} \\ . \\ e_{11n_2} \\ e_{121} \\ . \\ . \\ e_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} = \begin{bmatrix} y_{111} \\ y_{112} \\ y_{113} \\ . \\ y_{11n_2} \\ y_{121} \\ . \\ . \\ y_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} - \begin{bmatrix} \hat{y}_{111} \\ \hat{y}_{112} \\ \hat{y}_{113} \\ . \\ \hat{y}_{11n_2} \\ \hat{y}_{121} \\ . \\ . \\ \hat{y}_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} \tag{10}$$

with regards to the Eq. (8-10) if we minimize the sum of squared errors, we will have:

$$\begin{aligned} &\sum_{i=1}^{n} e_i^2 = \mathbf{e}'\mathbf{e} = (\mathbf{Y} - \hat{\mathbf{Y}})'(\mathbf{Y} - \hat{\mathbf{Y}}) \\ &= (\mathbf{Y}' - \hat{\mathbf{Y}}')(\mathbf{Y} - \hat{\mathbf{Y}}) = \mathbf{Y}'\mathbf{Y} - 2\mathbf{X}'\mathbf{Y}\hat{\beta} + \hat{\beta}'\mathbf{X}'\mathbf{X}\hat{\beta} \\ &\frac{\partial \mathbf{e}'\mathbf{e}}{\partial \hat{\beta}} = 0 - 2\mathbf{X}'\mathbf{Y} + 2\mathbf{X}'\mathbf{X}\hat{\beta} = 0 \end{aligned} \tag{11}$$

where,

- Y = The vector of observation of profiles variables
- X = The vector of observations of independent variables
- $\hat{\beta}$ = The vector of coefficients of the profile
- $\hat{Y}$ = The vector of predicted values of profile variable and e is the vector of the residuals (error terms)

So finally we have Eq. (12) for the estimate of the intercept and the slope of the profile:

$$\hat{\beta} = (x'\mathrm{x})^{-1} x'Y \tag{12}$$

In this study, changes in the parameters of the model are expressed with respect to the sensitivity analysis of Standard Average Run Length and with respect to the above equations we have obtained the coefficients, which are the intercept and the slope of the profile. In fact, because our study is in the second phase of control chart, at first we consider real profiles with default coefficients under control mode, then; according to the Eq. (12) we will estimate the coefficients after simulation.

Sensitivity analysis of average run length to the change in model parameters: As it was mentioned the research has been studied in the phase II of the control chart, at first we consider the coefficients of the model as given in Eq. (1), because the goal of the Phase II control chart is monitoring the process. So we want to achieve this important that: first, which coefficients of the profile are more sensitive to changes and secondly to investigate the changes of the coefficients and

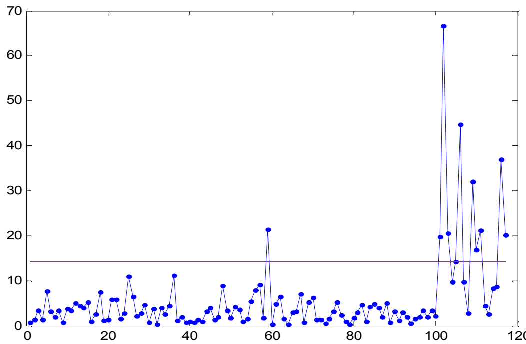

Fig. 2: Part of the chart T^2 for monitoring profiles coefficients

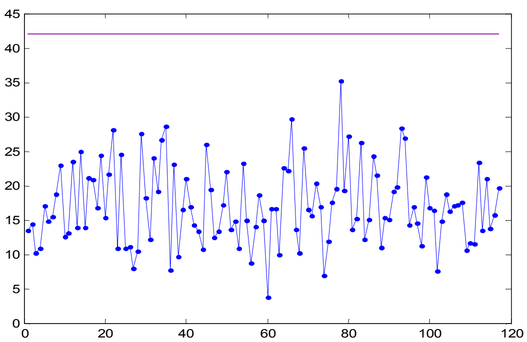

Fig. 3: Charts χ^2 for monitoring the residuals

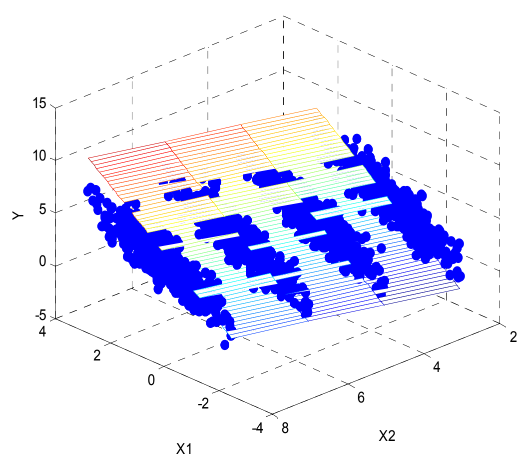

Fig. 4: View of fitted data against the actual data

finally, adopting the best performance of processes under changing situations. So for monitoring the quantitative characteristics of the first step, a graph $\bar{X}$ - R is used. For monitoring the profile parameters in second step, graph T^2 and for monitoring the residuals, graph χ^2 are used. Decision criterion is the standard average run length. According to the Eq. (1) it is assumed that the first stage qualitative features have standard normal distribution $x_1 \sim N(\mu_{x_1} = 0, \sigma^2_{x_1} = 1)$ and X_2 = [2 4 6 8] are constant with respect to the second phase assumptions of the model.

Since the research has been done in the second phase of the control chart, to control the plot, definite values for the coefficients of the profile should be considered. The values are: β_0 = 1, β_1 = 0.5 and γ = 1 under control mode. The process is considered to have a two-stage procedure as it is expressed in Eq. (1). Simulation has been made by MATLAB software with 10,000 repeats for every output of (ARL). At first, certain control limits for the control charts can be obtained with respect to the coefficients in the profiles. Then with the change in each of these coefficients it is possible that each of the graphs alerts to determine the changes. From the time of change to the time that at least one of the graphs recognizes the change is called run length. Then we will repeat this activity as many as 10,000 times to get an average run length. The same argument can be repeated for other changes in parameters. It should be noted that the type one error for each of the graphs is considered 0.0027 in this study and because there is no comparison between some control approaches, there is no need to consider a total error (type one) as 0.0027.

In addition to the items listed, in each step of the simulation other outputs can be obtained. They are given in the graph T^2 in Fig. 2 and the graph x^2 in Fig. 3.

The diagram shown in Fig. 2 is T^2 which is used to monitor profile coefficients β_0, γ, β_1 . As it can be seen there has been a change in one of the profile coefficients. At first this chart has sought to change in its sixtieth sample, then in its hundredth sample. With many simulations, number of samples between the two out-of-control ones can be obtained and then their average, gives the average run length in graph T^2.

The diagram shown in Fig. 3 is x^2 which has been used to monitor the residuals. As can be seen all the residuals of this 120 samples were under control mode. Between the actual values and the predicted values of the profile there is not any significant difference. If this chart alerts, at least one of the residuals is in out-of-control state, which means significant difference between the actual values and the predicted values of the profile.

However, before we get into the analysis of the profile coefficients, we check Fig. 4. In Fig. 4 the simulated numbers in three-dimensional form have been fitted against each other. In this diagram, the points are actual profile data with respect to the qualitative characteristic of the first stage of a process and the qualitative characteristic of the second stage of a process and the draw mesh shows the predicted values.

RESULTS

At this stage, the changes on the coefficients have been done in a way that β_0 = 1, β_1 = 0.5 and γ has changed from 0 to 2 to the size 0.05. Table 1 is defined

Table 1: Typical simulation to calculate ARL with change in γ

$\beta_0 = \&\beta_1 = 0.5$

SDRL	χ^2	T^2	R	$\bar{x}$	ARL	γ
0.870326	0.0008	0.9997	0.0077	0.0042	1.5029	0.00
0.989017	0.0007	0.9998	0.0084	0.0032	1.6102	0.05
1.148127	0.0013	0.9998	0.008	0.0049	1.7331	0.10
1.313174	0.0014	0.9994	0.007	0.0054	1.8949	0.15
1.569681	0.002	0.999	0.0109	0.0057	2.1151	0.20
1.851082	0.0019	0.9986	0.0106	0.0063	2.4173	0.25
2.225327	0.0019	0.9975	0.0118	0.0082	2.7926	0.30
2.776779	0.0026	0.9972	0.0139	0.0085	3.3187	0.35
3.521556	0.003	0.9946	0.0182	0.0099	4.0989	0.40
4.648166	0.0035	0.9884	0.0258	0.0157	5.1357	0.45
6.378787	0.0061	0.9801	0.0342	0.0157	6.9372	0.50
8.908278	0.0059	0.9614	0.0427	0.0288	9.3787	0.55
12.90161	0.0078	0.9381	0.0597	0.0375	13.3758	0.60
19.04557	0.0137	0.8778	0.0947	0.0537	19.494	0.65
27.87453	0.0209	0.8018	0.1316	0.0814	28.6233	0.70
40.8242	0.0269	0.6962	0.1927	0.1147	41.2335	0.75
54.80828	0.0334	0.5818	0.2553	0.1486	55.9128	0.80
70.70214	0.0482	0.4454	0.3287	0.1903	71.7146	0.85
81.69178	0.0576	0.3417	0.3833	0.2229	83.031	0.90
89.70223	0.0578	0.2765	0.4257	0.2455	91.2167	0.95
94.03508	0.0558	0.2576	0.4312	0.2588	93.8836	1.00
91.08521	0.0584	0.2786	0.4182	0.2493	90.5585	1.05
83.12354	0.0543	0.3399	0.3897	0.2239	83.5978	1.10
69.82313	0.045	0.4408	0.3365	0.1883	70.8991	1.15
54.27764	0.0375	0.568	0.2631	0.1500	55.5268	1.20
40.29191	0.0311	0.7025	0.1873	0.1068	41.0821	1.25
28.41613	0.0213	0.8004	0.1357	0.0753	28.6526	1.30
19.22876	0.0127	0.8829	0.0874	0.0553	19.5429	1.35
12.73306	0.0106	0.9351	0.0572	0.0381	13.3303	1.40
9.064944	0.0053	0.9649	0.0446	0.0246	9.5145	1.45
6.439476	0.0055	0.9792	0.0299	0.0190	6.9117	1.50
4.549363	0.0043	0.9889	0.0261	0.0120	5.2197	1.55
3.52011	0.0026	0.9949	0.0200	0.0100	4.0341	1.60
2.761958	0.0013	0.9977	0.0167	0.0082	3.3152	1.65
2.245368	0.0011	0.9986	0.0117	0.0069	2.7871	1.70
1.868048	0.0016	0.999	0.0105	0.0055	2.4356	1.75
1.578832	0.0018	0.999	0.0085	0.0051	2.1387	1.80
1.315524	0.0011	0.9995	0.0093	0.0063	1.8906	1.85
1.147195	0.0005	0.9997	0.008	0.005	1.7551	1.90
0.99605	0.0013	0.9994	0.0079	0.0051	1.6174	1.95
0.868572	0.0013	0.9993	0.0059	0.0039	1.5076	2.00

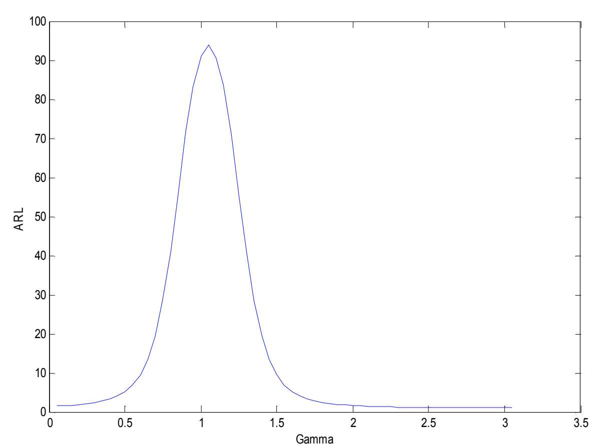

Fig. 5: Curve of average run length for changes in γ

as the best ARL when $\gamma = 1$. This state is well defined in Fig. 5.

Table 2: Typical simulation to calculate ARL with change in β_0

$\gamma = 1 \ \&\beta_1 = 0.5$

SDRL	χ^2	T^2	R	$\bar{x}$	ARL	β_0
0.492093	0.0007	0.9986	0.0043	0.0030	1.2029	0.00
0.625328	0.0005	0.9975	0.0064	0.0033	1.3066	0.05
0.78244	0.0008	0.9967	0.0077	0.0043	1.4197	0.10
1.002607	0.0004	0.9967	0.0076	0.0037	1.6126	0.15
1.249652	0.0011	0.9935	0.0076	0.0052	1.8526	0.20
1.610916	0.0016	0.9901	0.0092	0.0052	2.1863	0.25
2.133731	0.0013	0.9862	0.0130	0.0080	2.6888	0.30
2.766378	0.0022	0.9826	0.0145	0.0085	3.3603	0.35
3.850374	0.0027	0.9724	0.0205	0.0119	4.3372	0.40
5.319529	0.0050	0.9611	0.0262	0.0161	5.8672	0.45
7.405935	0.0041	0.9453	0.0350	0.0233	7.8547	0.50
10.46681	0.0088	0.9173	0.0498	0.0314	11.0033	0.55
15.14793	0.0100	0.8784	0.0741	0.0454	15.6363	0.60
22.19255	0.0124	0.8336	0.1025	0.0586	22.6325	0.65
30.89677	0.0230	0.7531	0.1493	0.0814	31.4723	0.70
44.27563	0.0275	0.6546	0.2063	0.1173	43.7413	0.75
58.62157	0.0387	0.5449	0.2702	0.1523	58.1268	0.80
72.22305	0.0492	0.4310	0.3317	0.1918	73.3339	0.85
81.80164	0.0524	0.3396	0.3927	0.2192	82.6581	0.90
94.29508	0.0635	0.2762	0.4192	0.2444	94.1105	0.95
93.15343	0.0594	0.2493	0.4464	0.2469	94.9933	100
91.58157	0.0590	0.2759	0.4225	0.2460	92.569	1.05
84.37356	0.0579	0.3255	0.3945	0.2269	84.2247	1.10
71.92771	0.0503	0.4250	0.3315	0.1990	72.9555	1.15
57.90471	0.0385	0.5477	0.2622	0.1571	58.6107	1.20
43.44442	0.0271	0.6473	0.2128	0.1183	44.1176	1.25
31.45686	0.0230	0.7471	0.1491	0.0868	32.047	1.30
22.16589	0.0113	0.8267	0.1047	0.0654	22.4464	1.35
15.32106	0.0101	0.8863	0.0744	0.0371	15.6415	1.40
10.72394	0.0062	0.9214	0.0487	0.0311	11.2406	1.45
7.495991	0.0041	0.9458	0.0372	0.0207	7.944	1.50
5.417517	0.0034	0.9638	0.0255	0.0145	5.8427	1.55
3.807253	0.0035	0.9741	0.019	0.0114	4.3663	1.60
2.784893	0.0022	0.9814	0.0146	0.0102	3.3196	1.65
2.09956	0.0014	0.9872	0.012	0.0063	2.6905	1.70
1.595567	0.0021	0.9903	0.0089	0.0074	2.1747	1.75
1.242577	0.0012	0.9925	0.0081	0.0059	1.8475	1.80
0.981617	0.0009	0.9950	0.0068	0.0049	1.6128	1.85
0.766215	0.0009	0.9964	0.0058	0.0035	1.4354	1.90
0.611295	0.0008	0.9984	0.0054	0.0040	1.2875	1.95
0.49493	0.0007	0.9985	0.0055	0.0030	1.2029	2.00

Column γ shows the change in terms of γ. It is important to note that $\gamma = 1$. In this study, is the control mode. For this reason, in the Fig. 5 for $\gamma = 1$ we have the highest ARL. Column ARL states that for each value of γ, on average, how many samples within the control charts have been drawn to observe a warning.

SDRL column is the standard deviation of the run length and other columns show the possibility of out of control range when the qualitative characteristics of the charts are controlled. For example, in Table 1, which is based on changes of γ, for line $\gamma = 0$ since the amount of γ was so much under the control of one, thus T^2 diagram is likely to be sensitive to these changes. This change is detected in the first sample with probability of 0.9997, but graph R show the error of out of control state with probability of 0.0077.

As it is shown in Fig. 5 with increase or decrease in the amount of γ an ARL value decreases. It means in case of $\gamma \leq 0.15$ and $\gamma \geq 0.85$ the figures will most likely show this change in the first example.

In the next step we perform changes on the β_0 coefficient. It means $\beta_1 = 0.5$ and $\gamma = 1$ and β_0 has changed from 0 to 2 to the size 0.05. In Table 2 the best

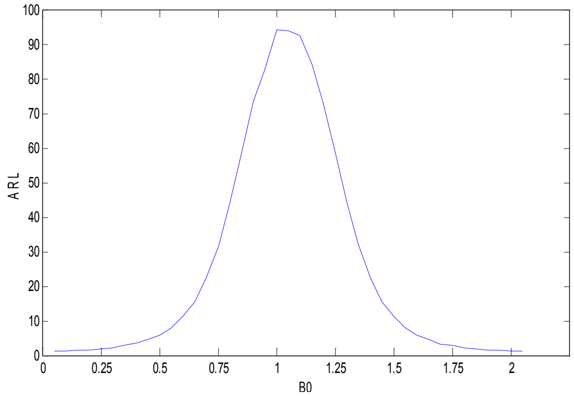

Fig. 6: Curve of average run length for changes in β_0

ARL is associated with β_0 = 1. This state is well defined in Fig. 6.

Column β_0 shows the change in terms of β_0 . It is important to note that in this study, β_0 = 1 is the control mode. For this reason, in Fig. 6 for β_0 = 1 we have the highest ARL.

Column ARL states that for each value of β_0 , on average, how many samples within the control charts have been drawn to observe a warning.

SDRL column is the standard deviation of the run length and other columns show the probability for values outside the control limits with respect to β_0.

As it is shown in Fig. 6 with increase or decrease in the amount of β_0 an ARL value decreases. It means in

Table 3: Typical simulation to calculate ARL with change in β_1

γ=1 &β_0 = 1

SDRL	χ^2	T^2	R	$\bar{x}$	ARL	β_1
0.0000	0.0001	1.0000	0.0044	0.0026	1.0000	0.10
0.0000	0.0004	1.0000	0.0042	0.0039	1.0000	0.11
0.0000	0.0005	1.0000	0.0050	0.0030	1.0000	0.12
0.0000	0.0008	1.0000	0.0040	0.0023	1.0000	0.13
0.0000	0.0007	1.0000	0.0055	0.0024	1.0000	0.14
0.0000	0.0004	1.0000	0.0038	0.0027	1.0000	0.15
0.0000	0.0004	1.0000	0.0050	0.0021	1.0000	0.16
0.0000	0.0010	1.0000	0.0044	0.0032	1.0000	0.17
0.0000	0.0006	1.0000	0.0044	0.0024	1.0000	0.18
0.01000	0.0004	1.0000	0.0043	0.0035	1.0001	0.19
0.00000	0.0006	1.0000	0.0046	0.0019	1.0000	0.20
0.014141	0.0008	1.0000	0.0034	0.0026	1.0002	0.21
0.019997	0.0009	1.0000	0.0051	0.0033	1.0004	0.22
0.039970	0.0005	0.9999	0.0049	0.0022	1.0016	0.23
0.053776	0.0006	1.0000	0.0042	0.0028	1.0029	0.24
0.069832	0.0005	0.9999	0.0034	0.0026	1.0049	0.25
0.096502	0.0007	1.0000	0.0057	0.0036	1.0094	0.26
0.134095	0.0006	0.9999	0.0052	0.0022	1.0179	0.27
0.189790	0.0006	0.9997	0.0058	0.0023	1.0344	0.28
0.231884	0.0004	0.9998	0.0050	0.0023	1.0523	0.29
0.307245	0.0005	0.9997	0.0046	0.0038	1.0855	0.30
0.405345	0.0005	0.9988	0.0061	0.0031	1.1404	0.31
0.520897	0.0008	0.9979	0.0070	0.0022	1.2177	0.32
0.669818	0.0009	0.9970	0.0073	0.0027	1.3333	0.33
0.873079	0.0008	0.9957	0.0073	0.0040	1.5031	0.34
1.141765	0.0015	0.9944	0.0074	0.0048	1.7516	0.35
1.488408	0.0014	0.9918	0.0098	0.0046	2.0668	0.36
1.971479	0.0014	0.9877	0.0108	0.0067	2.5454	0.37
2.733581	0.002	0.9828	0.0168	0.0070	3.2934	0.38
3.732067	0.0033	0.9746	0.0205	0.0100	4.2949	0.39
5.282656	0.0039	0.9593	0.0259	0.0185	5.9249	0.40
7.880373	0.007	0.9413	0.0369	0.0217	8.3115	0.41
11.75202	0.0065	0.9164	0.0529	0.0314	12.0799	0.42
17.35722	0.0113	0.8646	0.0822	0.0480	17.9683	0.43
25.3519	0.0166	0.8005	0.1198	0.0717	25.7964	0.44
38.37339	0.0234	0.7034	0.1725	0.1065	38.2665	0.45
52.61554	0.0365	0.5942	0.2335	0.1420	53.3013	0.46
68.82238	0.0458	0.4536	0.3185	0.1860	68.0664	0.47
79.89138	0.0486	0.3479	0.3847	0.2233	81.4841	0.48
89.53527	0.0605	0.2795	0.4223	0.2399	90.3109	0.49
93.66517	0.0625	0.2517	0.4358	0.2537	93.5173	0.50
90.94306	0.0634	0.2741	0.4213	0.2438	91.9487	0.51
81.50412	0.0521	0.3554	0.3777	0.2187	81.8941	0.52
67.67616	0.0484	0.4681	0.3166	0.1724	68.4684	0.53
53.39513	0.0329	0.5857	0.2435	0.1417	53.5049	0.54
38.10852	0.0244	0.7006	0.1797	0.1011	38.1596	0.55
25.55344	0.0156	0.8011	0.1187	0.0711	26.2429	0.56
17.31968	0.0129	0.8704	0.0806	0.0439	17.6712	0.57

Table 3: Continue

11.51443	0.0085	0.9094	0.0556	0.0346	12.0104	0.58
7.876571	0.0055	0.9469	0.0324	0.0221	8.4622	0.59
5.323364	0.004	0.9624	0.0265	0.0161	5.8661	0.60
3.768155	0.0024	0.9739	0.0206	0.0117	4.3055	0.61
2.73651	0.0023	0.9815	0.0169	0.0081	3.2794	0.62
1.977529	0.0015	0.9859	0.013	0.0075	2.5192	0.63
1.444344	0.0011	0.9916	0.0083	0.0060	2.0498	0.64
1.134439	0.0017	0.9926	0.0086	0.0041	1.7412	0.65
0.873654	0.0008	0.9966	0.0067	0.0030	1.5022	0.66
0.658386	0.0007	0.9976	0.0071	0.0033	1.3263	0.67
0.532183	0.0006	0.9987	0.0051	0.0027	1.2324	0.68
0.402149	0.0012	0.9987	0.0056	0.0021	1.1421	0.69
0.324056	0.0005	0.9992	0.0072	0.0031	1.0938	0.70
0.240468	0.0005	1.0000	0.0048	0.0028	1.0546	0.71
0.18898	0.0008	0.9995	0.0056	0.0027	1.0345	0.72
0.125892	0.0008	0.9999	0.0056	0.0029	1.0159	0.73
0.102414	0.0006	1.0000	0.0041	0.0031	1.0106	0.74
0.075287	0.0004	0.9999	0.0041	0.0029	1.0057	0.75
0.047906	0.0004	0.9999	0.0048	0.0033	1.0023	0.76
0.034622	0.0007	0.9999	0.005	0.0035	1.0012	0.77
0.02645	0.0008	1.0000	0.0055	0.0026	1.0007	0.78
0.019997	0.0008	1.0000	0.0037	0.0028	1.0004	0.79
0.0000	0.0009	1.0000	0.0053	0.0022	1.0000	0.80
0.0000	0.0012	1.0000	0.0047	0.0038	1.0000	0.81
0.0000	0.0010	1.0000	0.0051	0.0017	1.0000	0.82
0.0000	0.0008	1.0000	0.0057	0.0032	1.0000	0.83
0.0000	0.0002	1.0000	0.0038	0.0025	1.0000	0.84
0.0000	0.0005	1.0000	0.0053	0.0024	1.0000	0.85
0.0000	0.0009	1.0000	0.0041	0.0022	1.0000	0.86
0.0000	0.0011	1.0000	0.0029	0.0025	1.0000	0.87
0.0000	0.0008	1.0000	0.0053	0.0035	1.0000	0.88
0.0000	0.0004	1.0000	0.0038	0.0035	1.0000	0.89
0.0000	0.0005	1.0000	0.0046	0.0025	1.0000	0.90

case of β_0 ≤0.2 and β_0 ≥1.8 the figures will most likely show this change in the first example.

In the third step we perform changes on the β_1 coefficient. It means β_0 = 1 and γ = 1 and β_1 has changed from 0.1 to 0.9 to the size

0.01. In Table 3 the best ARL is associated with β_1 = 0.5. This state is well defined in Fig. 7.

Column β_1 shows the change in terms of β_1. It is important to note that in this study, β_1 = 0.5 is the control mode. For this reason, in Fig. 7 for β_1 = 0.5 we have the highest ARL. Column ARL states that for each value of β_1, on average, how many samples within the control charts have been drawn to observe a warning.

SDRL column is the standard deviation of the run length and other columns show the probability for values outside the control limits with respect to β_1.

As it is shown in Fig. 7 with increase or decrease in the amount of β_1 an ARL value decreases. It means in case of β_1≤0.35 and β_1≥0.65 the figures will most likely show this change in the first example.

With respect to the above simulation it is recognized that among profile coefficients, the variation of coefficients of γ, β_0, β_1 are not the same and the profile coefficients are less sensitive to β_1 coefficients. But γ, β_0 almost have uniform rates of change and have the same impact on the average run length.

According to Fig. 8, as can be seen, the average run length rate due to changes of β_0 and γ is the same. However, for a change in β_1 it is totally different.

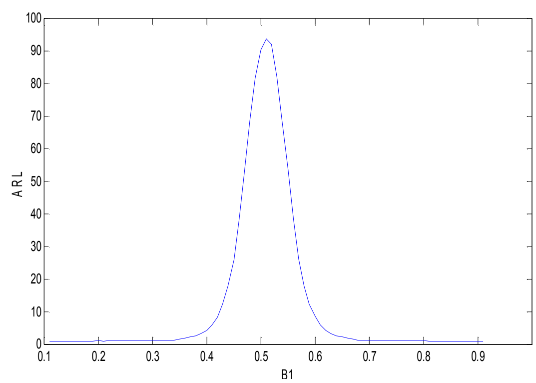

Fig. 7: Curve of average run length for changes in β_1

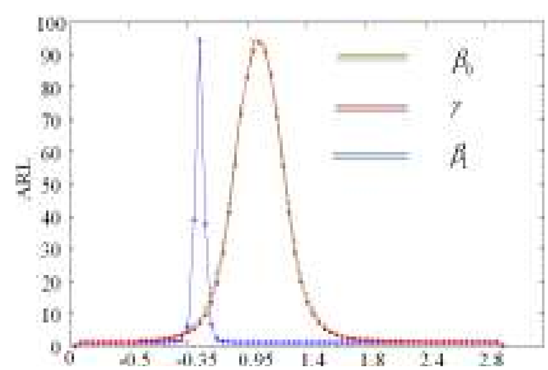

Fig. 8: Curve of coefficients of variation compared to average run lenght

This means that the qualitative characteristics of the first stage and their changes act as an intercept for the profile of the second stage. Also, control charts have different sensitivity to changes of β_1 related to β_0 and γ.

Also, with respect to the profile coefficients in control mode (β_0 = 1 and β_1 = 0.5 and γ =1) it can be seen that the worst changes are for 0.2≤γ≤1.8 and 0.2≤ β_0 ≤1.75 and 0.4 ≤ β_1 0.7.

Because graphs do not have the ability to detect these changes in the sample after the event. The output of this study is especially important in many industries that their product quality is a function of more than one phase and profile monitoring will be done in one of these phases. For example, products such as parts manufacturing, production of metals such as copper, textiles, etc. in which the product quality is not formed only in a particular stage and pre-processing steps which have an impact on the following processing steps which have the nature of the profile, is of the utmost importance.

CONCLUTION

In this study, the performance of the coefficients of a simple linear profile in monitoring a multi-stage process is being evaluated. Given that many of the processes have a few steps and such steps are often linked together, the qualitative specifications for monitoring product quality, which usually assessed in one stage does not form only on that same stage but in the different steps of the process. This subject is called cascade property in a multi-stage process, in statistical quality control. In such a case, care must be taken that this condition and not paying attention to it will cause the error in analysis of the control charts. However, in many situations, the quality of the process or the product is described by the relationship between one dependent variable and one independent variable. Thus at each stage of sampling, a set of data is collected which can be shown using a profile. In this study we examined the impact of the variation in profile coefficients on monitoring two-step process. It is understood that the variation of coefficients γ, $\beta_0$have approximately the same effect on monitoring a two-step process. However, the variation of coefficient β_1 , in narrower range, has some influence on monitoring a two-step process.

Given the originality of the topic of this research, the following cases can be considered in the future:

- To trigger the average of qualitative characteristics of the first stage and assessing the amount of this change on the rate of the average run length.
- The interaction between the existing independent qualitative characteristics in the profile equation
- Instead of using T^2 graph, using other multivariate graphs and compares the output with each other
- Assessing the performance of the coefficient of the simple linear profiles in monitoring process with more than two-step
- Assessing the performance of the coefficient of the simple linear profiles in monitoring process with two-step in a case that each step has a profile

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