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Research Article

Image De-Nosing Based on Non-Subsampled Contourlet Transform Domain in Multi-Bessel K Form Model

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Abstract: This study proposes a new image de-nosing algorithm based on Non-Subsampled Contourlet Transform (NSCT) domain in multi-Bessel k form model. Firstly, the noisy image is decomposed into a set of multi-scale and multidirectional frequency sub-bands by NSCT, according to BKF model to scale coefficient of intra-scale and inter-scale processing, fully considering correlation of internal and external scale. Lastly, the estimated coefficients are updated according to inverse non-subsampled Contourlet transformation is performed to get de-noised image. Experimental results show that out algorithm better than the other algorithms in peak signal-to-noise ratio, structural similarity and visual quality.

Keywords: Image de-nosing, multi-bessel K form model, Non-Subsampled Contourlet Transform (NSCT), Structural Similarity (SSIM)

INTRODUCTION

Image inevitably by noise pollution in the process of acquisition and transmission, noise have reduced the image resolution. Therefore, how to remove the noise effectively become the image processing of classic problem. Scholars put forward many wavelet processing image de-noising methods, for example, There is global threshold wavelet shrinkage de-noising method (Donoho, 1995), but this method makes the processed images too vague because of fixed threshold. Crouse *et al*. (1998) put forward hidden markov tree model de-noising algorithm, the algorithms' time complexity too high and noise can't achieve effective purify, Chang *et al*. (2000) is proposed based on generalized Gaussian distribution model Bayes Shrink de-noising algorithm, But lost too much high frequency coefficient lead to the de-noised effect is not ideal. With the limitation of the wavelet transform (lack of multi-directional selectivity and the sparse solution) is becoming increasingly obvious. So Multi-scale geometric analysis method appears, Contourlet transform is the most representative one (Do and Vetterli, 2005), the base function has multi-directional selectivity, can adaptive show optimal image. But as a result of Contourlet itself does not have translation invariance, so on the basis of the Contourlet appeared the Non-subsampled Contourlet Transform will be used in image de-noising and obtain better effects (Cunha *et al*., 2006).

Other image de-noising algorithms only consider intra-scale or inter-scale coefficients correlation usually result in de-noising effect is not very good. BKF model fully considering correlation of internal and external scale, Therefore, this study will be the Non-subsampled Contourlet transform and multiple BKF model combined, proposed a new image de-nosing algorithm based on Non-Subsampled Contourlet Transform (NSCT) domain in multi-Bessel k form model.

NON-SUBSAMPLED CONTOURLET TRANSFORM

Do put forward Contourlet Transform (CT) (Do and Vetterli, 2005), the transform using Laplacian Pyramid decomposition (LP) and Directional Filter group (DFB) realize a multi-resolution, multi-direction and multi-scale image representation method, But, Contourlet Transform (CT) in LP decomposition interlaced every column and every row subsampled on the image lead to Contourlet transform does not have the characteristics of translation invariance. Then, Cunha *et al*. (2006) put forward Non-Subsampled Contourlet Transform (NSCT), It is a multi-resolution, multi-scale, has the translation invariance redundant transformations compare to CT, This transformation the sampling to filter, to the signal filtering again. NSCT implementation by two major steps:

- Use the Non-subsampled pyramid filter to image multi-scale decomposition to a low-pass sub-band and a band-pass sub-band. In order to realize multistage decomposition structure, only the low frequency sub-band continue to iterative filter, finally the image decompose into a low-pass sub-band and multiple band-pass sub-band.

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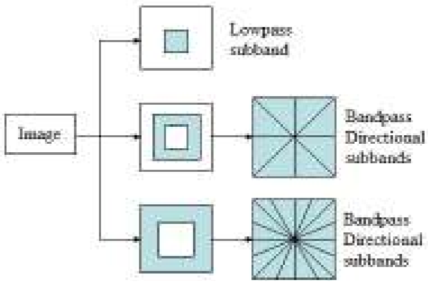


Fig. 1: Scheme of NSCT transform

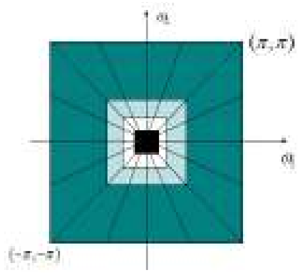


Fig. 2: Decomposed ideal frequency distribution

Figure 1 and 2 is NSCT transformation.

- At each level the Non-subsampled pyramid band-pass sub-bands direction decomposition with the Non-subsampled direction filter. The above two steps to complete the NSCT image decomposition.

Multi-BKF model: Assume that an image g by zero mean, variance is σ_n^2 Gaussian white noise n, f is the coefficients by noise. So we have the following equation:

$$f = g + n$$

After NSCT transformation we get:

$$Y = S + N \tag{1}$$

where,
Y = Wf
S = Wg
N = Wn
W = The Non-subsampled coutourlet transform operator

The spherically-contoured zero-mean d-dimensional BKF density can be written as:

$$p_s(s) = \frac{2}{(2\pi c)^{d/2}\Gamma(p)}\left(\frac{\sqrt{2c}}{\|s\|}\right)^{d/2-p} K_{d/2-p}\left(\sqrt{\frac{2}{c}}\|s\|\right) \tag{2}$$

With $s \in \mathbb{R}^d$, $K_\lambda(\mu)$ is the modified Bessel function of the second kind and c and p are the scale and shape parameters. $S = (u_i, S_i, S^p_i)$, S_i are estimate coefficients and S^p_i is S_i father, u_i are the center of the S_i 3×3 window eight wavelet coefficients. So we get:

$$\|s\| = \sqrt{\|u_i\|^2 + s_i^2 + (s_i^p)^2} \tag{3}$$

For any intra-scale wavelet coefficients y_i there is a vector y has:

$$\|y\| = \sqrt{\|v_i\|^2 + y_i^2 + (y_i^p)^2} \tag{4}$$

where,
v_i : The center of the y_i neighborhood coefficient
y_i^p : y_i father

With reference to (1), Maximum a Posteriori estimation (MAP) theory. $\hat{s}(y) = \arg max\ p_{s|y}\ (s|y)$ which is equivalent to:

$$\hat{s}(y) = \arg\max_s[\lg(p_n(y-s)) + \lg(p_s(s))] \tag{5}$$

Maximizing this expression for each component gives:

$$y_i = \hat{s}_i - \sigma_n^2 \frac{d}{d\hat{s}_i}\log p_s(\hat{s}), 1 \le i \le d \tag{6}$$

According to the literature (Khazron and Selesnick, 2008) the second term can be computed as:

$$\frac{d}{d\hat{s}_i}\log p_s(\hat{s}) = -\frac{\hat{s}_i}{\|\hat{s}\|}\sqrt{\frac{2}{c}}\frac{K_{d/2-p+1}(\sqrt{\frac{2}{c}}\|\hat{s}\|)}{K_{d/2-p}(\sqrt{\frac{2}{c}}\|\hat{s}\|)} \tag{7}$$

Therefore, the MAP estimator is:

$$y_i = \hat{s}_i\left[1 + -\frac{\sigma_n^2}{\|\hat{s}\|}\sqrt{\frac{2}{c}}\frac{K_{d/2-p+1}(\sqrt{\frac{2}{c}}\|\hat{s}\|)}{K_{d/2-p}(\sqrt{\frac{2}{c}}\|\hat{s}\|)}\right] \tag{8}$$

This estimator can be computed by successive substitution, namely, $\|\hat{s}\|^{(k+1)} = f\ \|\hat{s}\|^{(k)}$ obtain:

$$\hat{s}_i \approx \frac{y_i}{1 + \frac{\sigma_n^2}{\|y\|}\sqrt{\frac{2}{c}}\frac{K_{d/2-p+1}(\sqrt{\frac{2}{c}}\|y\|)}{K_{d/2-p}(\sqrt{\frac{2}{c}}\|y\|)}} \tag{9}$$

where,
$p = \frac{3}{Kurt(X)-3}, c = \frac{Var(X)}{p}, X$: The current estimated high frequency sub-bands
Var (X) : X variance
Kurt (X) : The kurtosis of a BKF random variable X
d : Equal to the vector y dimension

THE PROPOSED ALGORITHM

Due to the NSCT not the orthogonal transformation resulted in different directions sub-bands the noise variance is not equal, so in this study using Monte Carlo method estimated in each sub-band corresponding noise NSCT coefficients variance σ_n^2 (k). The following are the main steps:

Step 1: Compute four levels Non-Subsampled Contourlet Transform (NSCT) of noisy image.

Step 2: For each direction high frequency sub-band coefficients and combined with its neighborhood coefficients and the father coefficients, using (4) and (9).

Step 3: Estimate noise variance $\sigma_n^2(K)$ using Monte Carlo method and wavelet coefficient estimate the original image wavelet coefficients using Eq. (9).

Step 4: Compute the inverse Non-subsampled contourlet transform by estimated coefficients, get removed noise image.

SIMULATION RESULTS

In the simulation experiment, zero mean σ_n^2 variance Gaussian white noise is added to 512×512 Lena and Barbara image and we test our proposed algorithm on these images. This study first compare our de-noising method with db8 wavelet hard threshold, Do proposed Contourlet Transform (CT) with hard threshold and bayes risk minimum threshold method (BayesShrink) (Chang *et al*., 2000) and Mihcak *et al*. (1999) proposed LAWML (5×5), In order to give an objective comparison with other approaches, Peak Signal Noise Ratio (PSNR), structural similarity (Zhou *et al*., 2004) and visual effect are using as performance analysis. PSNR is defined by:

$$PSNR = 20\log_{10}\left(\frac{256}{MSE}\right)$$

And MSE is given by:

$$MSE = \sqrt{\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(x_{i,j} - y_{i,j})^2}$$

where,

X, Y : The original noisy image and de-noised image

N^2 : Image size

we give out the PSNR and de-noising image in Table 1.

From Table 1 to 3 show that our proposed method in this study better than other method and de-noising result has obvious advantages in PSNR and SSIM, From the Fig. 3 and 4, de-noising image based on this method that keep more detail information and visual effect is better. Compared with HT and CT, PSNR increased about 3 db, compared with Bayes Shrink PSNR is better. Because of this study fully considering correlation of internal and external scale, Bayes Shrink only considering the relationship of intra-scale coefficients, so our method consider more comprehensive, consequently to achieve higher PSNR, SSIM and also keep more details.

Table 1: PSNR values of de-noised Lena images for different variance

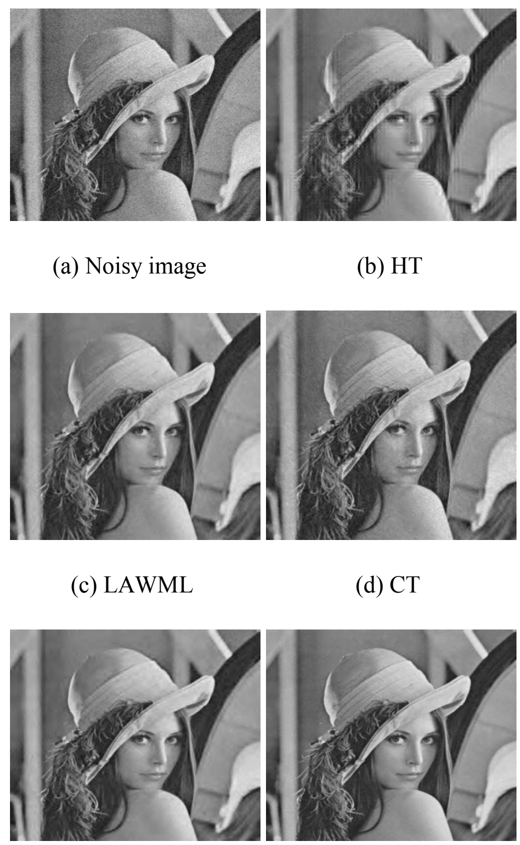
Lena	Noisy	HT	LAWML	CT	Bayes shrink	Our
$\sigma^2 = 10$	28.17	31.23	34.36	31.70	33.83	34.29
$\sigma^2 = 15$	24.64	29.28	32.24	28.84	31.29	31.66
$\sigma^2 = 20$	22.15	27.82	30.40	26.81	30.14	30.28
$\sigma^2 = 25$	20.19	26.43	29.25	25.11	29.22	29.44

Table 2: PSNR values of de-noised Barbara images for different variance

Barbara	Noisy	HT	LAWML	CT	Bayes shrink	Our
$\sigma^2 = 10$	28.17	28.73	32.59	30.92	31.90	34.19
$\sigma^2 = 15$	24.64	26.01	30.26	28.14	29.61	30.42
$\sigma^2 = 20$	22.15	25.54	27.62	26.11	27.07	27.91
$\sigma^2 = 25$	20.19	24.51	26.13	24.52	25.91	26.44

Table 3: SSIM values of de-noised Lena, Barbara images for different variance

Image	σ	HT	LAWML	CT	Bayes shrink	Our
Lena	10	0.9273	0.9625	0.9305	0.9531	0.9721
	15	0.8992	0.9390	0.8917	0.9322	0.9371
	20	0.8513	0.9136	0.8424	0.8975	0.9190
	25	0.8153	0.8903	0.7907	0.8865	0.8978
Barbara	10	0.9143	0.9642	0.9551	0.9677	0.9782
	15	0.8823	0.9454	0.9196	0.9465	0.9509
	20	0.8557	0.9110	0.8816	0.9121	0.9247
	25	0.8386	0.8919	0.8429	0.8903	0.8969



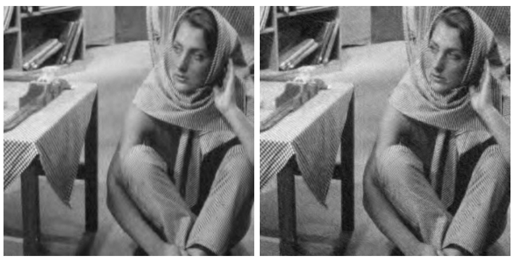
(a) Noisy image (b) HT

(c) LAWML (d) CT

(e) Bayes shrink (f) Proposed method

Fig. 3: Results of various de-noising methods of $\sigma_n^2 = 20$ Lena

(a) Noisy image (b) HT



(c) LAWML (d) CT

(e) Bayes shrink (f) Proposed method

Fig. 4: Results of various de-noising methods of $\sigma_n^2 = 20$ Barbara

CONCLUSION

This study proposes a new image de-nosing algorithm based on Non-subsampled Contourlet Transform domain in multi-Bessel k form model. First, using Non-subsampled contourlet transform advantages of translation invariance and direction, then using multi-Bessel k form correlation of internal and external scale coefficients. No matter in PSNR, SSIM and visual effect are better than many classical de-noising algorithms. But as mentioned in this study, program running time is longer and time complexity needs to be reducing.

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