Research Article
Fuzzy-Approximator-Based Adaptive Controller Design for Ship Course-Keeping Steering in Strict-Feedback Forms

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Abstract: Along with increasing marine transportation and logistics, the ship autopilot has become much important not only to lower the seaman's operating intentions, but also to reduce the seaman's deployment. It is still a challenge to design ship course-keeping controller because of ship's uncertain dynamics and time-varying environmental disturbance. This study focuses on backstepping adaptive course-keeping controller design for ship autopilot. Takagi-Sugeno (T-S) fuzzy approximator can formulate ship motion's uncertainties. Therefore, the proposed controller has no need of a priori knowledge about ship's system dynamics. Command filter can bypass the iterative differential manipulations in conventional ship course adaptive backstepping controller. The design can guarantee the ultimate uniform boundedness of the signals in closed-loop system. Finally, simulation study verifies the efficiency of the ship course-keeping design.

Keywords: Adaptive control, fuzzy system, ship course-keeping control, strict-feedback nonlinear system

INTRODUCTION

Nowadays, the large and high-speed ships have become more and more popular because of the increasing marine transportation and logistics. Thus, traffic density also increases during the past decades. To promote the economic profits, it is expected not only to lower the seaman’s operating intentions, but also to reduce the seaman’s deployment. In 1920s, classic control theories were applied to ship’s course-keeping controller design and Proportional-Integral-Derivative (PID) autopilot was invented. In 1970s, adaptive control theory was also applied. However, because of the complexities of ship’s dynamics, the randomness and unpredictability in environmental disturbances, these methods can’t handle the ship’s course control problem completely (Yang, 1999; Fossen, 1994). During recent years, kinds of the algorithms were applied to ship’s course control, such as model reference adaptive control, self-tuning control with minimal variance, neural network control, fuzzy control, variable structure control, robust control, generalized predictive control, intelligent control, etc (Roborts, 2008). Some of these algorithms had become the theoretical bases of recently developed autopilot. Furthermore, some studies had been published to study marine craft control (Roborts, 2008).

On the other hand, a great deal of attention has been received in the field of nonlinear control (Isidori, 1989; Slotine and Li, 1991). Many methods employ a synthesis approach where the controlled variable is chosen to make the time derivative of a negative definite Lyapunov candidate. The book (Krstić et al., 1995) develops the backstepping approach to the point of a step-by-step design procedure. Backstepping is a technique to control the nonlinear systems with parameter uncertainty, particularly those systems in which the uncertainties do not satisfy matching conditions. Adaptive backstepping is a powerful tool for the design of controllers for nonlinear systems in or transformable to the parameter strict-feedback form, where n ∈ ℝ, is the state, u ∈ ℝ is the control input and θ ∈ ℝ is an unknown constant vector. The adaptive backstepping approach utilizes stabilizing functions  and tuning functions  Calculation of these quantities utilizes the partial derivatives .

Motivated by the pioneering study (Farrell et al., 2009), a novel back stepping adaptive tracking fuzzy controller strategy is proposed for ship steering. The ship dynamics is formulated into a class of nonlinear system in strict-feedback form. The control objective is to force the ship’s course to track the output of the specified reference model. It is assumed that the ship motion’s dynamics are unknown. We use fuzzy logic system to approximate the unknown system functions. The proposed algorithm can guarantee the boundedness of all the signals in the closed-loop ship steering system. The main differences between our strategy and aforementioned methods (Du et al., 2007) are that, compensated tracking errors, not conventional tracking errors.
errors, are used to construct the course-keeping adaptive fuzzy controller. Then, the proposed design avoids the repeated differential of virtual control law completely, which make the controller structure quite simple and easy to implement in engineering. The simulation results demonstrate the effectiveness and usage of the proposed course-keeping controller.

**PROBLEM FORMULATION AND PRELIMINARIES**

This section will describe ship course-keeping control system modeling, some useful preliminaries and Takagi-Sugeno fuzzy system. They will be used in the later discussions.

**Ship motion modeling:** Japanese scholar Nomoto (Fossen, 1994) had put forward the 2nd-order linear mathematical model between ship’s rudder and course as follows:

$$T\dot{\psi} + \psi = K\delta$$  \hspace{1cm} (1)

where,

- $\psi$ = Ship course
- $\delta$ = Ship rudder angle
- $T$ = The ship’s straight-line stability index
- $K$ = The ship’s turning ability index

From the linearization theory of nonlinear system, only under the condition of small perturbations of ship’s motion state variables from its basis states can this linear model (1) be applied to. The basis states refers to that of constant forward speed from longitudinally middle section. If ship motion’s amplitude is very large or large rudder angle is utilized, nonlinear ship course control model should be used. Based on the Nomoto’s ship’s 2nd-order nonlinear model, Norrbin (Fossen, 1994) gave the following 2nd-order nonlinear model:

$$T\dot{\psi} + H_{Nom}(\psi) = K\delta$$ \hspace{1cm} (2)

$$H_{Nom}(\psi) = a_0\psi^3 + a_1\psi^2 + a_2\psi + a_0$$ \hspace{1cm} (3)

where, $a_0$, $a_1$, $a_2$ and $a_3$ are the uncertain coefficients. In this study, we will research the more general case, namely, it is assumed uncertain nonlinear part $H_{Nom}(\psi)$ in (2) is completely unknown. For the convenience, we introduce $x_1 = \psi$, $x_2 = \dot{\psi}$ and $u = \delta$. Then, the Norrbin model can be transformed into a class of nonlinear system in strict-feedback form as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x) + g(x)u
\end{align*}
\hspace{1cm} (4)
\]

where, $x = [x_1, x_2]^T$, $f(x) = H_{Nom}(\psi)$, and $g(x) = K/T$. In this study, we will design ship’s course-keeping adaptive fuzzy controller for the uncertain nonlinear system in the strict-feedback form in (4). The proposed adaptive fuzzy controller will guarantee ultimate uniform boundedness of the closed-loop system.

**Useful lemmas:** To proceed, the following lemmas play an important role in the manipulation of our main results on adaptive fuzzy controller design.

**Lemma 1:** (Young’s inequality) (Spooner et al., 2002) For scalar time functions $x(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$, it holds that:

$$2xy \leq \frac{x^2}{\omega} + \omega y^2$$ \hspace{1cm} (5)

for any $\omega > 0$.

**Lemma 2:** IF there exists:

$$u = \frac{AB^2}{|A|B + \varepsilon}$$ \hspace{1cm} (6)

where, $u$ is control input, $A$, $B \neq 0$, $A$, $B \in \mathbb{R}$ and $\varepsilon > 0$, then

$$Au + |A|B \leq \varepsilon$$ \hspace{1cm} (7)

will always holds.

**Proof:** Substitute (8) into (7) and we have

$$Au + |A|B \leq \frac{|A|B\varepsilon}{|A|B + \varepsilon} \leq \frac{|A|B\varepsilon + \varepsilon^2}{|A|B + \varepsilon} \leq \varepsilon$$

**Lemma 3:** (Zhou et al., 2005) Let $V: [0, \infty) \rightarrow \mathbb{R}$ satisfies the inequality:

$$V \leq -2a_0V + b_0, \quad t \geq 0,$$ \hspace{1cm} (8)

where $a_0$ and $b_0$ are positive constants. Then

$$V(t) \leq V(t_0)\exp[-2a_0(t-t_0)] + \frac{b_0}{2a_0}$$ \hspace{1cm} (9)

**Takagi-sugeno fuzzy system:** In this section, we introduce the structure of the Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno, 1985) in order to approximate unknown ship’s dynamics. T-S fuzzy rules
are a set of linguistic statements in the following form

\[ R_j : \text{IF } x_1 \text{ is } F_1^j \text{ and } x_2 \text{ is } F_2^j \text{ and } \cdots \text{ and } x_n \text{ is } F_n^j, \text{ THEN:} \]

\[ y_j = a_0^j + a_1^j x_1 + \cdots + a_n^j x_n, \quad j = 1, 2, \ldots, K, \]

where, \( a_i^j, i = 0, 1, \ldots, n \) = The unknown constants to be adapted

\( y_j \) = The output variable of the fuzzy system. In this study, it’s assumed that singleton fuzzifier and center-average defuzzifier are chosen. Then, \( f(x) \) can be expressed as the following:

\[
f(x) = \sum_{j=1}^{K} \frac{\zeta_j(x)}{\sum_{j=1}^{K} \mu_j^r(x_j)} y_j = \sum_{j=1}^{K} \zeta_j(x) y_j
\]

where,

\[
y_j = a_0^j + a_1^j x_1 + \cdots + a_n^j x_n,
\]

\[
\zeta_j(x) = \frac{\prod_{i=1}^{n} \mu_i^j(x_i)}{\sum_{j=1}^{K} \prod_{i=1}^{n} \mu_i^j(x_i)}.
\]

which is called fuzzy basis function. To proceed, the following lemma plays an important role on adaptive fuzzy controller design.

**Lemma 4:** (Universal Approximation Theorem) (Wang, 1994) Let the input universe of discourse \( U \) be a compact set in \( \mathbb{R}^n \). Then, for any given real continuous function \( h(x) \) on \( U \) and arbitrary \( d > 0 \), there exists a fuzzy system in the form of (11) such that \( \sup_{x \in U} |h(x) - f(x)| \leq d \).

Based on Lemma 4, it is well known that the aforementioned T-S fuzzy logic system is capable of uniformly approximating any well-defined nonlinear function over a compact set \( U_c \) to any degree of accuracy with triangular or Gaussian membership function.

The membership function \( \mu_{Fi}^r(x_i) \) in \( f(x) \) is denoted by some type of membership function, \( \zeta_j(x) \) is a known continuous function. So (11) can be restructured into as follows:

\[
f(x) = \zeta_j(x)A_2^j + \zeta_j(x)A_x^j x + d, \quad (13)
\]

where,

\[
\zeta_j(x) = [\zeta_j(x), \zeta_j(x), \ldots, \zeta_j(x)],
\]

\[
x = [x_1, x_2, \ldots, x_n]^T.
\]

\[
A_2^j = \left[ a_1^j, a_2^j, \ldots, a_n^j \right].
\]

\[
A_x^j = \left[ a_1^j, a_2^j, \ldots, a_n^j \right].
\]

**SHIP COURSE-KEEPING ADAPTIVE FUZZY CONTROLLER DESIGN**

Design procedures of model-reference adaptive fuzzy controller law will be presented in this section. The control objective is to steer ship course \( x_{ia} \) to track the output of the prescribed reference model and guarantee the ultimate uniform boundness for all the signals in the close-loop system.

**Step 1:** Define two tracking errors for the state \( x_1 \):

\[
x_{ia} = x_1 - x_{ic},
\]

\[
x_{ic} = \bar{x}_i - \xi_i
\]

where,

\( x_{ic} \) = The desired ship course

\( x_{ia} \) = Ship course tracking error

\( \bar{x}_i \) = The ship course’s compensated tracking errors

In this study, it is assumed that \( x_{ic} \) is continuous and has 1-order derivative:

\[
\dot{\xi}_i = -k_i \xi_i + (x_{ic} - x_{ic}^0)
\]

\[
x_{ic}^0 = \alpha_i - \xi_i
\]

where \( \xi_2 \) will be defined in the step 2, \( x_{ic} \) and \( x_{ic}^0 \) are obtained after the filtering of \( x_{ic}^0 \). \( \alpha_i \) is virtual control input, \( k_i > 0 \) is the constant to be chosen by the designer. Then, we obtain:

\[
\ddot{x}_i = \ddot{x}_2 + \alpha_i + k_i \zeta_i - \dot{x}_i
\]

Choose the following Lyapunov candidate function:

\[
V_i(t) = \frac{1}{2} \dot{x}_i^2
\]

Then, the derivative of Lyapunov candidate function (20) is given by:

\[
\dot{V}(t) = \bar{x}_i \dot{x}_i + \ddot{x}_i \alpha_i + k_i \dot{x}_i \zeta_i + \dot{x}_i \dot{x}_i
\]
We construct the following virtual control input $\alpha_1$ as follows:

$$\alpha_1 = -k_x \tilde{x}_1 - \dot{\tilde{x}}_1, \quad (21)$$

Substituting the virtual control input (21) into (20) results in:

$$\dot{V}_1(t) = -c_1 V_1(t) + \tilde{x}_1 \tilde{x}_2 \quad (22)$$

where, $c_1 = 2k_1$.

**Step 2:** Similar to Step 1, two tracking errors for $x_2$ is defined as follows:

$$\tilde{x}_2 = x_2 - x_{2e} \quad (23)$$

$$\tilde{x}_2 = \tilde{x}_2 - \tilde{\tilde{x}}_2 \quad (24)$$

where,

$\tilde{x}_2$ = Tracking error

$\tilde{\tilde{x}}_2$ = The subsequent descriptions

$x_{2e}$ = Compensated tracking error

$x_{2e}$ = Generated by the following filter

$$x_{2e}(t) = \frac{K_2}{K_2 + \kappa} \left[ x_{2e}(t) \right] \quad (25)$$

where $K_2 > 0$ is the constant to be chosen by the designer. Generally, there should be $K_2 \subseteq K_2$.

T-S fuzzy system (14) is used to approximate the unknown dynamics $f(x)$ in ship course control system. Then we obtain:

$$f(x) = \zeta(x) Q_0 x + \tilde{x}(x) Q_0 + \tilde{x}(x) \tilde{x}_1 + \tilde{\tilde{x}} \xi + d \quad (26)$$

where, $\tilde{x} = [\tilde{x}_1 \tilde{x}_2]^T$, $x_c = [x_{1c} x_{2c}]^T$, $\xi = [\xi_1 \xi_2]^T$. From (4) and (27), we obtain:

$$\tilde{x}_1 = g(x) u + \zeta(x) Q_0 \tilde{x} + \zeta(x) Q_0 \tilde{x}_1 + \tilde{x}_1 \xi + d$$

where, $\Omega$ is introduced for the reason of the convenience:

$$\Omega = \zeta(x) Q_0 \tilde{x} + \tilde{x}(x) Q_0 \tilde{x}_1 + \tilde{x}_1 \xi + d - \tilde{x}_1 \xi$$

$$\leq \|\zeta(x)\| Q_0 \tilde{x} + \|\tilde{x}(x)\| Q_0 \tilde{x}_1 + \|\tilde{x}_1\| \xi + d$$

$$\leq \|\zeta(x)\| + \|\tilde{x}_1\| \leq \chi' \beta (x)$$

Next, we introduce the definition:

$$\tilde{x}_2 = -k_2 \tilde{x}_2 + g(x)(u_c - u_c^0) \quad (29)$$

where $k_2 > 0$ is the constant to be chosen by the designer. $u_c$ is filtered from output $u_c^0$. Generally, we choose $u_c = u$.

Combining (24), (25), (28) and (29) yields:

$$\dot{V}_1(t) = -\tilde{x}_2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_2 + \tilde{\tilde{x}}_2 \tilde{\tilde{x}}_2 \quad (30)$$

Formulate the following Lyapunov candidate function:

$$V_1(t) = V_1(t) + \frac{1}{2} \tilde{x}_1^2 + \frac{1}{2} \tilde{\tilde{x}}_2$$

where $\theta = \theta^* - \theta$ and $\hat{\theta}$ are the estimated value of the adapted parameters $\theta^*$, $\Gamma > 0$ is chosen by the designer. The derivative of the Lyapunov candidate function is given by:

$$\dot{V}_1(t) = 2 \zeta(x) Q_0 \tilde{x} + \tilde{x}_2 \beta(x) + \tilde{\tilde{x}}_2 \beta(x)$$

The items $\tilde{x}_2 \zeta(x) Q_0 \tilde{x}$ and $\tilde{x}_2 \gamma(x)$ in (33) are discussed as follows, respectively. By use of Young’s inequality in Lemma 1, we obtain:

$$\tilde{x}_2 \zeta(x) Q_0 \tilde{x} \leq \frac{\sqrt{2}}{2w} \frac{\sqrt{2}}{2w} \tilde{x}_2 \zeta(x) \beta(x) \leq \frac{\sqrt{2}}{2w} \tilde{x}_2 \zeta(x) \beta(x)$$

$$\leq \tilde{x}_2 \zeta(x) \beta(x)$$

$$\leq \tilde{x}_2 \zeta(x) \beta(x)$$

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$$\leq \tilde{x}_2 \zeta(x) \beta(x)$$

where $\theta^\text{min} = \{\theta^2, \chi^*\}$ and $w > 0$ is chosen by the designer. Then by use of Lemma 2, we choose the following ship course-keeping control law:

$$\begin{align*}
\dot{x}_2 &= -k_2 \tilde{x}_2 + g(x)(u_c - u_c^0) \\
\dot{V}_1(t) &= -\tilde{x}_2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_2 + \tilde{\tilde{x}}_2 \tilde{\tilde{x}}_2 \\
\dot{V}_1(t) &= -\tilde{x}_2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_2 + \tilde{\tilde{x}}_2 \tilde{\tilde{x}}_2 \\
\dot{V}_1(t) &= -\tilde{x}_2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_2 + \tilde{\tilde{x}}_2 \tilde{\tilde{x}}_2 \\
\dot{V}_1(t) &= -\tilde{x}_2 \tilde{x}_2 + \tilde{x}_1 \tilde{x}_2 + \tilde{\tilde{x}}_2 \tilde{\tilde{x}}_2 \
\end{align*}$$
in Matlab 7.2, simulation experiments are carried out for the ship course controller design. Ship Yulong’s main particulars are as follows: design speed 14 knots, length between main particulars 126 m, breadth 20.8 m, draft 8 m, cubic coefficient 0.681, buoyant center position 0.25 meter, rudder area 18.8 m. From these parameters, \( K = 0.4343 \), \( T = 238.7592 \) could be calculated. The control objective is to force the ship course \( x_1 \) to track a reference signal \( x_{r,c} \), where is the output of the following transfer function:

\[
x_{r,c}(t) = \frac{0.0025}{s^2 + 0.08s + 0.0025} x_{r,c}(t)
\]

whose input \( x_{r,c}(t) \) is square wave with period 200 seconds and amplitude 30°.

We use total 9 IF-THEN rules to approximate the nonlinear system function \( f(x) \) in the ship steering control system. We select membership functions for ship course \( x_i \) and rate-of-turn \( x_j \) as follows:

\[
\mu_{positive}(x_i) = \frac{1}{1 + \exp[-4(x_i - \pi/4)]}
\]

\[
\mu_{zero}(x_i) = \frac{1}{\exp(-x_i^2)}
\]

\[
\mu_{negative}(x_i) = \frac{1}{1 + \exp[-4(x_i - \pi/4)]}
\]

We choose the virtual control input:

\[
a_t = -0.1i - \dot{x}_x
\]

We use the following filter:

\[
x_v(t) = \frac{10}{20} \left[ x_v(t) \right]
\]

We choose the ship course-keeping adaptive fuzzy controller as follows:

\[
u = \frac{T}{K} \left[ -4 \dot{x}_1 - 20 \dot{x}_1 \dot{\zeta}(x) \dot{\zeta}^T(x) - x_1 \right]
\]

\[
\dot{\theta}(x) \tanh \left( \frac{\dot{x}_1 \dot{\theta}(x)}{10} \right)
\]

equipped with adaptive laws:

\[
\dot{\gamma} = 2 \left[ 20 \dot{x}_1 \dot{\zeta}(x) \dot{\zeta}^T(x) + [\dot{x}_1] \beta(x) \right] - 0.05(\dot{\gamma} - 0.01) \]

\[
\dot{\dot{\gamma}} = \gamma^{'2} \gamma^'+ \gamma^{'2} \dot{\gamma} - \gamma^{''2} \gamma^'- \gamma^{''2} \dot{\gamma}
\]

Simulation Experiment

We takes Dalian Maritime University’s training ship “Yulong” as example. By use of Simulink Toolbox
CONCLUSION

In this study, ship course-keeping adaptive tracking fuzzy control scheme is proposed in the framework of the nonlinear system in strict-feedback form. T-S fuzzy system is employed to approximate the unknown dynamics. The proposed algorithm can guarantee the boundedness of all the signals in the closed-loop system. Compensated tracking errors, not tracking errors, are used to construct the controller. Based on compensated tracking error which is not the traditional tracking error, the proposed design avoids the repeated differential of virtual control law completely, which make the controller structure quite simple and easy to implement. Simulation experiment is implemented to demonstrate the effectiveness of the course-keeping control algorithm.

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