

## Research Article

### The Circular Interpolation Method of Formulating High-speed Train Diagram with Computer

Peng Yang

Department of Operations Management and Logistics, School of  
Management, Huazhong University of Science and Technology, China  
Department of Business Administration, Hunan University of Finance and  
Economics, China, Tel.: 13973169021

**Abstract:** Train diagram is the basis on which passengers make their choice on trains before going on a trip. It is directly related to the level of service to passengers. Based on passengers' travel demand and with the limitation of the starting time interval, station dwell time and other operation time intervals, this study constructs a high-speed train diagram optimization model, aiming to minimize the total expenses of passengers' travel. Meanwhile, with the goal to maximize reduction of passengers' travel expense incurred by the addition of every train to the working diagram, the study adds every train to the diagram circularly in different orders and then deletes it from the diagram so as to calculate the largest reduction of passengers' travel expense, based on which is proposed the circular interpolation method of formulating high-speed train diagram with computer. It's demonstrated through the analysis of the example that, formulating the high-speed train diagram with a computer based on circular interpolation method brings good convergence and optimization.

**Keywords:** Circular interpolation method, high-speed train, passenger flow at intervals, train diagram

## INTRODUCTION

Train diagram is the foundation for railway enterprises to manage passenger transport as well the basis for passengers to choose train before travel, therefore, its optimization is directly related to the level of service to passengers. With the increasing competition in passenger transport market and the enhancement of passengers' demand of transport service, it is now particularly important for railway enterprise to determine the start time of trains and to meet passengers' demand as much as possible according to the characteristics of passengers' travel demand and the time working standard of trains.

The optimization of train diagram is a complex and time-consuming work and considered as NP problem (Cai and Cheng, 1998). Usually, heuristic algorithm is used to work out the appropriate train diagram for practical railway networks within acceptable time. Shi (1996) proposes that the formulation of working diagram should be divided into two steps, i.e., determining the order of train operation and the stops of train, as well as the optimization of working diagram in fixed order and he also comes up with primal-dual algorithm for the optimization. Peng *et al.* (2001) puts forward the calculation method of train diagram optimization which adds railway sections constantly to the branch of railway network and gradually expands the range of working diagram, thus to work out the

diagram for the whole railway network. Zhou *et al.* (1998) uses a stratified node graph to represent railway network and represents train diagram as time sequence and then calculates the operation time of trains according to the priority level of train, the state of train at the both ends of railway block as well as the state transition of train within the railway block. Based on the discrete event model of train procession, Dorfman and Medanic (2004) propose train procession strategy according to local feedback. Xu *et al.* (2007) designs different strategies and algorithms of train diagram optimization on the basis of improved genetic algorithm. Burdett and Kozan (2010) proposed a novel hybrid job shop approach to create new train schedules. As discussed above, there are several areas where improvements would help to find a better method or solving algorithm.

This study chooses one train at a time on the order of optimization to conduct independent optimization so as to maximize the reduction of passengers' travel expense, thus, every train is optimized and added to the train diagram in a cyclic iterative way until the results of the successive two times are consistent, which is the circular interpolation method of formulating high-speed train diagram with computer. This method not only effectively reduces the difficulty of train schedule optimization, but also improves its optimization efficiency.

## DESCRIPTION OF THE OPTIMIZATION

As for high-speed rail which consists of upline and downline, train diagrams of the two lines can be optimized respectively because trains of the two lines proceeds separately without affecting each other. This study merely discusses the optimization of upline train diagram. Let the duplet set of the passenger flow OD on upline  $L = (S, E)$  be  $W = \{(r, s)\}$  and  $S = \{1, 2, \dots, l\}$  refers to the number of set of passenger station which are arrayed in order,  $E$  refers to the interval sets of the rail section and the operation time of the line in one day is  $[t_s, t_e]$ . The number of trains on this line is  $n$  and  $i = 1, 2, \dots, n$  represents the number of each train whose speed is  $v_i$ . The departure station and the terminal are  $d_i, a_i$  respectively. The sequence of stations in accordance with the proceeding direction of the train is  $Q_i$ . Let decision variable  $x_i^k$  be the starting time at which the train leaves the station while  $i$  refers to the number of train and  $k$  ( $d_i \leq k \leq a_i$ ) the number of the station and let decision variable  $y_i^k$  be the time at which the train arrives at the station while  $i$  means the number of train and  $k$  ( $d_i < k \leq a_i$ ) the number of the station.

The operation time of an arbitrary train in the railway block is the sum of additional time for start, running time and additional time for stop. Suppose the running time, additional time for start and additional time for stop of train  $i$  in railway block  $e$  ( $k, k+1$ ) are respectively  $t_i^{k,k+1}, t_i^{k,k+1}, t_i^{k,k+1}$  the stop sign of the two stations at either end of the railway block are respectively  $\delta_i^k, \delta_i^{k+1}$  ( $d_i \leq k < a_i$ ), if the train stops at station  $k$  ( $k+1$ ), then  $\delta_i^k = 1$  ( $\delta_i^{k+1} = 1$ ), otherwise,  $\delta_i^k = 0$ , ( $\delta_i^{k+1} = 0$ ), thus, the operation time of the train in the railway block can be expresses as:

$$y_i^{k+1} - x_i^k = t_i^{k,k+1} + \delta_i^k t_i^{k,k+1} + \delta_i^{k+1} t_i^{k,k+1} \quad (1)$$

$i = 1, 2, \dots, n; d_i \leq k < a_i$

In the case that trains at different speeds runs on the same line, except for the reason that passengers need to get off, a train at low speed may also stop at the station to let the high-speed train run first. For the former, the stop length is no shorter than the minimum time passengers need to get off the train in order to ensure passengers' safety. Suppose  $\tau_i^k$  stands for the minimum time passengers need to get off the train  $i$  at the station  $k \in Q_i$ , then:

$$x_i^k - y_i^k \geq \tau_i^k \quad i = 1, 2, \dots, n; k \in Q_i \quad (2)$$

As to the latter, the stop length is just no shorter than the minimum operation interval between trains so as to ensure the safety of the running train and the operating train at the station, therefore, the start time and arriving time of train  $i$  and train  $i'$  from station  $k$  toward railway block  $e$  ( $k, k+1$ ) should meet the following standard:

$$x_{i'}^k - x_i^k \geq t_{k,k+1}^d \quad \forall i \neq i' \quad (3)$$

$$\max\{d_i, d_{i'}\} \leq k < \min\{a_i, a_{i'}\}$$

$$y_{i'}^k - y_i^k \geq t_{k,k+1}^a \quad \forall i \neq i' \quad (4)$$

$$\max\{d_i, d_{i'}\} < k \leq \min\{a_i, a_{i'}\}$$

Among them,  $t_{k,k+1}^a$  represents the minimum arriving time interval between trains running from block  $e$  ( $k, k+1$ ) to station  $k+1$  and  $t_{k,k+1}^d$  refers to the minimum start time interval between trains running from station  $k$  toward railway block  $e$  ( $k, k+1$ ).

**Optimization model:** For a fixed upline train diagram, passengers' travel expense consists of the ticket, time spent on the train and in waiting for the train as well as some additional expense for setting out earlier, in which, additional expense for setting out earlier means additional costs produced because the passengers start much earlier than the train's departure time. In fact, these surcharges do not necessarily exist, especially for the travelers less demanding for travel time and its impact is not obvious. Thus, early travel surcharges are more significant as punitive cost to make the train schedule optimization more consistent throughout the day when the passengers travel and that is to meet the demand that travelers can depart punctually or close to the travel time point of departure. Not every passenger has to afford these four costs. The reason why we don't consider the passengers' train congestion costs are we previously assumed would not be limited by the train's capacity. What is worthy of note is that not every traveler has to bear the same four costs. In fact, according to different departure time of the train, their costs are different.

Considering the continuous distribution of the each OD's demand for travel time, the all-day line train operations period  $[t_s, t_e]$  is divided into smaller time periods  $G$  whose length is 1 min and denoted as  $w_1, w_2, \dots, w_G$ , respectively as Fig. 1 shows. For the smaller time period  $w_g$ , the start time is  $t_s + \delta(g-1)$ , the end of time is  $t_s + g\delta$ . The passengers' travel time for the period  $w_g$  of  $(r, s) \in F_i$  in OD refers to the middle of that period  $t_g = t_s + g\delta/2$  and the corresponding travel demand quantity is  $q_{rs}^g$ .

For the passenger stream  $q_{rs}^g$  travel in time  $t$ , the fare and travel time for train  $i$  are as follows:

$$p_{rs}^i(t_g) = \mu_i \sum_{k=r}^{s-1} l_{k,k+1} \quad (5)$$

$$b_{rs}^i(t_g) = y_i^s - x_i^r \quad (6)$$

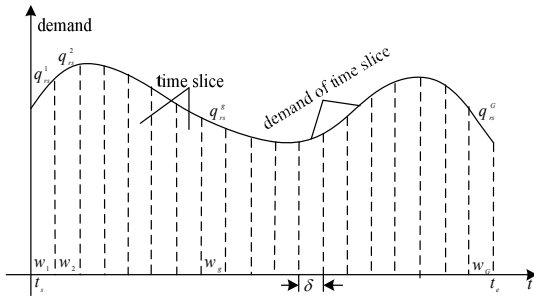


Fig. 1: The discrete distribution of the each OD's demand for travel time

Among them,  $\mu_i$  is the train fare rates, which is the unit mileage fares;  $l_{k,k+1}$  is the mileage for the line interval  $e(k, k+1)$ .

If the train's departure time is later than the passenger's travel time  $t_g$ , the additional cost the passengers afford because of the early start is 0 and the waiting time is:

$$w_{rs}^i(t_g) = x_i^r - t_g \quad (7)$$

Otherwise, the cost the passengers bear for waiting is 0, the additional cost the passengers afford because of the early start is:

$$d_{rs}^i(t_g) = \lambda \cdot (t_g - x_i^r) \quad (8)$$

To unify the 4 kinds of fare cost dimension, travel time parameter  $\alpha$  and punishment coefficient  $\beta$  are introduced. Then, passenger's total travel cost  $c_{rs}^i(t_g)$  can be expressed as:

$$c_{rs}^i(t_g) = p_{rs}^i(t_g) + \alpha \cdot (b_{rs}^i(t_g) + w_{rs}^i(t_g)) + \beta \cdot d_{rs}^i(t_g) \quad (9)$$

Passenger flow  $q_{rs}^g$  will choose the train cost least in the alternative train set  $\Omega_{rs}$ . Denote passenger flow  $q_{rs}^g$  will choose train  $i^*$  and the corresponding minimum travel cost  $c_{rs}^*(t_g)$  is:

$$c_{rs}^*(t_g) = \min \{c_{rs}^i(t_g) | i \in \Omega_{rs}\} \quad (10)$$

The common objective of the passenger train operation optimization diagram is to minimize train's operating time. In order to more fully reflect the levels of passenger travel services, this study aimed to minimize the passengers' travel cost as optimization objectives and the time the train operates in each region, stops at every station and departure time interval and arrival time interval are taken as constraints to build uplink train diagram optimization model, which goes as follows:

$$\min z = \sum_{(r,s) \in W} \sum_{g=1}^G q_{rs}^g c_{rs}^*(t_g) \quad (11)$$

$$y_i^{k+1} - x_i^k = t_i^{k,k+1} + \delta_i^k t_i'^{k,k+1} + \delta_i^{k+1} t_i''^{k,k+1} \quad (12)$$

$$i = 1, 2, \dots, n; d_i \leq k < a_i \quad (13)$$

$$x_i^k - y_i^k \geq \tau_i^k \quad i = 1, 2, \dots, n; k \in Q_i \quad (14)$$

$$x_{i'}^k - x_i^k \geq t_{k,k+1}^d \quad \forall i \neq i' \quad (15)$$

$$y_{i'}^k - y_i^k \geq t_{k,k+1}^a \quad \forall i \neq i'; \max\{d_i, d_{i'}\} < k \leq \min\{a_i, a_{i'}\} \quad (16)$$

$$t_s \leq x_i^k, y_i^k \leq t_e \quad i = 1, 2, \dots, n; d_i \leq k \leq a_i \quad (17)$$

In the model Eq. (11) to (16), the objective function of formula (11) is the minimized total travel cost for the passengers. The conditional constraint formula (12) is time constraint of the range. The conditional formula (13) is the stopping time which must be no less than the minimum time needed for the passengers to get on and off the train. The constraints of the conditional formulas (14) and (15) are the departure time interval and the arrival time interval respectively. The conditional formula (16) is the time constraint for the train's departure and arrival, which means the train's departure and arrival should be within the train's operating time period.

## THE PROPOSED ALGORITHM

The train diagram Optimization is a NP-hard problem. Considering the difficulty of solving the problem, this study chooses one train at a time on the order of optimization to conduct independent optimization so as to maximize the reduction of passengers' travel expense, thus, every train is optimized and added to the train diagram in a cyclic iterative way until the results of the successive two times are consistent. First, sort the trains according to their running mileage descending order. If there are two or more trains have the same running mileage, then sort them according to their operating speed from high to low. Thus, the optimized sequence of the trains is obtained. Denote  $o_i$  as the ranking position of the train  $i$ .

Denote the current selection of the train to be optimized is  $i$ , the existing train operating diagram  $(X, Y)$ ,  $F_i = \{(r, s)\}$  is the pair set for train  $i$  in OD collection. For passenger flow  $q_{rs}^g$  in time  $g$  of  $(r, s) \in F_i$

in OD, denote  $c_{rs}(t_g)$  as the smallest travel expenses before  $i$  is added. Then,  $i$  is added in the train operating diagram  $(X, Y)$  in time  $(X_i, Y_i)$ . If the travel expense  $c_{rs}^i(t_g)$  is lower,  $i$  is chosen to save the expenses.

Obviously, if  $c_{rs}^i(t_g) < c_{rs}(t_g)$ , the passenger  $q_{rs}^g$  will choose train  $i$  to travel. The corresponding travel costs reduced is:

$$\Delta c_{rs}^i(t_g) = c_{rs}(t_g) - c_{rs}^i(t_g) \quad (17)$$

Otherwise, if  $c_{rs}^i(t_g) \geq c_{rs}(t_g)$ , the passenger  $q_{rs}^g$  will continue to choose the original train to travel and travel costs remain unchanged. The travel cost saved is 0.

According to this method, the total amount of savings after  $i$  is added in  $F_i$  for all OD is determined and accumulated to get the total savings after  $i$  is added:

$$\Delta c^i = \sum_{(r,s) \in F_i} \sum_{g=1}^G \Delta c_{rs}^i(t_g) \quad (18)$$

Most reasonable running time for train  $i$  is to maximize the total saving  $\Delta c^i$  after  $i$  is added in  $(X, Y)$ :

$$\max \Delta c^i = \sum_{(r,s) \in F_i} \sum_{g=1}^G \Delta c_{rs}^i(t_g) \quad (19)$$

Meanwhile, all trains must meet each station's arrival time constraint (12) to (16).

Before the train  $i$  is added to train operating schedule  $(X, Y)$ , the trains depart from station  $d_i$  has divided the train operating period  $[t_s, t_e]$  into  $H$  parts, denoted as  $[t_1^f, t_2^d], [t_2^f, t_2^d], \dots, [t_H^f, t_H^d], \dots$ . Denote  $[t_{hf}^y, t_{hd}^y]$  as the train's operating time in range  $e(d_i, d_i + 1)$  at time  $h = 1, 2, \dots, H$ 's start and ending. Thus, for time  $[t_h^f, t_h^d]$ , as long as the following conditions are met,  $i$  can be added train and satisfy the departure and arrival time constraints in each range:

$$t_h^d - \max(0, t_i^e - t_{hd}^y) - t_h^f - \max(0, t_{hf}^y - t_i^e) \leq 2t_e^d \quad (20)$$

Denote  $R_i$  as the departure time domain of all time that train  $i$  can depart in.

To determine each train's time, we first need to choose  $n_i$  from  $R_i$  that satisfy the requirement of depart interval as the departure time of each train  $i$ . The specific method is to calculate the passengers travel costs savings of train  $i$  departs in  $R_i$ . Take the time  $\hat{t}_i$  the largest savings corresponds as the departure time of train  $i$ . Second, obtaining the departure and arrival time  $x_i^r$  and  $y_i^r = x_i^r + t_i^e$  of train  $i$  in the first operation

range, we will further identify the train's departure and arrival time along the line section by section. For any train  $i$ , the departure time  $x_i^{k+1}$  should not be earlier than  $y_i^r + t_i^{k+1}$  in range  $e(r+1, r+2)$  and the final decision for the time is related to the departure and arrival time the range already has. Obviously, if there is no other train running in the range, the departure time of the train will be  $y_i^r + t_i^{k+1}$ , but if there are other train running, you need to determine the smallest  $\Delta t$  to make  $x_i^{k+1} + \Delta t$  satisfy the formula (20)'s time period (which is divided according to the running trains' departure time) and then  $x_i^{k+1} + \Delta t$  will be denoted as the train's departure time. And all the departure time will be calculated in range  $e(s-1, s)$ .

According to the above method to determine the order in which train to optimize each train run time and optimize to get the trains running time is added to the schedule. At this point, the line passengers travel cost savings for that. Thereby generate the initial train diagram.

As the optimization of running time of the initial train diagram are based on certain existing train schedules, once the train schedule is changed, the corresponding passenger boarding choice and their travel expenses may also change and its original optimized quality will not be assured. The optimization adjustment to the train running time will be repeatedly made according to the sequence of the train optimization. Denote  $i$  as the current optimized train. First, delete train  $i$  from the schedule  $(X, Y)$  and  $\Delta \bar{c}^i$  as the amount of the increase in total travel cost at this time; Second, re-optimize the train  $i$ 's running time and  $\Delta c^i$  is obtained as the passengers' travel savings. Thus,  $z = z + \Delta \bar{c}^i - \Delta c^i$  is the total cost of the passengers travel in this line.

With the increase of the optimization adjustment, the magnitude of optimal adjustment of each train is getting smaller and smaller. Once the results of the former and later adjustment are the same, the optimization and adjustment will stop and the train operating schedule  $(X^*, Y^*)$  is what we are seeking for this line.

**Algorithm:** The circular interpolation method of formulating high-speed train diagram

Input the uplink  $L$ , train  $i = 1, 2, \dots, n$ , OD pair set  $W$

Output  $(X^*, Y^*)$  the schedule of the line,  $z^*$  passenger total travel costs

beginning

Determine the optimal sequence of passenger trains  $o_i (i = 1, 2, \dots, n)$

Let  $(X, Y) = \phi$  and  $z = M$  (A large enough value)

According the optimal sequence  $o_i$  (1, 2, ..., n), update  
 Optimize the train  $i$ 's running time, set  $(X, Y) = (X, Y) \cup (X_i, Y_i)$   
 Compute the passengers' travel savings  $\Delta c^i$ , let  $z = z + \Delta c^i$ , Let  $(X', Y') = (X, Y)$   
 According the optimal sequence  $o_i = 1, 2, \dots, n$  update  $(X, Y) = (X, Y) / (X_i, Y_i)$   
 Compute the amount of the increase in total travel cost  $\Delta c^{-i}$ , let  $z = z + \Delta c^{-i}$   
 Re-optimize the train  $i$ 's running time, set  $(X, Y) = (X, Y) \cup (X_i, Y_i)$   
 Compute the passengers' travel savings  $\Delta c^i$ , let  $z = z - \Delta c^i$   
 If  $(X, Y) \neq (X', Y')$ , then repeat the above cycle  
 Set  $(X^*, Y^*) = (X, Y)$ ,  $z^* = z$   
 End

### NUMERICAL EXAMPLE

The validity of the model and algorithm will be verified by the Wuhan-Guangzhou high-speed railway train diagram. The high-speed railway is made up by 17 bus terminals, 16 operating ranges, with the length of 969 km, among which every range's mileage is shown in Table 1. The operating period is 6:00-22:00o'clock. The current trains running on the line are three high-speed train from Wuhan to Northern Guangzhou, Wuhan to Southern Changsha and southern Changsha to northern Guangzhou. High-speed trains are focused when the line is designed. Some medium-speed trains are allowed to run on this line. Collinear run situation is considered in this situation. The pure time the high-speed and medium-speed trains needs for running in each range is shown in Table 1. The additional time

needed is 2 min for the high-speed trains to start and stop in each range and for the medium-speed trains the additional start time is 2 min and stop time 1 min. The minimum operating time between trains in the a station is 5 min.

The proposed method clusters the image at first and takes the information of each group to apply intensity allocation. The mechanism is to adaptively enhance the image by its own characteristic. The advantage of it is that the detailed texture can be displayed more clearly than other methods. In addition, the proposed method causes over-enhancement with lower probability than other methods, so the enhanced images are more natural than the results of other methods.

On line OD, the whole day's passenger flow is divided into 10 min time periods, which are generated randomly and whose maximum value cannot exceed 100/10 min. Passenger travel time value is 15 Yuan/h, which is unified. Passengers' early travel punitive cost factor is 1.2. Passengers' unit mileage fare rate is 0.35 Yuan/km.

The diagram of Wuhan-Guangzhou uplink high-speed railway is automatically programmed using train C # programming language algorithm in Visual Studio Net development platform. All of the train operating diagram designs are done by microcomputer with T2330/1.6 GHz basic frequency, 1G processor and memory and Windows XP operating system. Computer renderings of the high-speed train effect diagram is shown in Fig. 2.

(20, 25 and 30) trains are designated on the Wuhan-Guangzhou line and the convergence effect is shown in Fig. 3. The figure shows that the algorithm only needs 5 times iterations optimization and the train running time convergence can be achieved, which shows that the algorithm converges well.

Table 1: Some indexes of the Wuhan-Guangzhou high-speed railway

No	Beginning station	Terminal station	Distance/km	The pure time/min	
				High speed	Medium speed
1	Wuhan	Xianning North	85	16	21
2	Xianning North	Chibi North	42	8	11
3	Chibi North	Yueyang East	82	16	20
4	Yueyang East	Miluo East	70	14	17
5	Miluo East	Changsha South	68	13	17
6	Changsha South	Zhuzhou West	40	8	10
7	Zhuzhou West	Hengshan West	68	13	17
8	Hengshan West	Hengyang East	41	8	10
9	Hengyang East	Leiyang West	56	11	14
10	Leiyang West	Chenzhou West	77	15	19
11	Chenzhou West	Lechang East	83	16	20
12	Lechang East	Shaoguan	47	9	12
13	Shaoguan	Yingde West	68	13	17
14	Yingde West	Qingyuan	59	12	15
15	Qingyuan	Guangzhou North	37	7	9
16	Guangzhou North	Guangzhou South	46	9	12

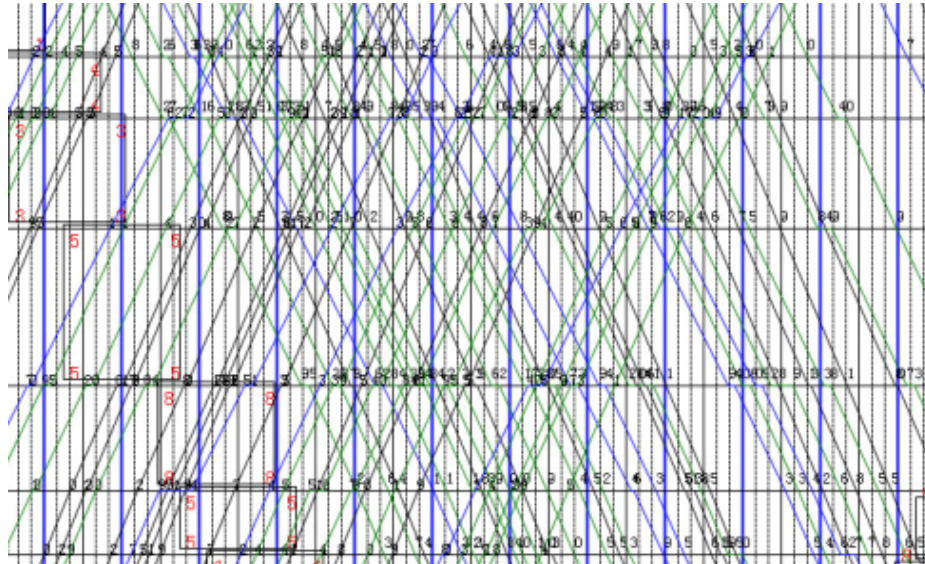


Fig. 2: Part of train operation diagram

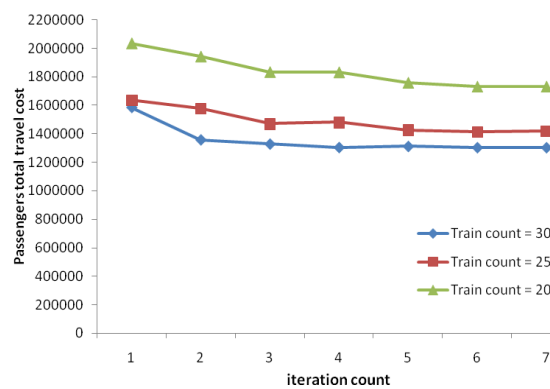


Fig. 3: Convergence performance of the algorithm

## CONCLUSION

Since the intense competition the passenger transport market faces, it is the high time for the railway enterprises to improve the quality of the design of the passenger train diagram to meet the passengers' travel requirements maximally. Based on passengers' travel demand and with the limitation of the starting time interval, station dwell time and other operation time intervals, this study constructs a high-speed train diagram optimization model, aiming to minimize the total expenses of passengers' travel. Meanwhile, according to the trains' all-day departure time in the station, OD's passenger flow is divided into multiple different lengths time periods. Every train's operating time will be determined, meeting the trains' operating time standard and considering the characteristics of passengers' travel demands with the objective of minimizing the total cost of travel and the trains' running time interval, stop time and a variety of operating

interval are the constraints for building this model thereby reducing the passenger travel expenses. Based on the initial train operation diagram, each train will be circulated in order, deleted from and added to the diagram to increase the travel savings until the two optimization results are consistent, according to which the high-speed railway cycle column interpolation will be constructed. Each period the largest target row by row to determine the train's departure time and can infer from the train in the other run sent to the time interval and access to each of the train operation time. On the basis of the initial train diagram, repeated cycles train removed from the running figure, increased passenger travel in order to obtain the amount of cost savings until the results remain the same before and after two iterations optimized far, in order to construct a computer preparation the cycle of the high-speed train diagram shown interpolation. It's demonstrated through the analysis of the example that, formulating the high-speed train diagram with a computer based on circular interpolation method brings good convergence and optimization.

## REFERENCES

- Burdett, R.L. and E. Kozan, 2010. A disjunctive graph model and framework for constructing new train schedules. *Eur. J. Oper. Res.*, 200(1): 85-98.
- Cai, X.Q. and T.C.E. Cheng, 1998. Multi-machine scheduling with variance minimization. *Discrete Appl. Math.*, 84(1): 55-70.
- Dorfman, M.J. and J. Medanic, 2004. Scheduling trains on a railway network using a discrete event model of railway traffic. *Transport. Res. B-Meth.*, 38(1): 81-98.

- Peng, Q.Y., S.N. Zhu and P. Wang, 2001. Study on a general optimization model and its solution for railway network train diagram. *J. China Railway Soc.*, 23(1): 1-8.
- Shi, F., 1996. A primal-dual algorithm for making train diagram fixed order on single-track rail lines. *J. China Railway Soc.*, 18(1): 8-20.
- Xu, H., J.J. Ma and J.C. Long, 2007. Research on the model and algorithm of the train working diagram of dedicated passenger line. *J. China Railway Soc.*, 29(2): 1-7.
- Zhou, L.S., S.J. Hu and J.J. Ma, 1998. Research on train working diagram system of railway netted lines worked out with computer. *J. China Railway Soc.*, 20(5): 15-21.