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Research Article

Learning from Adaptive Neural Control of Electrically-Driven Mechanical Systems

Yu-Xiang Wu, Jing Zhang and Cong Wang
College of Automation Science and Engineering, South China University of Technology, Guangzhou 510640, China

Abstract: This study presents deterministic learning from adaptive neural control of unknown electrically-driven mechanical systems. An adaptive neural network system and a high-gain observer are employed to derive the controller. The stable adaptive tuning laws of network weights are derived in the sense of the Lyapunov stability theory. It is rigorously shown that the convergence of partial network weights to their optimal values and locally accurate NN approximation of the unknown closed-loop system dynamics can be achieved in a stable control process because partial Persistent Excitation (PE) condition of some internal signals in the closed-loop system is satisfied. The learned knowledge stored as a set of constant neural weights can be used to improve the control performance and can also be reused in the same or similar control task. Numerical simulation is presented to show the effectiveness of the proposed control scheme.

Keywords: Adaptive neural control, deterministic learning, electrically-driven mechanical systems, high-gain observer, RBF network

INTRODUCTION

The motion tracking control of uncertain mechanical systems described by a set of second-order differential equations has attracted the interest of researchers over the years. For the mechanical systems without the actuator dynamics, many approaches have been introduced to treat the motion tracking control problem and various adaptive control algorithms have been found (Ge *et al*., 1997; Zhang *et al*., 2008; Wai, 2003; Lee and Choi, 2004; Chang and Yen, 2005; Sun *et al*., 2001; Xu *et al*., 2009; Chang and Chen, 2005). However, as pointed out by Tarn *et al*. (1991) in order to construct high-performance tracking controllers, especially in the cases of high-velocity movements and high varying loads, the inclusion of the actuator dynamics in mechanical systems was very significant (Tarn *et al*., 1991). The incorporation of the actuator dynamics into the mechanical model complicates considerably the equations of motion. In particular, the electrically-driven mechanical systems were described by third-order differential equations (Tarn *et al*., 1991) and the number of degrees of freedom was larger than the number of control inputs.

Many works addressing the tracking problem of mechanical systems with actuator dynamics have been described in Dawson *et al*. (1998), Su and Stepanenko (1996, 1998), Chang (2002) and Driessen (2006). These works were based on the integrator backstepping technique. In backstepping design procedures, regression matrix is required and the procedures will become very tedious for mechanical systems with multiple degrees of freedom. Based on the universal approximation ability of Neural Networks (NNs) and fuzzy neural networks, adaptive neural/fuzzy neural control schemes have been developed to treat the tracking control of uncertain electro-mechanical systems (Kwan *et al*., 1998; Huang *et al*., 2003, 2008; Kuc *et al*., 2003; Wai and Chen, 2004, 2006; Wai and Yang, 2008). In Kwan *et al*. (1998), two-layer NNs were used to approximate two very complicated nonlinear functions with the NN weights being tuned on-line, the designed controller guaranteed the Uniformly Ultimately Bounded (UUB) stability of tracking errors and NN weights with some conditions. In Huang *et al*. (2003), a NN controller was developed to further reduce the conditions in Kwan *et al*. (1998) for the stability. In Huang *et al*. (2008), an adaptive NN control algorithm was proposed for reducing the dimension of NN inputs. In Kuc *et al*. (2003), employing three neural networks, the designed controller implemented the global asymptotic stability of the learning control system. In Wai and Chen (2004, 2006), robust neural fuzzy network control was derived for robot manipulators including actuator dynamics, favorable tracking performance was obtained for complex robot systems. In Wai and Yang (2008), an adaptive FNN controller with only joint position information was designed to cope with the problem caused by the assumption of all system state variables

Corresponding Author: Yu-Xiang, College of Automation Science and Engineering, South China University of Technology, Guangzhou 510640, China, Tel.: 13512757116

to be measurable in Wai and Chen (2004, 2006). In the proposed adaptive neural control schemes above, NNs were used to approximate the nonlinear components in the electrically-driven mechanical systems and Lyapunov stability theory was employed to design closed-loop control systems. However, the learning ability of the approximation-based control is actually very limited and the problem of whether the neural networks employed in adaptive neural controllers indeed implement their function approximation ability has been less investigated. As a consequence, most of the adaptive neural controllers have to recalculate the control parameters even for repeating the same control task.

Recently, deterministic learning approach was proposed for identification and adaptive control of nonlinear systems (Wang and Hill, 2006, 2009). By using the localized RBF network, a partial PE condition, i.e., the PE condition of a certain regression subvector constructed out of the RBFs along the recurrent trajectory, is proven to be satisfied. This partial PE condition leads to exponential stability of the closed-loop error system which is in the form of a class of Linear Time-Varying (LTV) systems. Consequently, accurate NN approximation of the unknown closed-loop system dynamics is achieved within a local region along the recurrent trajectory. The deterministic learning approach provides an effective solution to the problem of learning in dynamic environments and is useful in many applications (Wang and Chen, 2011; Wu *et al*., 2012; Zeng *et al*., 2012; Dai *et al*., 2012).

This study addresses learning from adaptive neural control of the unknown electrically-driven mechanical systems. An adaptive neural control algorithm is proposed using RBF networks. Partial PE condition of some internal signals in the closed-loop system is satisfied during tracking control to a recurrent reference trajectory. Consequently, the convergence of partial neural weights to their optimal values and learning of the unknown closed-loop system dynamics are implemented in the closed-loop control process. The learned knowledge stored as a set of constant neural weights can be used to improve the control performance and can also be reused in the same or similar control task. Compared with back stepping scheme, the designed adaptive neural controller only uses one RBF network, which significantly reduces the complexity of controller, so that our proposed controller can be easily implemented in practice.

PROBLEM FORMULATION

Consider the following electrically-driven mechanical systems (Tarn *et al*., 1991; Dawson *et al*., 1998; Su and Stepanenko, 1996):

$$\begin{aligned} &M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}+G(q)+F(\dot{q})=K_T i \\ &L\frac{di}{dt}+R(i,\dot{q})=u \\ &y=q \end{aligned} \tag{1}$$

where, $q(t)\in R^n$ denotes the angle displacement variable vector, $M(q)\in R^{n\times}$ is the symmetric positive ndefinite inertia matrix; $V_m(q, \dot{q})\in R^{n\times n}$ is the Coriolis and centripetal forces matrix; $G(q)\in R^n$ is the gravity vector; $F(\dot{q})\in R^n$ is the dynamic frictional force vector; $i(t)\in R^n$ sis the motor armature current vector and $u(t)\in R^n$ is the control input voltage vector, $y \in R^n$ is the measurement output vector. Multi-axes mechanical systems are driven by the same motor, $K_T \in R^{n\times n}$ is the positive definite constant diagonal matrix which characterizes the electro-mechanical conversion between current and torque and $K_T = k_t\, l_{n\times n}$; $L = R^{n\times n}$ is a positive definite constant diagonal matrix denoting the electrical inductance and $L = Il_{n\times n}$; $R(I, \dot{q})\in R^n$ represents the electrical resistance and the motor back-electromotive force vector; and $M(q)$, $V_m(q, \dot{q})$, $G(q)$, K_T, L, $R(I, \dot{q})$, are all unknown. Some fundamental properties of mechanical system dynamics are stated as follows (Tarn *et al*., 1991; Dawson *et al*., 1998; Su and Stepanenko, 1996).

Property 1: The Coriolis and centripetal forces matrix can always be selected so that the matrix $[\dot{M}(q) - 2V_m(q, \dot{q})]$ is skew symmetric.

Property 2: The inertia matrix M(q) is symmetric, uniformly positive definite, for some positive constants $m_2 \geq m_1 > 0$, $m_1 I\, M(q) \leq m_2 I$, $\forall_q \in R^n$.

Let, $x_1 = q$, $x2 = q$, $\dot{x}_1 = \dot{q}$, $x_3 = i$, then Eq.1 can be transformed as follows:

$$\begin{aligned} &\dot{x}_1 = x_2 \\ &\dot{x}_2 = F_2(x_1,x_2)+G_2(x_1,x_2)x_3 \\ &\dot{x}_3 = F_3(x_1,x_2,x_3)+G_3(x_1,x_2,x_3)u \\ &y = x_1 \end{aligned} \tag{2}$$

where, $x = [x^T{}_1, x^T{}_2, x^T{}_3]^T$, $F_2 = - M^{-1}(V_m\, x_2 + G + F)$, $G2 = M^{-1} K_T$, $F_3 = - L^{-1} R$, $G_3 = L^{-1}$, ,.

Property 3*:* According to Property 2, $G_2(\cdot)$ is positive definite, symmetric and bounded, for some positive constants $g_2 \geq g_1 > 0$, $g_2 I \geq G_2(\cdot) \geq g_1 I$, $\forall x \in \Omega \subset R^{2n}$.

The objective of the paper is: given a bounded and smooth recurrent reference output $y_d(t)$, to design an adaptive neural controller using the localized RBF networks for the system (1) such that output y track the desired output y_d and both control and learning can be achiseved. It is assumed that $y_d(t)$ and its derivatives up to the 3th order are uniformly bounded and known smooth recurrent orbits.

In the following, we show that Eq. (2) can be transformed into the normal form with respect to the newly defined state variables.

Let $z_1 = y$, $z_2 = \dot{z}_2$, $z_3 = \dot{z}_2 = F_2(x_1, x_2) + G2(x_1, x_2)x_3$. The derivative of z_3 is derived as:

$$\begin{aligned}\dot{z}_3 &= \sum_{j=1}^{2}\frac{\partial F_2}{\partial x_j}\dot{x}_j + \sum_{j=1}^{2}\frac{\partial (G_2 x_3)}{\partial x_j}\dot{x}_j + G_2\dot{x}_3 \\ &= \sum_{j=1}^{2}(\frac{\partial F_2}{\partial x_j} + \frac{\partial (G_2 x_3)}{\partial x_j})\dot{x}_j + G_2(F_3 + G_3 u) \\ &\overset{\Delta}{=} F_Z(x) + G_Z(x)u\end{aligned} \tag{3}$$

where, $F_Z(x) = G_2F_3 + \sum_{j=1}^{2}((\partial F_2/\partial x_j) + (\partial(G_2x_3)/\partial x_j))\dot{x}_j$,

$G_Z(x) = G_2G_3$.

Therefore, the electrically-driven mechanical systems defined by Eq.1 can be described as the following normal form with respect to the new state variables:

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= F_Z(x) + G_Z(x)u \\ y &= z_1\end{aligned} \tag{4}$$

It should be noted that apart from the fact that functions $F_z(x)$ and $G_z(x)$ are functions of x, they are completely unknown.

Property 4: From Property 3, it is also noted that there exist constants $\bar{g}_1 \geq \bar{g}_2 > 0$ such that $\bar{g}_1 I \geq G_Z(x) \geq \bar{g}_2 I$, $\forall x \in \Omega \subset R^{3n}$.

Property 5: According to property 2, $\dot{G}_Z^{-1} = LK_T^{-1}\dot{M}$ is bounded, for a positive constant $g_3 > 0$, $\|\dot{G}_Z^{-1}(x)\| \leq g_3$, $\forall x \in \Omega \subset R^{3n}$.

Remark 1: For mechanical systems, the components of $M(q)$ are only the linear combination of constants and trigonometric function of q, so the components of $\dot{M}(q)$ are only the combination of $\dot{q}$ and trigonometric function of q, $\dot{M}$(q)q is bounded. According to $\dot{M}$(q) $\dot{M}^{-1}$(q) = l, $\dot{M}^{-1}$(q) is bounded. Gz(x) = $M^{-1}K_T L^{-1}$, so $\dot{G}_z$(x) and $\dot{G}_z^{-1}$(x) are also bounded.

ADAPTIVE NEURAL CONTROL AND LEARNING

High gain observer design: From Eq.4, we noted that z_i is incomputable. Since $F_2(x)$, $F_3(x)$, and $G_2(x)$, $G_3(x)$ are unknown nonlinear functions. The HGO used to estimate the state z_i is the same as the one in (Ge *et al*., 1999; Eehow *et al*., 2010) and is described by the following equations:

$$\begin{aligned}\varepsilon\dot{\xi}_1 &= \xi_2 \\ \varepsilon\dot{\xi}_2 &= \xi_3 \\ &\vdots \\ \varepsilon\dot{\xi}_n &= -d_1\xi_n - d_2\xi_{n-1} - \cdots - d_{n-1}\xi_2 - \xi_1 + y\end{aligned} \tag{5}$$

where, ε is a small positive design constant and parameters d_j(i = 1, … , n -1) are chosen such that the polynomial s^n + d1 s^{n-1} =+ … + $d_{n-1}$$^{s+1}$ is Hurwitz. Then, there exist positive constants h and t^* such that ∀ > t^* we have:

$$|\hat{z} - z| \leq \varepsilon h, \hat{z} = [z_1, \frac{\xi_2}{\varepsilon}, \frac{\xi_3}{\varepsilon^2}, \cdots, \frac{\xi_n}{\varepsilon^{n-1}}]^T \tag{6}$$

To prevent peaking (Khalil, 2002), saturation functions can be employed on the observer signals whenever they are outside the domain of set Ω, as follows:

$$\begin{aligned}&\xi_{i,j}^s = S_{i,j}\phi(\frac{\xi_{i,j}}{S_{i,j}}), S_{i,j} \geq \max_{z\in\Omega}(\xi_{i,j}) \\ &\phi(a) = \begin{cases} -1 & for \quad a < -1 \\ a & for \quad |a| \leq 1 \\ 1 & for \quad a > 1\end{cases}\end{aligned} \tag{7}$$

for $i = 1,2,3,\ j = 1,2,\cdots,n$.

Adaptive neural controller design: For the system defined by Eq. (4) and recurrent reference orbit y_d(t), an adaptive neural controller using RBF networks is designed as follows. Vector Y_d, E and filtered tracking error vector r are defined as:

$$Y_d = [y_d^T, \dot{y}_d^T, \ddot{y}_d^T]^T \tag{8}$$

$$E = z - Y_d \tag{9}$$

$$r = \ddot{e} + \lambda_1\dot{e} + \lambda_2 e = [K^T \quad I]E \tag{10}$$

$$e = y - y_d = z_1 - y_d \tag{11}$$

where, E = $[e^T, \dot{e}^T, \ddot{e}^T]^T$, e = y - y_d is the output tracking error vector, K = $[\lambda_2\ I,\ \lambda_1 I]^T$ is appropriately chosen such that polynomial $s^2 + \lambda_1 s + \lambda_2$ is Hurwitz:

$$\hat{E} = \hat{z} - Y_d \tag{12}$$

$$\hat{r} = [K^T \quad I]\hat{E} \tag{13}$$

Differentiating r:

$$\dot{r} = \dot{z}_3 - y_d^{(3)} + [0 \quad K^T]E = F_Z(x) + G_Z(x)u + \hat{v} \tag{14}$$

where, $\hat{v} = -y_d^{(3)} + [0 \quad K^T]\hat{E} + [0 \quad K^T]\widetilde{E}$, $\widetilde{E} = E - \hat{E}$.

Choosing the control:

$$u = -k_v\hat{r} - \hat{W}^T S(X) \tag{15}$$

where, $k_v > 0$ is the control gain, S(X) = diag(S_1(X), …, S_n(X)), $\hat{W}^T S(X) = [\hat{W}_1^T S_1(X), .., \hat{W}_n^T S_n(X)]^T$ are used to approximate the unknown functions:

$$\psi(X) = G_Z^{-1}(x)(F_Z(x) + \hat{v}) = W^{*T} S(X) + \varepsilon(X) \quad (16)$$

where, $W^{*T} S(X) = \left[W_1^{*T} S_1(X), \ldots, W_n^{*T} S_n(X)\right]^T$, X = $[x^T, \hat{v}^T]^T \in \Omega_x$ ------- R^{4n} are the NN inputs, W^* is the optimal NN weight vector and ε(X) are the NN approximation errors, with $\|\varepsilon(X)\| < \varepsilon^*$ ($\varepsilon^* > 0$), $\forall X \in \Omega_x$

The weight update law is given by:

$$\dot{\hat{W}} = \Gamma S(X)\hat{r} - \sigma\Gamma\|\hat{r}\|\hat{W} \quad (17)$$

where, $\Gamma = \Gamma^T > 0$ is a constant design matrix, $\sigma > 0$ is a small positive constant.

The overall closed-loop system consisting of systems defined by Eq.1, filtered tracking error defined by Eq. (10), the controller defined by Eq. (15) and the NN adaptive law defined by Eq. (17) can be summarized into the following form:

$$\begin{cases} \dot{r} = G_Z(x)[-k_v\hat{r} - \widetilde{W}^T S(X) + \varepsilon] \\ \dot{\hat{W}} = \dot{\widetilde{W}} = \Gamma S(X)\hat{r} - \sigma\Gamma\|\hat{r}\|\hat{W} \end{cases} \quad (18)$$

where, $\widetilde{W} = \hat{W} - W^*$, $\widetilde{W}^T S(X) = \left[\widetilde{W}_1^T S_1(X), \ldots, \widetilde{W}_n^T S_n(X)\right]^T$.

Theorem 1: Consider the closed-loop system defined by Eq. (18). For any given recurrent reference orbit starting from initial condition $y_d(0) \in \Omega_d$(Ω_d is a compact set) and with initial condition y (0) $\in \Omega_0$ (Ω_0 is a compact set) and $\widehat{W}(0) = 0$ we have that:

- All signals in the closed-loop system remain ultimately uniformly bounded.
- There exists a finite time T_1 ($T_1 > t^*$)such that the state tracking errors E= $[e^T, \dot{e}^T, \ddot{e}^T]^T$ converge to a small neighborhood around zero for all $t \geq T_1$ by appropriately choosing design parameters.

Proof:

- **Consider the following Lyapunov function:**

$$V_w = \hat{W}^T \Gamma^{-1} \hat{W} / 2 \quad (19)$$

Differentiating V_w:

$$\begin{aligned} \dot{V}_w &= \hat{W}^T \Gamma^{-1} \dot{\hat{W}} = \hat{W}^T (S(X)\hat{r} - \sigma\|\hat{r}\|\hat{W}) \\ &\leq -\|\hat{W}\|\|\hat{r}\|(\sigma\|\hat{W}\| - \|S(X)\|) \end{aligned} \quad (20)$$

Thus, it follows that if $\|\widehat{W}\| > s^* / \sigma$, then $\dot{V}_w > 0$ s^* is the upper bound of $\|S(X)\|$ (see reference literature (Slotine and Li, 1991). This leads the UUB of $\|\widehat{W}\|$ as $\|\widehat{W}\| \leq s^* / \sigma$. According to $\widetilde{W}$ $\widehat{W}$ = W^*,we have that $\widetilde{W}$ is UUB as follows:

$$\|\widetilde{W}\| \leq \|W^*\| + \|\hat{W}\| \leq \|W^*\| + s^* / \sigma =: \widetilde{W}^* \quad (21)$$

Take the Lyapunov function $V_r = r^T G^{-1}{}_z r/2$. Differentiating V_r, we have:

$$\begin{aligned} \dot{V}_r &= -r^T k_v r + r^T (k_v (r - \hat{r}) - \widetilde{W}^T S(X) + \varepsilon) + \frac{1}{2} r^T \dot{G}_Z^{-1} r \\ &\leq -k_{v1}\|r\|^2 + \|r\|(k_v c_\lambda \varepsilon h + \varepsilon^* + \widetilde{W}^* s^*) \end{aligned} \quad (22)$$

where, $k_{v1} = k_v - g_3 / 2$, $c_\lambda = \left\|[K^T \quad I]\right\|$. Note that the equality $\widetilde{E} = \widetilde{z}$ can be easily induced from Eq.9 and Eq. (12) as follows:

$$\widetilde{E} = E - \hat{E} = z - \hat{z} = \widetilde{z} \leq \varepsilon h \quad (23)$$

Let $c = \widetilde{W}^* s^* + \varepsilon^*$, Eq. (22) is further derived as:

$$\dot{V}_r \leq -k_{v1}\|r\|(\|r\| - c_\lambda \varepsilon h - c / k_v) \quad (24)$$

This implies that the filtered tracking error vector r is UUB as follows:

$$\|r\| \leq c_\lambda \varepsilon h + c / k_v \quad (25)$$

Whose boundary can be made small enough by increasing the control gain k_v and decreasing ε.

Because r = $\ddot{e}$ + $\lambda\dot{e}$ + λ_2 e is stable by appropriately choosing design parameters λ_1, λ_2 and y_d, $\dot{y}$d, $\ddot{y}_d$ are bounded, then z_1, z_2, z_3 are bounded. S(X) is bounded for all values of X, we conclude that control u is also bounded. Thus, all the signals in the closed-loop system remain ultimately uniformly bounded.

- **Consider the following Lyapunov function:**

$$V_r = r^T G_Z^{-1} r / 2 \quad (26)$$

The derivative of V_r is:

$$\begin{aligned} \dot{V}_r &= r^T \dot{r} + r^T \dot{G}_Z^{-1} r / 2 \\ &\leq -r^T k_v r + r^T k_v [K^T \quad I]\widetilde{z} + r^T \varepsilon - r^T \widetilde{W}^T S(X) + r^T \dot{G}_Z^{-1} r / 2 \end{aligned} \quad (27)$$

Let, $k_v = 2\bar{k}_{v1} + 3\,\bar{k}_{v2} + g_3/2$, using the inequality $2\alpha^T \beta \leq \eta\alpha^T \alpha + (1/\eta)\beta^T \beta$ ($\eta > 0$), we have:

$$r^T \varepsilon \leq \varepsilon^{*2} / (4\bar{k}_{v2}) + \bar{k}_{v2}\|r\|^2 \quad (28)$$

$$-r^T \widetilde{W}^T S(X) \leq \widetilde{W}^{*2} s^{*2} / (4\bar{k}_{v2}) + \bar{k}_{v2}\|r\|^2 \quad (29)$$

$$r^T k_v [K^T \quad I]\widetilde{z} \leq \bar{k}_{v2}\|r\|^2 + \bar{c}_\lambda^2 / (4\bar{k}_{v2})$$
(30)

where, $\bar{c}\lambda = k_v c_\lambda \|\widetilde{z}\| = O(\varepsilon h)$, s^* is the upper bound of $\|S(X)\|$, $\widetilde{W}^*$is the upper bound of $\|\widetilde{W}\|$which is given in Eq. (21). Then Eq. (27) becomes:

$$\dot{V}_r \leq -r^T \bar{k}_{v1} r + (\widetilde{W}^{*2} s^{*2} + \varepsilon^{*2} + \bar{c}_\lambda^2) / (4\bar{k}_{v2}) \quad (31)$$

Let, $\delta = (\widetilde{W}^{*2} s^{*2} + \varepsilon^{*2} + \bar{c}^2\lambda)/ (4\bar{k}_{v2})$, it is clear that δ can be made small enough using large enough k_v, so we have:

$$\dot{V}_r \leq -r^T \bar{k}_{v1} r + \delta \leq -2\bar{k}_{v1}\bar{g}_2 V_r + \delta \tag{32}$$

Let $c = \bar{k}_{v1}\bar{g}_2, p = \delta / 2_c > 0$ then V_r satisfies:

$$0 \leq V_r(t) < p + (V_r(0) - p)\exp(-2ct) \tag{33}$$

that is:

$$r^T r < 2\bar{g}_2 p + 2\bar{g}_2 V_r(0)\exp(-2ct) \tag{34}$$

The above equation implies that given $\beta > \sqrt{2\bar{g}_2 p} = \sqrt{\delta / \bar{k}_{v1}}$, there exists a finite time T_1, determined by δ and $\bar{k}_{v1}$, such that for all $t \geq T_1$, the filtered tracking error r satisfy:

$$\|r\| < \beta \tag{35}$$

where, β is the size of a small residual set that can be made small enough by appropriately choosing $\bar{k}_{v1}, \bar{k}_{v2}$. By choosing a large k_v, the filtered tracking error r can be made small enough $\forall t \geq T_1$ That is to say, there exists a T_1 such that the state tracking errors $E = [e^T, \dot{e}^T, \ddot{e}^T]^T$ converge to a small neighborhood around zero for all $t \geq T_1$ by appropriately choosing design parameters (Slotine and Li, 1991)' so that the tracking states $z(t)|_{t\geq T1}$. follow closely to $Y_d(t)|_{t\geq T1}$.

Remark 2: Theorem 1 indicated that the system orbits $z(t)$ will become as recurrent as $Y_d(t)$ that after time $T_1(T_1 > t^*)$, so x(t) will also become recurrent. $\hat{\upsilon}$ converges to a small neighborhood around y(3)d $y_d^{(3)}$ after $T \geq T_1$, which indicates that $\hat{V}$ is as recurrent as $y^{(3)}_d$. Since $X = [x^T, \hat{\upsilon}^T]^T$are selected as the RBF networks inputs, according to theorem 2.7 in Wang and Hill (2009), S(X) will satisfy the partial PE condition, i.e., along $Y_d(t)|_{t\geq T1}$., $S_\xi(X)$ satisfies the PE condition.

Remark 3: The key aspect of the proposed method is that electrically-driven mechanical systems are transformed into the affine nonlinear system in the normal form with state transformation. Thus, learning and stability analysis avoid using virtual control terms and their time derivatives, which require complex analysis and computing. In our proposed approach, only one RBF network is employed to approximate the unknown lumped system nonlinear dynamics, which shows the superiority of our proposed learning control scheme.

Learning from adaptive neural control stability of a class of LTV systems: For deterministic learning from adaptive neural control of nonlinear systems with unknown affine term, the associated LTV system is extended in the following form Liu *et al*. (2009):

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{\eta} \end{bmatrix} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{A(t)} & \begin{matrix} 0 \\ S^T(t) \end{matrix} \\ \hline 0 & -\Gamma S(t)G(t) & 0 \end{array} \right] \begin{bmatrix} e_1 \\ e_2 \\ \eta \end{bmatrix} \tag{36}$$

where, $e_1 \in R^{(n-q)}, e_2 \in R^q, \eta \in R^p$, $A(\cdot):[0,\infty) \to R^{n\times n}$, $S(\cdot):[0,\infty) \to R^{p\times q}$, $G(\cdot):[0,\infty) \to R^{q\times q}$ and $\Gamma = \Gamma^T > 0$

Define $e := [e_1^T, e_2^T]^T, B(t) := [0 \quad S(t)] \in R^{p\times n}$, $H(t) := block\text{-}diag\{I \quad G(t)\} \in R^{n\times n}$, where diag refers to block diagonal form and $C(t) := \Gamma B(t)H(t)$.

Assumption 1: (Loría and Panteley, 2002). There exists a $\emptyset_M$ < 0such that, for all $t \geq 0$, the following bound is satisfied:

$$\max\left\{ \|B(t)\| \quad , \quad \left\| \frac{dB(t)}{dt} \right\| \right\} \leq \phi_M \tag{37}$$

Assumption 2: (Liu *et al*., 2009). There exist symmetric matrices P((t) and Q(t) such that $-Q(t) = P(t) + A(t)\dot{P}(t) + A^T(t)P(t)$. Furthermore, $\exists p_m, q_m, p_M$ and $q_M > 0$ such that, $p_m I \leq P(t) \leq p_M I$ and $q_m I \leq Q(t) \leq q_M I$.

Lemma 1: (Liu *et al*., 2009). With assumption 3.1 and 3.2 satisfied in a compact set Ω, system defined by Eq.36 is uniformly exponentially stable in the compact set Ω if S(t) satisfies the PE condition.

Learning from adaptive neural control : Using the localization property of RBF network, after time T_1, system defined by Eq.18 can be expressed in the following form along the tracking orbits $X(t)|_{t\geq T1}$. as:

$$\begin{cases} \dot{r} = G_Z(x)[-k_v\hat{r} - \widetilde{W}_\xi^T S_\xi(X) + \varepsilon_\xi] \\ \dot{\hat{W}}_\xi = \dot{\widetilde{W}}_\xi = \Gamma_\xi S_\xi(X)\hat{r} - \sigma\Gamma_\xi \|\hat{r}\| \hat{W}_\xi \end{cases} \tag{38}$$

$$\dot{\hat{W}}_{\bar{\xi}} = \dot{\widetilde{W}}_{\bar{\xi}} = \Gamma_{\bar{\xi}} S_{\bar{\xi}}(X)\hat{r} - \sigma\Gamma_{\bar{\xi}} \|\hat{r}\| \hat{W}_{\bar{\xi}} \tag{39}$$

where, $\widetilde{W}_\xi^T S_\xi(X) = \left[\widetilde{W}_{1\xi}^T S_{1\xi}(X), \ldots, \widetilde{W}_{j\xi}^T S_{j\xi}(X)\right]^T$, $S_\xi(X)$ is a subvector of S(X); $\hat{W}_\xi$ is the corresponding weight subvector; the subscript $\bar{\xi}$ stands for the region far away from the trajectories $X(t)|_{t\geq T1}$.; ε_ξ are the local approximation errors, $\|\varepsilon_\xi\|$ is small.

Theorem 2: Consider the closed-loop system defined by Eq. (38). For any given recurrent reference orbit starting from initial condition $y_d(0) \in \Omega_d$ and with initial condition $y_d(0) \in \Omega_d$, $\hat{W}(0) = 0$ and control parameters appropriately chosen, we have that

Along the tracking orbits $X(t)|_{t \geq T}$, neural weight estimates $\hat{W}_\xi$ converge to a small neighborhood of the optimal values W_ξ^* and locally accurate approximation of the unknown closed-loop system dynamics $\psi(X)$ are obtained by $\hat{W}^T S(Z)$ and $\overline{W}^T S(Z)$, where $\overline{W}$ is obtained from:

$$\overline{W} = \underset{t \in [t_a, t_b]}{mean}(\hat{W}(t)) \tag{40}$$

where, $[t_\alpha, t_b]$ ($t_b > t_\alpha > T$, T_1) represents a time segment after transient process of $\hat{W}$.

Proof: Let $\theta = G^{-1}_z(x)r$, and $\eta = \tilde{W}_\xi$, then system defined by Eq.38 is transformed into:

$$\begin{cases} \dot{\theta} = -[k_v G_Z(x) - \dot{G}_Z^{-1}(x) G_Z(x)]\theta \\ \quad -\eta^T S_\xi(X) + \varepsilon_\xi(X) + k_v(r - \hat{r}) \\ \dot{\eta} = \Gamma_\xi S_\xi(X) G_Z(x)\theta - \sigma\Gamma_\xi \|\hat{r}\| \hat{W}_\xi \end{cases} \tag{41}$$

Rewrite Eq. (41) in matrix, we have:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} A(t) & B(t) \\ -C(t) & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \eta \end{bmatrix} + \begin{bmatrix} \varepsilon_\xi + k_v(r - \hat{r}) \\ -\sigma\Gamma_\xi \|\hat{r}\| \hat{W}_\xi \end{bmatrix} \tag{42}$$

Because $\|k_v(r-\hat{r})\| \leq k_v c_\lambda(\varepsilon h) = O(\varepsilon h)$, so $\|\varepsilon_\xi + k_v(r-\hat{r})\|$ and $\|\sigma\Gamma_\xi \|\hat{r}\| \hat{W}_\xi\|$ are small, system defined by Eq.42 can be considered as a perturbed system (Wang and Chen, 2011).

where,

$$A(t) = -k_v G_Z(x) + \dot{G}_Z^{-1}(x) G_Z(x) \tag{43}$$

$$B(t) = -S_\xi^T(X) \tag{44}$$

$$C(t) = -\Gamma_\xi S_\xi(X) G_Z(x) \tag{45}$$

Introducing $P(t) = G_z(x)$, we have:

$$\dot{P} + PA + A^T P = -2G_Z(k_v - \dot{G}_Z^{-1})G_Z + \dot{G}_Z \, . \tag{46}$$

The satisfaction of Assumption 3.1 can be easily checked. With $G_z(.)$ and $\dot{G}_z(.)$,$\dot{G}^{-1}{}_z(.)$,being bounded, k_v can be designed such that $2G_z(k_v = \dot{G}_z^{-1})Gz = \dot{G}_z$ is strictly positive definite and the negative definite of $\dot{P}$ + PA + $A^T P$, is guaranteed. Thus, Assumption 2 is satisfied.

After time T_1, the NN inputs X(t) follow recurrent orbits and the partial PE condition (Wang and Hill, 2006, 2009) can be satisfied by the regression subvector $S_\xi(X)$, which consists of RBF networks with centers located in a small neighborhood of the tracking orbits $X(t)|_{t \geq T1}$. Thus, uniformly exponentially stability of the nominal system of system defined by Eq. (42) is guaranteed by Lemma 1. For the perturbed system defined by Eq. (42), using Lemma 4.6 in Khalil (2002), the parameter errors $\eta = \tilde{W}_\xi$ converge exponentially to a small neighborhood of zero in a finite time $T(T > T_1)$, with the size of the neighborhood being determined by the NN approximation ability and state tracking errors.

The convergence of $\hat{W}_\xi$ to a small neighborhood of W^*_ξ implies that along the tracking trajectories $X(t)|_{t \geq T}$, the unknown closed-loop system dynamics $\psi(X)$ can be represented by regression subvector $S_\xi(X)$ with small error, i.e.,:

$$\psi(X) = \hat{W}_\xi^T S_\xi(X) + \varepsilon_\xi \tag{47}$$

where, $\hat{W}_\xi^T = [\hat{w}_{j1}, \cdots, \hat{w}_{j\xi}]^T$ is the subvector of $\hat{W}$, $\hat{W}_\xi^T S_\xi(X) = \left[\hat{W}_{1\xi}^T S_{1\xi}(X), \ldots, \hat{W}_{j\xi}^T S_{j\xi}(X)\right]^T$ and $\left|\|\varepsilon_\xi\| - \varepsilon^*\right|$ is small. Choosing $\overline{W}$ according to $\overline{W} = \underset{t \in [t_a, t_b]}{mean}(\hat{W}(t))$, Eq. (47) can be expressed as:

$$\psi(X) = \overline{W}_\xi^T S_\xi(X) + \bar{\varepsilon}_\xi \tag{48}$$

where, $\overline{W}$ $\overline{W}_\xi^T = [\overline{w}_{j1}, \cdots, \overline{w}_{j\xi}]^T$ Tξ = is the subvector of $\overline{W}$, $\overline{W}_\xi^T S_\xi(X) = \left[\overline{W}_{1\xi}^T S_{1\xi}(X), \ldots, \overline{W}_{j\xi}^T S_{j\xi}(X)\right]^T$ and $\bar{\varepsilon}_\xi$ are errors using $\overline{W}^T{}_\xi$ $S_\xi(X)$to approximate the unknown closed-loop system dynamics. After the transient process, $\left\|\bar{\varepsilon}_\xi\right\| - \left\|\varepsilon_\xi\right\|$ is small.

According to the localization property of RBF network, both $S_{\bar{\xi}}(X)$ and $\overline{W}_{\bar{\xi}}^T S_{\bar{\xi}}(X)$ are very small along the tracking trajectories $X(t)|_{t \geq T}$. This means that the entire RBF networks $\overline{W}^T S(X)$ can approximate the unknown functions $\psi(X)$ along tracking trajectories $X(t)|_{t \geq T}$, as:

$$\begin{aligned} \psi(X) &= \overline{W}_\xi^T S_\xi(X) + \overline{W}_{\bar{\xi}}^T S_{\bar{\xi}}(X) + \varepsilon_1 \\ &= \overline{W}^T S(X) + \varepsilon_2 \end{aligned} \tag{49}$$

where, $\overline{W}^T S(X) = \left[\overline{W}_1^T S_1(X), \ldots, \overline{W}_n^T S_n(X)\right]^T$, $\|\varepsilon_2\|$ - $\|\varepsilon_2|$ |is small. It is seen that $\overline{W}^T S(X)$ can approximate the unknown nonlinear functions $\psi(X)$ along the tracking trajectories $X(t)|_{t \geq T}$,.

Remark 4: Theorem 2 reveals that deterministic learning (i.e., parameter convergence) can be achieved during tracking control to a recurrent reference orbit. The learned knowledge can be stored in the constant RBF networks $\bar{W}^T S(X)$, but it is generally difficult to represent and store the learned knowledge using the time-varying neural weights. Through the deterministic learning, the representation and storage of the past experiences become a simple task.

SIMULATION

A single-link robotic manipulator coupled to a DC motor is considered. The dynamic equations of the system are:

$$\begin{aligned} &M\ddot{q} + B\dot{q} + N\sin(q) = K_T I \\ &L\dot{I} + RI + K_B\dot{q} = u \end{aligned} \tag{50}$$

where, $M = J + \frac{1}{3}mL_0^2 + \frac{2}{5}M_0L_0^2R_0^2, N = mgL_0 + M_0gL_0$

q = The angular position
J = The inertia of the actuator's rotor
L_0 = The length of the link
m = The mass of the link
M_0 = Payload mass
R_0 = The radius of the payload
g = The gravitational constan
K_T = The torque constant
K_B = The back-EMF constant
R = The armature resistance
L = The armature inductance
I = The armature current
u = The armature voltage

Eq. (50) can be expressed in the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u \end{aligned} \tag{51}$$

where, $f_2 = - M^{-1}(Bx_2 + N \sin_{x1})$, $g_2 = M^{-1} K_T$, $f_3 = - L$-1, $(Rx_3 + K_Bx_2)$, $g3 = L^{-1}$.

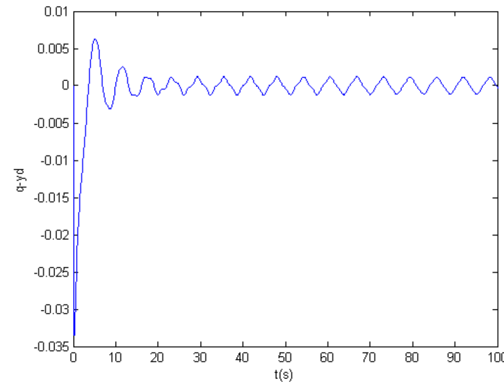
The initial state values are $x_1(0) = 0x_2(0)$ and $x_3(0) = 0$ and the desired output is set to $y_d = 0.8\text{sint}$.

Our proposed controller design procedure is summarized as follows. First, define the three-order HGO as:

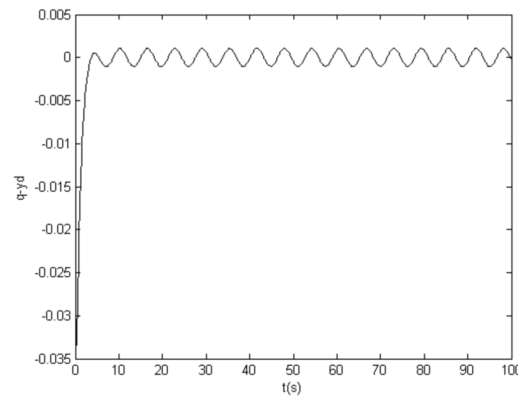
$$\begin{aligned} \varepsilon\dot{\xi}_1 &= \xi_2 \\ \varepsilon\dot{\xi}_2 &= \xi_3 \\ \varepsilon\dot{\xi}_3 &= -d_1\xi_3 - d_2\xi_2 - \xi_1 + y \end{aligned} \tag{52}$$

Further, the state estimation vector and error vector are $\hat{\mathbf{z}} = [z_1, \frac{\xi_2}{\varepsilon}, \frac{\xi_3}{\varepsilon^2}]^T$ and $\hat{E} = \hat{\mathbf{z}} - Y_{\mathbf{d}}$. The control input is determined as:

$$u = -K_v\hat{r} - \hat{W}^T S(X), \quad X = [x_1, x_2, x_3, \hat{v}]^T \tag{53}$$



(a)



(b)

Fig. 1: Output tracking error; (a): q tracking error using $\hat{W}^T$S(X); (b): q tracking error using $\bar{W}^T$S(X)

where, $\hat{v} = -\ddot{y}_d + [0 \;\; \lambda_1 \;\; \lambda_2]\hat{E} + [0 \;\; \lambda_1 \;\; \lambda_2]\tilde{E}$ and $\hat{r} = [\lambda_1 \;\; \lambda_2 \;\; 1]\hat{E}$.
The neural weight is updated by:

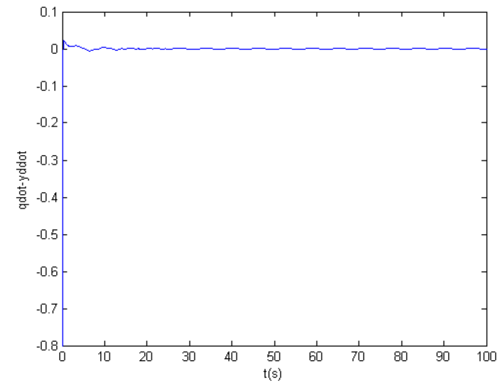
$$\dot{\hat{W}} = \Gamma S(X)\hat{r} - \sigma\Gamma\|\hat{r}\|\hat{W} \tag{54}$$

The parameters of the single-link robotic system (Huang *et al.*, 2008) are J = 1625 × 10^{-3}(kgm^2), R = 5.0(Ω), K_T = 0.9(Nml A), K_B = 0.9(Nml A) K_B = 0.9(Nml A), B = 1625 × 10^{-3} (Nml/rad), L_0 = (0.305(m)), m = 0.596(kg), M0 = 0.434(kg) M_0 = 0.434(kg), R_0 = 0.023(m) and L = 0.025(H)

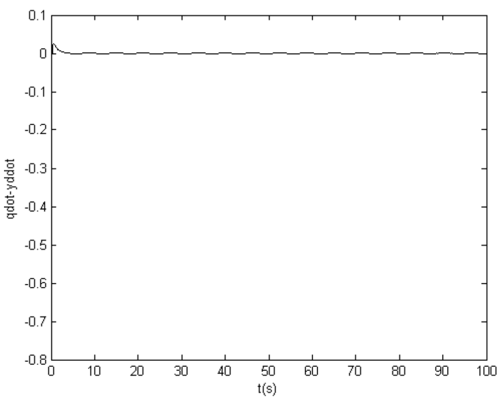
The design parameters of the above controller are K_v = 50, $\lambda_1 = \lambda_2$ = 25, Γ = diag[20], , σ = 0.0001. The HGO parameters are d_1 = 3, d_2 = 3 , $\xi(0) = [0, 0, 0]^T$ and ε_λ = 1/100. The

RBF networks $\hat{W}^T$S(Z) contain 5 × 5 × 7× 11 = 1925 nodes (i.e., N = 1925), the centers C_i(I = 1, 2, … , N) are evenly spaced on [-1.5 1.5] [-1.5 1.5] × [-3.5 3.5] × [-25 25], with width η_i = 0.75, $\hat{W}(0) = 0$.

Figure 1 to 5 show the simulation results. The tracking performance of system is shown to be good in Fig. 1 and 2 and the tracking performance become



(a)



(b)

Fig. 2: Speed tracking error; (a): $\dot{q}$ tracking error using $\hat{W}^{T}S(X)$; (b): $\dot{q}$ tracking error using $\bar{W}^{T}S(X)$

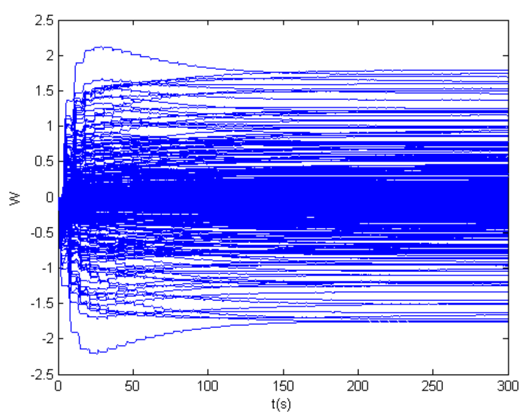
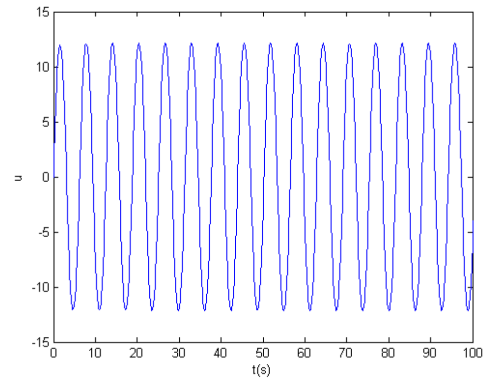


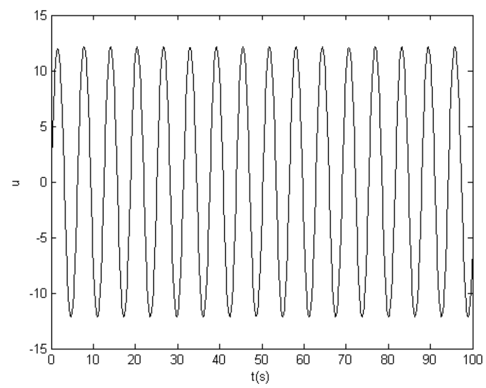
Fig. 3: Partial parameter convergence $\hat{W}_{\xi}$

better using the learned knowledge. In Fig. 3, it is seen that the convergence of $\hat{W}_{\xi}$ is obtained. The control signal u and state observer error are presented in Fig. 4 and 5, respectively.

From the simulation results, we can see clearly that our proposed neural control algorithm has not only implemented output tracking perfectly, but also achieved deterministic learning of the unknown closed-loop system dynamics during tracking control to a recurrent reference orbit. The learned knowledge stored as a set of constant neural weights can be used to improve the control performance of system.



(a)

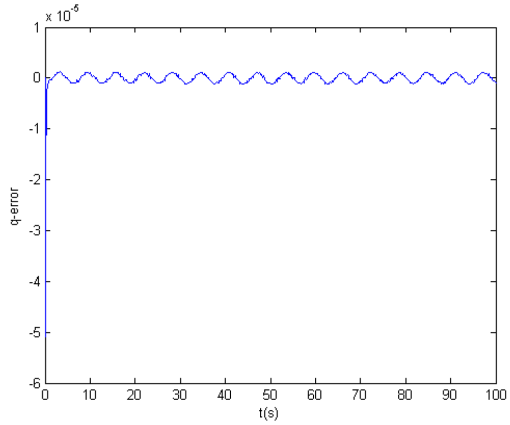


(b)

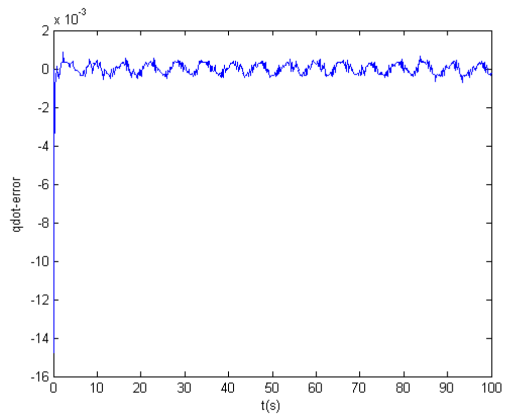
Fig. 4: Control input u; (a): control input u using $\hat{W}^{T}$ S(X); (b): control input u using $\bar{W}^{T}S(X)$

CONCLUSION

In this study, we have investigated deterministic learning from adaptive neural control of electrically-driven mechanical systems with completely unknown system dynamics. Compared with back stepping scheme, the key factor of the proposed method is that the electrically-driven mechanical systems are transformed into the affine nonlinear systems in the normal form, which avoids back stepping in controller design. Only one RBF network was used to approximate the unknown lumped nonlinear function, which shows the superiority of our proposed control algorithm. The designed controller has not only implemented the UUB of all signals in the closed-loop system, but also achieved learning of the unknown closed-loop system dynamics during the stable adaptive control process. The learned knowledge stored as a set of constant neural weights can be used to improve the



(a)



(b)

Fig. 5: State observer error; (a): q observer error; (b): $\dot{q}$ observer error

control performance and can also be reused in the same or similar control task so that the electrically-driven mechanical systems can be easily controlled with little effort.

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