Research Article

Learning from Adaptive Neural Control of Electrically-Driven Mechanical Systems

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Abstract: This study presents deterministic learning from adaptive neural control of unknown electrically-driven mechanical systems. An adaptive neural network system and a high-gain observer are employed to derive the controller. The stable adaptive tuning laws of network weights are derived in the sense of the Lyapunov stability theory. It is rigorously shown that the convergence of partial network weights to their optimal values and locally accurate NN approximation of the unknown closed-loop system dynamics can be achieved in a stable control process because partial Persistent Excitation (PE) condition of some internal signals in the closed-loop system is satisfied. The learned knowledge stored as a set of constant neural weights can be used to improve the control performance and can also be reused in the same or similar control task. Numerical simulation is presented to show the effectiveness of the proposed control scheme.

Keywords: Adaptive neural control, deterministic learning, electrically-driven mechanical systems, high-gain observer, RBF network

INTRODUCTION

The motion tracking control of uncertain mechanical systems described by a set of second-order differential equations has attracted the interest of researchers over the years. For the mechanical systems without the actuator dynamics, many approaches have been introduced to treat the motion tracking control problem and various adaptive control algorithms have been found (Ge et al., 1997; Zhang et al., 2008; Wai, 2003; Lee and Choi, 2004; Chang and Yen, 2005; Sun et al., 2001; Xu et al., 2009; Chang and Chen, 2005). However, as pointed out by Tarn et al. (1991) in order to construct high-performance tracking controllers, especially in the cases of high-velocity movements and high varying loads, the inclusion of the actuator dynamics in mechanical systems was very significant (Tarn et al., 1991). The incorporation of the actuator dynamics into the mechanical model complicates considerably the equations of motion. In particular, the electrically-driven mechanical systems were described by third-order differential equations (Tarn et al., 1991) and the number of degrees of freedom was larger than the number of control inputs.

Many works addressing the tracking problem of mechanical systems with actuator dynamics have been described in Dawson et al. (1998), Su and Stepanenko (1996, 1998), Chang (2002) and Driessen (2006). These works were based on the integrator backstepping technique. In backstepping design procedures, regression matrix is required and the procedures will become very tedious for mechanical systems with multiple degrees of freedom. Based on the universal approximation ability of Neural Networks (NNs) and fuzzy neural networks, adaptive neural/fuzzy neural control schemes have been developed to treat the tracking control of uncertain electro-mechanical systems (Kwan et al., 1998; Huang et al., 2003, 2008; Kuc et al., 2003; Wai and Chen, 2004, 2006; Wai and Yang, 2008). In Kwan et al. (1998), two-layer NNs were used to approximate two very complicated nonlinear functions with the NN weights being tuned on-line, the designed controller guaranteed the Uniformly Ultimately Bounded (UUB) stability of tracking errors and NN weights with some conditions. In Huang et al. (2003), a NN controller was developed to further reduce the conditions in Kwan et al. (1998) for the stability. In Huang et al. (2008), an adaptive NN control algorithm was proposed for reducing the dimension of NN inputs. In Kuc et al. (2003), employing three neural networks, the designed controller implemented the global asymptotic stability of the learning control system. In Wai and Chen (2004, 2006), robust neural fuzzy network control was derived for robot manipulators including actuator dynamics, favorable tracking performance was obtained for complex robot systems. In Wai and Yang (2008), an adaptive FNN controller with only joint position information was designed to cope with the problem caused by the assumption of all system state variables
to be measurable in Wai and Chen (2004, 2006). In the proposed adaptive neural control schemes above, NNs were used to approximate the nonlinear components in the electrically-driven mechanical systems and Lyapunov stability theory was employed to design closed-loop control systems. However, the learning ability of the approximation-based control is actually very limited and the problem of whether the neural networks employed in adaptive neural controllers indeed implement their function approximation ability has been less investigated. As a consequence, most of the adaptive neural controllers have to recalculate the control parameters even for repeating the same control task.

Recently, deterministic learning approach was proposed for identification and adaptive control of nonlinear systems (Wang and Hill, 2006, 2009). By using the localized RBF network, a partial PE condition, i.e., the PE condition of a certain regression subvector constructed out of the RBFs along the recurrent trajectory, is proven to be satisfied. This partial PE condition leads to exponential stability of the proposed adaptive neural control schemes above, NNs are useful in many applications (Wang and Chen, 2011; Su and Stepanenko, 1996). As a consequence, most of the adaptive neural controllers have to recalculate the control parameters even for repeating the same control task.

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bounded, for a positive constant 
\( q = l \), \( 1999; \) Eehow following equations:

\[ z = \frac{\partial F_x}{\partial x_i} \xi_i + G_z(x) \]

\[ z = \sum_{j=1}^{n} \left( \frac{\partial F_z}{\partial x_j} \right) \xi_j + G_z(x) \]

where, \( F_z(x) = G_z(x) \).

Therefore, the electrically-driven mechanical systems defined by Eq.1 can be described as the following normal form with respect to the new state variables:

\[ \dot{z}_1 = z_2 \]
\[ \dot{z}_2 = z_3 \]
\[ \dot{z}_3 = F_z(x) + G_z(x)u \]
\[ y = z_1 \]

It should be noted that apart from the fact that functions \( F_z(x) \) and \( G_z(x) \) are functions of \( x \), they are completely unknown.

**Property 5:** According to property 2, \( G_z^{-1} = LK_zM \) is bounded, for a positive constant \( g_1 > 0 \), \( \left\| G_z^{-1} (x) \right\| \leq g_1 \), \( \forall x \in \Omega \subset R^n \).

**Remark 1:** For mechanical systems, the components of \( M(q) \) are only the linear combination of constants and trigonometric function of \( q \), so the components of \( M(q)M^{-1}(q) \) are bounded. According to \( M(q)M^{-1}(q) = I \), \( M^{-1}(q) \) is bounded. \( G_z(x) = M^1K_T L^1 \), so \( G_z(x) \) and \( G_z^{-1}(x) \) are also bounded.

**ADAPTIVE NEURAL CONTROL AND LEARNING**

**High gain observer design:** From Eq.4, we noted that \( z_i \) is incomputable. Since \( F_z(x) \), \( F_z(x) \), and \( G_z(x) \), \( G_z(x) \) are unknown nonlinear functions. The HGO used to estimate the state \( z_i \) is the same as the one in (Ge et al., 1999; Eehow et al., 2010) and is described by the following equations:

\[ \dot{\xi}_i = \frac{\partial F_z}{\partial x_i} \xi_i + \sum_{j=1}^{n} \left( \frac{\partial G_z(x)}{\partial x_j} \right) \xi_j + G_z(x) \]

where, \( \epsilon \) is a small positive design constant and parameters \( d_i(i = 1, \ldots, n - 1) \) are chosen such that the polynomial \( s^2 + d_1 s + d_2 \) is Hurwitz. Then, there exist positive constants \( h \) and \( t \) such that \( \forall \) we have:

\[ |\ddot{y} - \ddot{z}| \leq \epsilon \ddot{z} + \sum_{j=1}^{n} \left( \frac{\partial G_z(x)}{\partial x_j} \right) \xi_j + G_z(x) \]

To prevent peaking (Khalil, 2002), saturation functions can be employed on the observer signals whenever they are outside the domain of set \( \Omega \), as follows:

\[ \dot{\xi}_i = S_i, \phi \left( \frac{\xi_i}{S_i} \right), S_i \geq \max \left( \xi_i \right) \]

for \( i = 1, 2, 3, j = 1, 2, \ldots, n \).

**Adaptive neural controller design:** For the system defined by Eq.4 and recurrent reference orbit \( y_{\hat{q}}(t) \), an adaptive neural controller using RBF networks is designed as follows. Vector \( y_{\hat{q}} \), \( E \) and filtered tracking error vector \( r \) are defined as:

\[ Y_d = \left[ y_{\hat{q}}^T, y_{\hat{q}}^e, y_{\hat{q}}^\varepsilon \right]^T \]
\[ E = z - Y_d \]
\[ r = \dot{e} + \lambda_1 e + \lambda_2 e = \left[ K^T, I \right] E \]
\[ e = y - y_{\hat{q}} = z_i - y_{\hat{q}} \]

where, \( E = \left[ e^T, \dot{e}^T, \varepsilon \right]^T \), \( e = y - y_{\hat{q}} \) is the output tracking error vector, \( K = [\lambda_2 I, \lambda_1 I] \) is appropriately chosen such that polynomial \( s^2 + \lambda_1 s + \lambda_2 \) is Hurwitz:

\[ \hat{E} = \dot{z} - Y_d \]
\[ \hat{r} = \left[ K^T, I \right] \hat{E} \]

Differentiating \( r \):

\[ \dot{r} = \dot{z}_i - y_{\hat{q}}^e + [0, K^T] E = F_z(x) + G_z(x) \dot{v} + \hat{v} \]

Choosing the control:

\[ u = -k_0 \hat{r} - \hat{W}^T S(X) \]

where, \( k_0 > 0 \) is the control gain, \( S(X) = \text{diag}(S_i(X), \ldots, S_n(X)) \), \( \hat{W}^T S(X) = \left[ \hat{W}_1^T S(X), \ldots, \hat{W}_n^T S(X) \right]^T \) are used to approximate the unknown functions.
where, \( w^T S(X) = w^T \Phi(X) \) and \( \varepsilon(X) \) are the NN approximation errors, with \( \varepsilon(X) < \varepsilon_0 \) for all \( X \in \Omega_x \). The weight update law is given by:

\[
\dot{\hat{W}} = \Gamma S(X) \hat{\varepsilon} - \sigma \| \hat{\varepsilon} \| \hat{\varepsilon}
\]

where, \( \Gamma = \Gamma^T > 0 \) is a constant design matrix, \( \sigma > 0 \) is a small positive constant.

The overall closed-loop system consisting of systems defined by Eq. (1), filtered tracking error defined by Eq. (10), the controller defined by Eq. (15) and the NN adaptive law defined by Eq. (17) can be summarized into the following form:

\[
\begin{align*}
\dot{\hat{r}} &= G \hat{r} + [k_r (r - \hat{r}) - \hat{W}^T S(X) + \varepsilon] \\
\dot{\hat{W}} &= \Gamma S(X) \hat{\varepsilon} - \sigma \| \hat{\varepsilon} \| \hat{\varepsilon}
\end{align*}
\]

where, \( \dot{\hat{r}} = \hat{r} - W^T \Phi(X) \).

**Theorem 1:** Consider the closed-loop system defined by Eq. (18). For any given recurrent reference orbit starting from initial condition \( y_0 \in \Omega_0 (\Omega_0 \) is a compact set) and with initial condition \( y(0) \in \Omega_0 \) (\( \Omega_0 \) is a compact set) and \( \hat{W}(0) = 0 \) we have that:

- All signals in the closed-loop system remain ultimately uniformly bounded.
- There exists a finite time \( T_f > 0 \) such that the state tracking errors \( E = [e^T, \dot{e}^T, \hat{\varepsilon}^T]^T \) converge to a small neighborhood around zero for all \( t \geq T_f \) by appropriately choosing design parameters.

**Proof:**

**Consider the following Lyapunov function:**

\[
V_1 = \hat{W}^T \Gamma^{-1} \hat{W} / 2
\]

Differentiating \( V_1 \):

\[
\dot{V}_1 = \hat{W}^T \Gamma^{-1} \hat{W} = \hat{W}^T \dot{S}(X) \hat{\varepsilon} - \sigma \| \hat{\varepsilon} \| \hat{\varepsilon}
\]

Thus, it follows that if \( \| \hat{W} \| > s^* / \sigma \), then \( \dot{V}_1 > 0 \) is the upper bound of \( \| S(X) \| \) (see reference literature (Slotine and Li, 1991). This leads the UUB of \( \| \hat{W} \| \) as \( \| \hat{W} \| \leq s^* / \sigma \). According to \( \hat{W} \), \( \hat{W} = W \), we have that:

\[
\| \hat{W} \| \leq \| W \| + \sigma \leq \| W \| + s^* / \sigma = \tilde{s}^*
\]

Take the Lyapunov function \( V_i = r^T G^{-1} r / 2 \). Differentiating \( V_i \), we have:

\[
\dot{V}_i = -r^T \dot{r} + r^T \dot{G}^T r + k_r (r - \hat{r}) - \hat{W} S(X) + \varepsilon + \frac{1}{2} r^T \dot{G}^T r
\]

where, \( k_r = k_r - g_s / 2 \), \( c_s = \| K \| / 2 \). Note that the equality \( \tilde{E} = \tilde{z} \) can be easily induced from Eq. (9) and Eq. (12) as follows:

\[
\tilde{E} = E - \tilde{E} = \tilde{z} - \tilde{z} = \tilde{z} \leq \varepsilon_h
\]

Let \( c = \tilde{W}^T s^* + \varepsilon^* \), Eq. (22) is further derived as:

\[
\dot{V}_i = -k_r \| \| - c_s \varepsilon_h - c / k_r
\]

This implies that the filtered tracking error vector \( r \) is UUB as follows:

\[
\| r \| \leq c_s \varepsilon_h + c / k_r.
\]

Whose boundary can be made small enough by increasing the control gain \( k_r \) and decreasing \( c \).

Because \( r = \hat{r} + \lambda \dot{r} + \lambda_2 e \) is stable by appropriately choosing design parameters \( \lambda_1 \), \( \lambda_2 \) and \( c_\lambda \), \( y_d \), \( \hat{y}_d \) are bounded, then \( z_1 \), \( z_2 \), \( z_3 \) are bounded. \( S(X) \) is bounded for all values of \( X \), we conclude that control \( u \) is also bounded. Thus, all the signals in the closed-loop system remain ultimately uniformly bounded.

**Consider the following Lyapunov function:**

\[
V_i = r^T G^{1/2} r / 2
\]

The derivative of \( V_i \) is:

\[
\dot{V}_i = r^T \dot{r} + r^T \dot{G}^T r + k_r (r - \hat{r}) - \hat{W} S(X) + \varepsilon + \frac{1}{2} r^T \dot{G}^T r
\]

Let, \( k_r = 2 \bar{k}_1 + 3 \bar{k}_2 + g_s / 2 \), using the inequality \( 2a^T \beta \leq \eta a^T \alpha + (1 / \eta) \beta^T \beta (\eta > 0) \), we have:

\[
r^T \varepsilon \leq \varepsilon^2 / (4 \bar{k}_1) + \bar{k}_1 \| \|
\]

\[
r^T \dot{r} \leq \dot{s}^2 / (4 \bar{k}_2) + \bar{k}_1 \| \|
\]

\[
r^T k_r [K^T I] \| z \| \leq \bar{k}_2 \| \| + \| \| / (4 \bar{k}_2)
\]

where, \( \bar{c} = k_r, \| \| = O(\varepsilon), s^* \) is the upper bound of \( \| S(X) \| \), \( \hat{W} \) is the upper bound of \( \| \hat{W} \| \) which is given in Eq. (21). Then Eq. (27) becomes:

\[
\dot{V}_i \leq -k_r \| \| + (\tilde{W}^2 s^2 + \varepsilon^2 + \varepsilon^2) / (4 \bar{k}_2)
\]
Let $\delta = \frac{(\overline{W}^2 s^2 + e^2 + e^2)}{(4\overline{k}_c)}$, it is clear that $\delta$ can be made small enough using large enough $k_c$, so we have:

$$\dot{V}_r \leq -r^2\overline{k}_r + \delta \leq -2\overline{k}_r\overline{S}_r V_r + \delta$$ (32)

Let $c = \overline{k}_r|g|/p = \delta / 2c > 0$ then $V_r$ satisfies:

$$0 \leq V_r(t) < p + (V_r(0) - p)\exp(-2ct)$$ (33)

that is:

$$r^2r < 2\overline{k}_r p + 2\overline{k}_r V_r(0)\exp(-2ct)$$ (34)

The above equation implies that given $\beta > \sqrt{2\overline{k}_r p = \sqrt{\delta / \overline{k}_r}}$, there exists a finite time $T_1$, determined by $\delta$ and $\overline{k}_r$, such that for all $t \geq T_1$, the filtered tracking error $r$ satisfy:

$$|r| < \beta$$ (35)

where, $\beta$ is the size of a small residual set that can be made small enough by appropriately choosing $\overline{k}_r$, $\overline{k}_c$.

By choosing a large $k_c$, the filtered tracking error $r$ can be made small enough for all $t \geq T_1$ that is to say, there exists a $T_1$ such that the state tracking errors $E = [e^T, e_{ti}^T, e_{ti}^T]^T$ converge to a small neighborhood around zero for all $t \geq T_1$ by appropriately choosing design parameters (Slotine and Li, 1991) so that the tracking states $z(t)_{t \geq T_1}$ follow closely to $Y_d(t)_{t \geq T_1}$.

**Remark 2:** Theorem 1 indicated that the system orbits $z(t)$ will become as recurrent as $Y_d(t)$ that after time $T_1(T > t)$, so $x(t)$ will also become recurrent. $\hat{v}$ converges to a small neighborhood around $y(3)_{y_d(3)}$ after $T > T_1$, which indicates that $\hat{\varphi}$ is as recurrent as $y_d(3)$. Since $X = [x^T, \hat{v}^T]^T$ are selected as the RBF networks inputs, according to theorem 2.7 in Wang and Hill (2009), $S(X)$ will satisfy the partial PE condition, i.e., along $Y_d(t)_{t \geq T_1}$, $S_d(X)$ satisfies the PE condition.

**Remark 3:** The key aspect of the proposed method is that electrically-driven mechanical systems are transformed into the affine nonlinear system in the normal form with state transformation. Thus, learning and stability analysis avoid using virtual control terms and their time derivatives, which require complex analysis and computing. In our proposed approach, only one RBF network is employed to approximate the unknown lumped system nonlinear dynamics, which shows the superiority of our proposed learning control scheme.

**Learning from adaptive neural control stability of a class of LTV systems:** For deterministic learning from adaptive neural control of nonlinear systems with unknown affine term, the associated LTV system is extended in the following form Liu et al. (2009):

$$\frac{d}{dt} q = F(X, q; \phi)$$ (36)

where, $q_i \in R^{n_i}, i \in R^m, \eta \in R^m, A(\cdot) : [0, x) \rightarrow R^{n_x}, S(\cdot) : [0, x) \rightarrow R^{n_x}, G(\cdot) : [0, x) \rightarrow R^{n_x}$ and $\gamma = I > 0$.

Define $e := [e_1^T, e_2^T]$, $B(t) := [0 \quad S(t)] \in R^{n_x}$, $H(t) = block-diag[0, G(t)] \in R^{n_x}$, where diag refers to block diagonal form and $C(t) := \Gamma B(t) H(t)$.

**Assumption 1:** (Loria and Panteley, 2002). There exists a $\Phi_{M_1} < 0$ such that, for all $t \geq 0$, the following bound is satisfied:

$$\max \left[ |B(t)|, \frac{|dB(t)|}{dt} \right] \leq \phi_{M_1}$$ (37)

**Assumption 2:** (Liu et al., 2009). There exist symmetric matrices $P(t)$ and $Q(t)$ such that $-Q(t) = P(t) + A(t)P(t) + A^T(t)P(t)$.

Furthermore, $\|p_a_q\|, p_a_q$ and $q_a$ such that:

$$p_a_q \leq |P(t)| \leq p_a_q I \quad and \quad \eta_a \leq |Q(t)| \leq \eta_a I$$.

**Lemma 1:** (Liu et al., 2009). With assumption 3.1 and 3.2 satisfied in a compact set $\Omega$, system defined by Eq.36 is uniformly exponentially stable in the compact set $\Omega$ if $S(t)$ satisfies the PE condition.

**Learning from adaptive neural control:** Using the localization property of RBF network, after time $T_1$, system defined by Eq.18 can be expressed in the following form along the tracking orbits $X(t)_{t \geq T_1}$ as:

$$\dot{\hat{v}} = G \hat{v}(x)[k_{\hat{v}} \hat{v}^T S(X, \eta) + e_{\xi}]$$ (38)

$$\hat{\hat{v}} = \hat{\hat{v}} = \Gamma S(X, \eta) r - \sigma \|r\| \hat{\hat{v}}$$ (39)

where, $\hat{\hat{v}} = [\hat{v}, \hat{v}, \hat{v}, \hat{v}, \hat{v}, \hat{v}, \hat{v}]^T$, $S_d(X)$ is a subvector of $S(X)$; $\hat{\hat{v}}$ is the corresponding weight subvector; the subscript $\xi$ stands for the region far away from the trajectories $X(t)_{t \geq T_1}$; $\xi$ are the local approximation errors, $\|\xi\|$ is small.

**Theorem 2:** Consider the closed-loop system defined by Eq. (38). For any given recurrent reference orbit starting from initial condition $y_d(0) \in \Omega_d$ and with
initial condition \(y_d(0) \in \Omega_{y_d}\), \(\dot{y}(0) = 0\) and control parameters appropriately chosen, we have that

Along the tracking orbits \(X(t)|_{t \geq T}\), neural weight estimates \(\hat{W}_\xi\) converge to a small neighborhood of the optimal values \(\hat{W}_\xi^*\) and locally accurate approximation of the unknown closed-loop system dynamics \(\psi(X)\) are obtained by \(\dot{\hat{W}}^T S(Z)\) and \(\hat{P}^T S(Z)\), where \(\hat{P}\) is obtained from:

\[
\hat{P} = \text{mean}(\hat{W}(t))
\]

where, \([t_u, t_b]\) \((t_b > t_u > T, T_i)\) represents a time segment after transient process of \(\hat{W}\).

**Proof:** Let \(\theta = G^{-1}(X)\), and \(\eta = \hat{W}_\xi\), then system defined by Eq.38 is transformed into:

\[
\begin{aligned}
\dot{\theta} &= -[kG_x(x) - \hat{G}_x(x)G_x(x)]\theta \\
\eta &= \Gamma S_x(x)G_x(x)\theta - \sigma \hat{P}_\xi
\end{aligned}
\]

Rewrite Eq. (41) in matrix, we have:

\[
\begin{bmatrix}
\dot{\theta} \\
\eta
\end{bmatrix} = 
\begin{bmatrix}
A(t) & B(t) \\
-C(t) & 0
\end{bmatrix}
\begin{bmatrix}
\theta \\
\eta
\end{bmatrix} + 
\begin{bmatrix}
\varepsilon \xi \\
-k\xi
\end{bmatrix}
\]

where,\(A(t) = -kG_x(x) + \hat{G}_x(x)G_x(x)\)

\(B(t) = -S_x(x)\)

\(C(t) = -\Gamma S_x(x)G_x(x)\)

Introducing \(P(t) = G_x(x)\), we have:

\[
\dot{P} + PA + A^T P = -2G_x(k - \hat{G}_x)G_x + \hat{G}_x.
\]

The satisfaction of Assumption 3.1 can be easily checked. With \(G_x(x)\) and \(\hat{G}_x(x)\), being bounded, \(k\) can be designed such that \(2G_x(k - \hat{G}_x)G_x = \hat{G}_x\) is strictly positive definite and the negative definite of \(\dot{P} + PA + A^TP\), is guaranteed. Thus, Assumption 2 is satisfied.

After time \(T_1\), the NN inputs \(X(t)\) follow recurrent orbits and the partial PE condition (Wang and Hill, 2006, 2009) can be satisfied by the regression subvector \(S_x(X)\), which consists of RBF networks with centers located in a small neighborhood of the tracking orbits \(X(t)|_{t \geq T}\). Thus, uniformly exponentially stability of the nominal system of system defined by Eq. (42) is guaranteed by Lemma 1. For the perturbed system defined by Eq. (42), using Lemma 4.6 in Khalil (2002), the parameter errors \(\eta = \hat{W}_\xi\) converge exponentially to a small neighborhood of zero in a finite time \(T(T > T_1)\), with the size of the neighborhood being determined by the NN approximation ability and state tracking errors.

The convergence of \(\hat{W}_\xi\) to a small neighborhood of \(W^*_\xi\) implies that along the tracking trajectories \(X(t)|_{t \geq T}\), the unknown closed-loop system dynamics \(\psi(X)\) can be represented by regression subvector \(S_x(X)\) with small error, i.e.,

\[
\psi(X) = \dot{\hat{W}}^T S_x(X) + \varepsilon
\]

Choosing \(\hat{W}\) according to \(\hat{P} = \text{mean}(\hat{W}(t))\), Eq. (47) can be expressed as:

\[
\psi(X) = \hat{W}_\xi^T S_x(X) + \varepsilon
\]

where, \(\hat{W}_\xi^T = [\hat{w}_{11}, \cdots, \hat{w}_{1n}]^T\) is the subvector of \(\hat{W}\), \(\hat{W}_\xi^T S_x(X) = [\hat{w}_{11}S_x(x), \cdots, \hat{w}_{1n}S_x(x)]^T\) and \(\|\varepsilon\|\) is small.

Remark 4: Theorem 2 reveals that deterministic learning (i.e., parameter convergence) can be achieved
during tracking control to a recurrent reference or bit. The learned knowledge can be stored in the constant RBF networks \( \mathbf{W}^T S(\mathbf{x}) \), but it is generally difficult to represent and store the learned knowledge using the time-varying neural weights. Through the deterministic learning, the representation and storage of the past experiences become a simple task.

**SIMULATION**

A single-link robotic manipulator coupled to a DC motor is considered. The dynamic equations of the system are:

\[
M\ddot{q} + B\dot{q} + N\sin(q) = K_T I \\
L\dot{I} + RI + K_Bq = u
\]

(50)

where,

- \( M = J + \frac{1}{3}mL^2 + \frac{2}{5}M_EL_R^2 \)
- \( q \) = The angular position
- \( J \) = The inertia of the actuator’s rotor
- \( m \) = The mass of the link
- \( M_0 \) = Payload mass
- \( R_0 \) = The radius of the payload
- \( g \) = The gravitational constant
- \( K_T \) = The torque constant
- \( K_B \) = The back-EMF constant
- \( R \) = The armature resistance
- \( L \) = The armature inductance
- \( I \) = The armature current
- \( u \) = The armature voltage

Eq. (50) can be expressed in the following form:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\
\dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u
\end{align*}
\]

(51)

where,

- \( f_2 = -M^{-1}(Bx_2 + N \sin x_1) \)
- \( g_2 = M^{-1}K_T \)
- \( f_3 = -L^{-1}(Rx_3 + K_Bx_2) \)
- \( g_3 = L^{-1} \)

The initial state values are \( x_1(0) = 0 \) and \( x_2(0) = 0 \) and the desired output is set to \( y_d = 0.8 \sin(t) \).

Our proposed controller design procedure is summarized as follows. First, define the three-order HGO as:

\[
\begin{align*}
\dot{e}_1 &= \xi_2 \\
\dot{e}_2 &= \xi_3 \\
\dot{e}_3 &= -d_1e_1 - d_2e_2 - \xi_1 + y
\end{align*}
\]

(52)

Further, the state estimation vector and error vector are \( \hat{\mathbf{z}} = [\hat{x}_1, \frac{\mathbf{e}}{\mathbf{e}}, \frac{\mathbf{e}}{\mathbf{e}}]^T \) and \( \hat{\mathbf{E}} = \hat{\mathbf{z}} - Y_d \). The control input is determined as:

\[
u = -K\hat{r} - \mathbf{W}^T S(\mathbf{x}) \quad X = [x_1, x_2, x_3, \hat{\mathbf{E}}]^T
\]

(53)

The parameters of the single-link robotic system (Huang et al., 2008) are \( J = 1.625 \times 10^{-3} (\text{kgm}^2) \), \( R = 5.0 (\Omega) \), \( K_T = 0.9 (\text{Nml A}) \), \( K_B = 0.9 (\text{Nml A}) \), \( M_0 = 0.434 (\text{kg}) \), \( R_0 = 0.023 (\text{m}) \) and \( L = 0.025 (\text{H}) \).

The design parameters of the above controller are \( K_v = 50, \lambda_1 = \lambda_2 = 25, \Gamma = \text{diag}[20], \sigma = 0.0001 \). The HGO parameters are \( d_1 = 3, d_2 = 3, \xi(0) = [0, 0, 0]^T \) and \( \eta_i = 1/100 \). The RBF networks \( \mathbf{W}^T S(\mathbf{Z}) \) contain \( 5 \times 5 \times 7 \times 11 = 1925 \) nodes (i.e., \( N = 1925 \)), the centers \( \mathbf{C}_i (i = 1, 2, \ldots, N) \) are evenly spaced on \([-1.5 1.5] [-1.5 1.5] \times [-3.5 3.5] \times [-25 25] \), with width \( \eta_i = 0.75 \), \( \mathbf{W}(0) = 0 \).

Figure 1 to 5 show the simulation results. The tracking performance of system is shown to be good in Fig. 1 and 2 and the tracking performance become
Fig. 2: Speed tracking error; (a): $\dot{q}$ tracking error using $\hat{W}^T S(X)$; (b): $\dot{q}$ tracking error using $\hat{W}^T S(X)$

Fig. 3: Partial parameter convergence $\hat{W}_H$

better using the learned knowledge. In Fig. 3, it is seen that the convergence of $\hat{W}_H$ is obtained. The control signal $u$ and state observer error are presented in Fig. 4 and 5, respectively.

From the simulation results, we can see clearly that our proposed neural control algorithm has not only implemented output tracking perfectly, but also achieved deterministic learning of the unknown closed-loop system dynamics during tracking control to a recurrent reference orbit. The learned knowledge stored as a set of constant neural weights can be used to improve the control performance of system.

CONCLUSION

In this study, we have investigated deterministic learning from adaptive neural control of electrically-driven mechanical systems with completely unknown system dynamics. Compared with back stepping scheme, the key factor of the proposed method is that the electrically-driven mechanical systems are transformed into the affine nonlinear systems in the normal form, which avoids back stepping in controller design. Only one RBF network was used to approximate the unknown lumped nonlinear function, which shows the superiority of our proposed control algorithm. The designed controller has not only implemented the UUB of all signals in the closed-loop system, but also achieved learning of the unknown closed-loop system dynamics during the stable adaptive control process. The learned knowledge stored as a set of constant neural weights can be used to improve the
control performance and can also be reused in the same or similar control task so that the electrically-driven mechanical systems can be easily controlled with little effort.

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