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Research Article

Seepage Characteristics Study on Rabinowitch Fluid through Porous Media

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Abstract: Fractal expressions for flow rate, velocity and effective permeability of Rabinowitch fluid flow in porous media are derived based on the fractal properties of porous media and capillary model. Each parameter in the proposed expressions does not contain any empirical constant and has clear physical meaning and the proposed fractal models relate the flow properties of Rabinowitch fluid with the structural parameters of porous media. The presented analytical expressions help to understand the physical principles of Rabinowitch fluid through porous media.

Keywords: Effective permeability, fractal, porous media

INTRODUCTION

The Rabinowitch fluid is one of non-Newtonian fluids and its constitutive equation is Chen (1992):

$$\dot{\gamma} = \frac{\tau}{\mu} + a\tau^3 \tag{1}$$

where,
$\dot{\gamma}$ = Shear rate
τ = Shear stress

μ and a = Material constants that describe the properties of fluids

The Rabinowitch fluid model may be reduced to the Newtonian fluid model when $a = 0$.

It has been shown that porous media in nature are fractal objects and fractal geometry theory has been proven to be powerful for analysis of porous media (Xiao *et al*., 2009; Cai *et al*., 2012; Yun and Zheng, 2012a, b). In this study, fractal model for Rabinowitch fluid is derived based on the fractal characters of porous media.

FRACTAL MODEL DERIVATION

The cumulative size distribution of pores in porous media follows the fractal scaling law (Yu and Cheng, 2002):

$$N(L \ge r) = \left(\frac{r_{\max}}{r}\right)^{D_f} \tag{2}$$

where,
r = The radius of pore/capillary
$r_{\max}$ = The maximum pore radius
D_f = The fractal dimension for pores

Differentiating Eq. (2) with respect to r results in the number of pores whose radii are within the infinitesimal range r to r +dr:

$$-dN = D_f r_{\max}^{D_f} r^{-(D_f+1)} dr \tag{3}$$

In Eq. (3), −dN>0, which implies that the number of pores decreases with the increase of pore sizes.

The fractal scaling law for the tortuous capillaries in porous media is Yu (2005):

$$L_t = L_0^{D_T} (2r)^{1-D_T} \tag{4}$$

where, L_t is the tortuous length along the flow direction and L_0 is the representative length. Due to the tortuous nature of the capillary, $L_t \ge L_0$. D_T is the fractal dimension for tortuosity, $D_T = 1$ represents a straight capillary and $L_t = L_0$. Eq. (4) also shows that the larger

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the capillary r, the shorter the capillary length L_t. This is consistent with the physical situation.
For a given capillary, differentiating Eq. (4) yields:

$$dL_t = L_0^{D_T-1}(2r)^{1-D_T} D_T dL_0 \quad (5)$$

The flow rate for Rabinowitch fluid flow in a straight capillary is given by (Mohos, 2010):

$$q(r) = \frac{\pi r^4}{8\mu}(-\frac{dp}{dL_0}) + \frac{\pi a r^6}{48}(-\frac{dp}{dL_0})^3 \quad (6)$$

where, dp/dL_0 is the pressure gradient. Since in real porous media, the capillaries are usually tortuous, Eq. (6) is modified by replacing dL_0 by dL_t as:

$$q(r) = \frac{\pi r^4}{8\mu}(-\frac{dp}{dL_t}) + \frac{\pi a r^6}{48}(-\frac{dp}{dL_t})^3 \quad (7)$$

where, dL_t is determined by Eq. (5).

It has been shown that the pore size distribution in porous media follows the fractal power law, so, the total flow rate Q through a cross-section can be obtained by integrating Eq. (7) over the entire range of pore sizes from the minimum pore size to the maximum pore size:

$$\begin{aligned} Q &= -\int_{r_{\min}}^{r_{\max}} q(r) dN(r) \\ &= \frac{\pi D_f r_{\max}^{3+D_T} \Delta p}{2^{4-D_T} \mu D_T L_0^{D_T}(3+D_T-D_f)}[1-(\frac{r_{\min}}{r_{\max}})^{3+D_T-D_f}] \\ &+ \frac{\pi a D_f r_{\max}^{3+3D_T} \Delta p^3}{3\times 2^{7-3D_T} D_T^3 L_0^{3D_T}(3+3D_T-D_f)}[1-(\frac{r_{\min}}{r_{\max}})^{3+3D_T-D_f}] \end{aligned} \quad (8)$$

where, $r_{\min}$ and $r_{\max}$ are the minimum and maximum radii of pores, respectively. since $1 < D_T < 2$ and $0 < D_f < 2$ in two dimensions, the exponent $3 + D_T - D_f > 1$ and $3 + 3D_T - D_f > 1$, in general $r_{\min}/r_{\max} \sim 10^{-2}$, thus

$$(\frac{r_{\min}}{r_{\max}})^{3+D_T-D_f} \ll 1$$

And

$$(\frac{r_{\min}}{r_{\max}})^{3+3D_T-D_f} \ll 1$$

It follows that Eq. (8) can be reduced to:

$$Q = \frac{\pi D_f r_{\max}^{3+D_T} \Delta p}{2^{4-D_T} \mu D_T L_0^{D_T}(3+D_T-D_f)} + \frac{\pi a D_f r_{\max}^{3+3D_T} \Delta p^3}{3\times 2^{7-3D_T} D_T^3 L_0^{3D_T}(3+3D_T-D_f)} \quad (9)$$

The total pore area A_p can be obtained by:

$$A_p = -\int_{r_{\min}}^{r_{\max}} \pi r^2 dN(r) = \frac{\pi D_f r_{\max}^2}{2-D_f}[1-(\frac{r_{\min}}{r_{\max}})^{2-D_f}] \quad (10)$$

Due to Yu and Li (2001)

$$\phi = (\frac{r_{\min}}{r_{\max}})^{2-D_f} \quad (11)$$

where, ϕ is the porosity of porous media.
Thus, the cross-sectional area of porous media is:

$$A = \frac{A_p}{\phi} = \frac{\pi D_f r_{\max}^2 (1-\phi)}{\phi(2-D_f)} \quad (12)$$

Dividing Eq. (9) by Eq. (12) gives the average flow velocity for Rabinowitch fluid in porous media:

$$V = \frac{Q}{A} = \frac{\phi(2-D_f) r_{\max}^{1+D_T} \Delta p}{2^{4-D_T} \mu D_T (1-\phi) L_0^{D_T}(3+D_T-D_f)} + \frac{a\phi(2-D_f) r_{\max}^{1+3D_T} \Delta p^3}{3\times 2^{7-3D_T}(1-\phi) D_T^3 L_0^{3D_T}(3+3D_T-D_f)} \quad (13)$$

When $a = 0$ in Eq. (13), the Rabinowitch model reduces to the Newtonian fluid model:

$$V = \frac{\phi(2-D_f) r_{\max}^{1+D_T} \Delta p}{2^{4-D_T} \mu D_T (1-\phi) L_0^{D_T}(3+D_T-D_f)} \quad (14)$$

For flow in a tube/capillary, the shear stress at wall is Cavatorta and Tonini (1987):

$$\tau_w = -\frac{r}{2}\frac{dp}{dL_t} \quad (15)$$

where, τ_w is the shear stress at wall, the negative sign in Eq. (15) implies that the direction of the shear stress is opposite to that of the pressure gradient.
The total shear stress at all walls can be found from:

$$\tau = -\int_{r_{\min}}^{r_{\max}} \tau_w dN = \frac{D_f r_{\max}^{D_T}[1-(\frac{r_{\min}}{r_{\max}})^{D_T-D_f}]\Delta p}{2^{2-D_T} L_0^{D_T} D_T (D_T-D_f)} \quad (16)$$

According to Eq. (1), the apparent viscosity of Rabinowitch fluid can be found to:

$$\mu_a = \frac{\tau}{\dot{\gamma}} = \frac{1}{\frac{1}{\mu} + a\tau^2} \quad (17)$$

The apparent viscosity μ_α and effective permeability Ke are incorporated in the generalized Darcy law (Kong *et al*., 1999):

$$V = \frac{K_e}{\mu_a}\frac{\Delta p}{L_0} \quad (18)$$

According to Eq. (13), (16), (17) and (18), the effective permeability for Rabinowitch fluid in porous media can be found to be:

$$K_e = \left[\frac{\phi(2-D_f){r_{\max}}^{1+D_T}}{2^{4-D_T}\mu D_T(1-\phi){L_0}^{D_T-1}(3+D_T-D_f)} + \frac{a\phi(2-D_f){r_{\max}}^{1+3D_T}\Delta p^2}{3\times 2^{7-3D_T}(1-\phi){D_T}^3{L_0}^{3D_T-1}(3+3D_T-D_f)}\right] \cdot \left[\frac{1}{\mu} + \frac{a{D_f}^2{r_{\max}}^{2D_T}\left[1-\left(\frac{r_{\min}}{r_{\max}}\right)^{D_T-D_f}\right]^2\Delta p^2}{2^{4-2D_T}{L_0}^{2D_T}{D_T}^2(D_T-D_f)^2}\right]^{-1} \tag{19}$$

When $a=0$ in Eq. (19), the effective permeability for the Rabinowitch model reduces to that for the Newtonian model:

$$K_e = \frac{\phi(2-D_f){r_{\max}}^{1+D_T}}{2^{4-D_T}D_T(1-\phi){L_0}^{D_T-1}(3+D_T-D_f)} \tag{20}$$

Equation (20) is the same as that for Newtonian fluid by Zhang *et al*. (2006).

For straight capillaries $D_T = 1$, Eq. (20) can be further reduced to:

$$K_e = \frac{\phi(2-D_f){r_{\max}}^2}{8(4-D_f)(1-\phi)} \tag{21}$$

Equation (21) is in accord with the result by Yu and Cheng (2002).

RESULTS AND DISCUSSION

It is seen from Eq. (9) and (13) that the flow rate Q and average velocity V for Rabinowitch fluid in porous media are related not only to the structural parameters of porous media (r_{max}, D_T, D_f, L_0 and ϕ) but also to the fluid characteristic parameters (μ and *a*). Traditionally, the volume average method was used to study flow velocity, microstructure of porous media was ignored, so that is not profound to understand flow mechanism. It is seen from Eq. (19) that the effective permeability for Rabinowitch fluid in porous media is related not only to the fluid characteristic parameters and pressure drop across porous media but also to the structural parameters of porous media, but we can see from Eq. (20) that effective permeability for Newtonian fluid is only related to structural parameters of porous media.

Based on the fractal geometry theory, the fractal expressions for flow rate, flow velocity and effective permeability for Rabinowitch fluid flow in porous media have been derived. The proposed fractal models are expressed as a function of fluid characteristic parameters, pressure drop and structural parameters of porous media. The obtained models have clear physical meaning and relate the properties of Rabinowitch fluid to the structural parameters of porous media, so they are help to understand flow mechanism for non-Newtonian fluid through porous media.

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