

Research Article

A Novel D-S Combination Method for Interval-valued Evidences

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Abstract: D-S evidence theory is widely applied to the fields of information fusion, pattern recognition, decision analysis and other fields. When information are given as interval numbers with high conflicting evidences, traditional combinations rules may result in antinomy that some evidences are discarded while conflict factor is zero. To avoid this inconsequence, a method of disjointing algorithm is proposed to reallocate the focal elements and their basic probability assignments for intersecting intervals. Then the weighted average method is used to combine the evidences. Simulation results show that the proposed method can get more reasonable results in combining interval-valued evidences.

Keywords: Combination rules, D-S evidence theory, disjointing algorithm, interval numbers

INTRODUCTION

D-S evidence theory was initially developed by Dempster (1967) and formalized by Shafer (1976). Evidence theory is a convenient framework for modeling imperfection in data and for combining information. As a theory of uncertain information processing, evidence theory has been widely used in data fusion (Sevastianov and Dymova, 2009; Deng *et al.*, 2011), pattern recognition (Perrin *et al.*, 2004; Liu *et al.*, 2008), decision analysis (Kanoksri and Miroslav, 2008; Ludmila and Pavel, 2010), uncertainty quantification (Aven, 2011; Xie *et al.*, 2010) and other fields (He *et al.*, 2007; Basir and Yuan, 2007; Thomas and Olivier, 2011).

Combination rule plays a central role in evidence theory. It can combine the information from different sources and obtain more reliable results. The most famous combination rule is Dempster's rule. But researchers found it had some weaknesses, particularly in the conflict evidence combination. Therefore, many scholars put forward their solutions (Yager, 1989; Inagaki, 1991; Lefevre and Colot, 2002; Murphy, 2000; Huynh and Murai, 2006). However, the combination of interval-valued evidences was very few reported. Most of the studies were dealing with focal elements of point sets. Only in reference (Kari, 2003), combination of interval-valued evidences was discussed. But in Kari (2003), the high conflicting problem for interval numbers was not studied. In reality, interval number is an important type of evidence in probability analysis, reliability evaluation and logistics supply chain

management (Jim and Jonathan, 2004; Oberguggenberger and Fellin, 2008; Wang and Hsu, 2010), etc.

When focal elements are interval numbers, the relationship between them is not only equality, containment, but also intersection in some occasions. Then it may fall into contradiction that conflict coefficient is zero while the combination results discard some evidences, which details will be discussed in rests of the paper. That means the existing combination rules are not suitable completely for interval focal elements, especially when there are conflicts among the evidences. So the motivation of this research work is to propose a more robust method to solve the conflicting problem of interval-valued evidences.

EVIDENCE THEORY

Evidence theory is a convenient framework for modeling imperfection in data and for combining information. In this section, the basic notations of evidence theory are introduced and the main concepts that are necessary to understand the rest of the paper are briefly recalled.

A basic measure in evidence theory is a BPA (Basic Probability Assignment) (Dempster, 1967). Let Θ be a finite set of mutually exclusive and exhaustive hypotheses, called the frame of discernment. Let 2^Θ be the set of all subsets of Θ . For a given evidential event A , BPA is represented by $m(A)$, which defines a mapping of 2^Θ to the interval between 0 and 1, i.e.,

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$m:2^\Theta \rightarrow [0,1]$. $m(A)$ expresses the proportion of all relevant and available evidence that supports the claim that a particular element of Θ belongs to the set A and has no additional claims about any subsets of A . The value of $m(A)$ only belongs to the set A and makes no additional claim about any subsets of A . For example, if $B \subset A$, then $m(B)$ is another BPA. Generally, $m(A)$ must satisfy the following constraints:

$$m(A) \geq 0 \text{ for all } A \in 2^\Theta \tag{1}$$

$$m(\emptyset) = 0 \tag{2}$$

$$\sum_{A \in 2^\Theta} m(A) = 1 \tag{3}$$

Evidence theory uses two measures, belief (Bel) and plausibility (Pl), to characterize uncertainty. For a set A , Bel is defined as the sum of all the BPAs of the proper subsets (B) of the set of interest (A) ($B \subset A$). Pl is the sum of all the BPAs of the sets (B) that intersect the set of interest (A) ($B \cap A \neq \emptyset$). For a set, A , in the power set ($A \in 2^\Theta$):

$$Bel(A) = \sum_{B \subset A} m(B) \tag{4}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{5}$$

These two functions can be derived from each other. For example, belief function can be derived from plausibility function in the following way:

$$Bel(A) = 1 - Pl(\bar{A}) \tag{6}$$

The relationship between belief and plausibility functions is:

$$Bel(A) \leq Pl(A) \tag{7}$$

Equation (4) shows that as a measure of “event A is true”, if $P(A)$ is the true value of the measure of set $\{A \text{ is true}\}$, then $Pl(A)$ is the upper bound of $P(A)$ and $Bel(A)$ is the lower bound. So:

$$Bel(A) \leq P(A) \leq Pl(A) \tag{8}$$

CONFLICTING PROBLEM OF COMBINATION RULES FOR INTERVAL NUMBERS

Dempster’s rule is the classical approach for evidence combination, but it is not ideal for conflicting evidences. Yager, Inagaki and other researchers made

Table 1: BPAs of x

Evidence source	A	$m(A)$
1	[1.0,1.5]	0.9
	[1.0,2.0]	0.1
2	[1.0,1.5]	0.9
	[1.0,2.0]	0.1
3	[1.0,1.5]	0.9
	[1.0,2.0]	0.1
4	[1.4,1.6]	0.9
	[1.4,2.0]	0.1

Table 2: The combination results of Dempster’s rule

K	A	$m(A)$
0	[1.4,1.5]	0.9990
	[1.4,2.0]	0.0001
	[1.4,1.6]	0.0009

some improvements on Dempster’s rule and could combine conflicting evidences with more reasonable results. However, all of above methods have weak points when combine interval-valued evidences.

Example 1: There 4 independent evidence sources to support parameter x , as shown in Table 1.

Combination rules of Dempster, Yager and Inagaki will be described below. Then 4 evidence sources will be combined with these rules to discover the shortages of them.

Dempster’s rule: Let m_1, m_2, \dots, m_n be n BPAs on a frame of discernment, Θ , which focal elements are $A_i (i=1,2,\dots,N)$. Then (Dempster, 1967; Shafer, 1976):

$$m_p(A) = \begin{cases} \frac{1}{1-K} \sum_{\cap_{i=1}^n A_i \subseteq A} \prod_{i=1}^n m_i(A_i) & A \neq \emptyset \\ 0 & A = \emptyset \end{cases} \tag{9}$$

where, $K = \sum_{\cap_{i=1}^n A_i = \emptyset} \prod_{i=1}^n m_i(A_i)$ is the conflict coefficient. If $K = 0$, then there are no conflicts among the evidences; on the other hand, if $K \rightarrow 1$, then there have high level conflicts among them.

Eq. (9) is the Dempster’s rule, in which $m(A)$ is the confidence level for event A . Now it is used to combine the 4 evidences in example 1 and get the conflict coefficient $K = 0$, which shows there are no conflicts among the evidences. The combining results are shown in Table 2.

From Table 1 and 2 it can be seen that:

- The first three evidence sources argue that parameter x is included in the interval [1.0, 2.0]. However, the fourth evidence source declares that $x \in [1.4, 2.0]$. So the combined result is $x \in [1.4, 2.0]$ and the interval [1.0, 1.4] is discarded directly.

Obviously, the result is inconsistent with that the conflict coefficient K is 0

- The combined result is reasonable in the respect of enhancing the confidence level for $x \in [1.4, 1.5]$.

Even most of the evidences declare that x is falling in the interval $[1.0, 1.4]$ with some probability, the declaration is rejected even though only one evidence source is not agree with the others. This phenomenon is so called “one-vote negation”. It may be reasonable in some cases, but in some occasions, such as risk evaluation, decision analysis, such “one-vote negation” may result in unsafe results.

Yager’s rule: To solve the problem that Dempster’s rule involved counter-intuitive behaviors when evidence highly conflicts, Yager made some improvements on Dempster’s rule (Yager, 1989).

Let m_1, m_2, \dots, m_n be n BPAs on a frame of discernment, Θ , which focal elements are $A_i (i=1, 2, \dots, N)$. Then:

$$m_y(A) = \begin{cases} \sum_{\cap A_i = A, 1 \leq i \leq N} \prod m_i(A_i) & A \neq \emptyset, \Theta \\ \sum_{\cap A_i = A, 1 \leq i \leq N} \prod m_i(A_i) + K & A = \Theta \\ 0 & A = \emptyset \end{cases} \quad (10)$$

where, $K = \sum_{\cap A_i = \emptyset, 1 \leq i \leq N} \prod m_i(A_i)$ is the conflict coefficient. If $K = 0$, then there are no conflicts among the evidences; on the other hand, if $K \rightarrow 1$, then there have high level conflicts.

If there are no conflicts, the combination results are same as Dempster’s. But in Yager’s rule, instead of normalizing out the conflict in Dempster’s rule, it ultimately attributes conflict to the universal set Θ through the conversion of the ground probability assignment to the basic probability assignments. The interpretation of the BPA of Θ is the degree of ignorance. Yager deemed that as it could not make a reasonable choice for conflict evidences, it should take them as uncharted territory. This is an epistemologically honest interpretation of the evidence as it does not change the evidence by normalizing out the conflict.

The combination results of Yager’s rule are shown in Table 3.

Obviously the results are the same as that of the Dempster’s rule and cannot solve the issue of conflict.

Inagaki’s rule: Inagaki defined a continuous parameterized class of combination rules which

Table 3: The combination results of Yager’s rule

K	A	$m(A)$
0	[1.4, 1.5]	0.9990
	[1.4, 2.0]	0.0001
	[1.4, 1.6]	0.0009

Table 4: The combination results of Inagaki’s rule

K	A	$m(A)$
0	[1.4, 1.5]	0.999
	[1.4, 2.0]	0.0001
	[1.4, 1.6]	0.0009

subsumes both Dempster’s rule and Yager’s rule (Inagaki, 1991).

If m_1, m_2, \dots, m_n are n BPAs on a frame of discernment, Θ , which focal elements are $A_i (i=1, 2, \dots, N)$, then Inagaki’s rule, the combined ground probability assignment is defined as the combination of n BPAs as:

$$G(A) = \sum_{\cap A_i = A, 1 \leq i \leq N} \prod m_i(A_i) \quad (11)$$

Then Inagaki argued that every combination rule could be expressed as:

$$m_i(A) = \begin{cases} 0 & A = \emptyset \\ G(A)(1+kG(\emptyset)) & A \neq \emptyset, \Theta \\ G(A)(1+kG(\emptyset)) + (1+kG(\emptyset)-k)G(\Theta) & A = \Theta \end{cases} \quad (12)$$

where, parameter k was used for normalization and satisfied the following constraint:

$$0 \leq k \leq \frac{1}{1-G(\emptyset)-G(\Theta)} \quad (13)$$

In Inagaki’s rule, conflicting and unknown evidences are all reassigned and the reassigned factors depend on the parameter k . Generally, k can be determined through experimental data, simulation, or the expectations of an expert in the context of a specific application. When $k = 0$, the unified combination rule coincides with Yager’s rule. When $k = 1/(1-G(\emptyset))$, the rule corresponds to Dempster’s rule. When k is equal to its upper bound, then the most extreme rule availed by the formulation is:

$$m_i(A) = \begin{cases} 0 & A = \emptyset \\ G(A) \left(\frac{1-G(\Theta)}{1-G(\Theta)-G(\emptyset)} \right) & A \neq \emptyset, \Theta \\ G(A) & A = \Theta \end{cases} \quad (14)$$

If Inagaki’s rule is used for the combination of example 1, then $G(\emptyset) = 0$ and combination results are shown in Table 4, which are the same as that of the Dempster’s rule and Yager’s rule and also cannot solve the issue of conflict.

A NOVEL COMBINATION RULE FOR INTERVAL-VALUED EVIDENCE

As discussed above, Dempster's rule, Yager's rule and Inagaki's rule all fall into the contradiction that conflict coefficient is zero while the combination results discard some evidences when combine the evidences in the example 1. All the combination results are not satisfying. Generally, evidences are described as point sets. Evidences in different sources are always inclusion or equal relations, then the combination results with Yager's rule or Inagaki's rule are always reasonable. However, in interval-valued evidences, the upper and lower bounds of interval focal elements are often not equal and they are always intersection relations. In these cases, using abovementioned three combination rules may get unreasonable results. This is the root cause of the contradiction in the combination of conflicting evidences.

To get more reasonable combination results for interval-valued evidences with the existing rules, focal elements with intersection relations should be translated to equal or inclusion relations first.

If there are n evidence sources and the focal elements of the i -th source are $A_i^1, A_i^2, \dots, A_i^{r_i}$, respectively, where $A_i^j = [a_i^{l-j}, a_i^{u-j}]$. Then the disjointing algorithm is described as follows:

- Comparing the j -th focal element A_1^j in the first evidence source with k -th focal element A_2^k ($1 \leq k \leq r_2$) in the secondary evidence source. If $A_1^j \not\subset A_2^k$ and $A_2^k \not\subset A_1^j$, denote $A_1^j \cap A_2^k = [b, a_i^{u-j}]$ (or $A_1^j \cap A_2^k = [a_i^{l-j}, b]$), in which $b \in [a_i^{l-j}, a_i^{u-j}]$, then A_1^j is divided into two focal elements, $A_1^{j1} = [a_i^{l-j}, b]$ and $A_1^{j2} = [b, a_i^{u-j}]$, which BPAs are:

$$m(A_1^{j1}) = (b - a_i^{l-j}) \cdot m(A_1^j) / (a_i^{l-j} - a_i^{u-j}) \tag{15}$$

$$m(A_1^{j2}) = (a_i^{u-j} - b) \cdot m(A_1^j) / (a_i^{l-j} - a_i^{u-j}) \tag{16}$$

- Repeating the step 1 for $j = 1$ to $j = r_1$, then the disjointing operations between the first and second evidence sources are completed
- Repeating the step 1 and 2, do the disjointing operations for the first evidence source to the other $n-1$ sources
- Do the disjointing operations for the second evidence source to the other $n-2$ sources and so on. Then the disjointing operations are done for all the interval focal elements.

After the disjointing operations, the relation between any two focal elements is equation or inclusion and the BPA of each focal element is reallocated.

Then, with the concept of distance between evidences which proposed by Jousselme *et al.* (2001), evidences with high conflicts are modified and more credible results are obtained.

If m_i and m_j are two BPAs in Θ , which focal elements are A_i and B_j then the distance between m_i and m_j are:

$$d(m_i, m_j) = \sqrt{\frac{1}{2}(\mathbf{m}_i - \mathbf{m}_j) \mathbf{D} (\mathbf{m}_i - \mathbf{m}_j)} \tag{17}$$

where, $\mathbf{m}_i = [m_i(A_1), m_i(A_2), \dots, m_i(A_{2^n})]$ and \mathbf{D} is a matrix with $2^n \times 2^n$ elements, each element is:

$$\mathbf{D}(A_i, B_j) = \frac{|A_i \cap B_j|}{|A_i \cup B_j|} \quad i, j = 1, 2, \dots, 2^n \tag{18}$$

where, $|A_i \cap B_j|$ shows the conflict and similarity between A_i and B_j . If $|A_i \cap B_j| = 0$, then the similarity between A_i and B_j is 0 and the conflict is the biggest; otherwise, if $|A_i \cap B_j| = 1$, then the similarity between A_i and B_j is 1 and there is no conflict between them.

From Eq. (17) and (18), the formula of distance between m_i and m_j is:

$$d(m_i, m_j) = \sqrt{\frac{1}{2}(\|\mathbf{m}_i\|^2 + \|\mathbf{m}_j\|^2 - 2\langle \mathbf{m}_i, \mathbf{m}_j \rangle)} \tag{19}$$

where,

$$\|\mathbf{m}_i\|^2 = \langle \mathbf{m}_i, \mathbf{m}_i \rangle, \quad \|\mathbf{m}_j\|^2 = \langle \mathbf{m}_j, \mathbf{m}_j \rangle \quad \text{and} \quad \langle \mathbf{m}_i, \mathbf{m}_j \rangle$$

is the inner product:

$$\langle \mathbf{m}_i, \mathbf{m}_j \rangle = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_i(A_i) m_j(B_j) \frac{|A_i \cap B_j|}{|A_i \cup B_j|} \tag{20}$$

If there are r evidence sources, then all the distances between each two evidences is a matrix of:

$$\mathbf{D}_M = \begin{bmatrix} 0 & d_{12} & \dots & d_{1r} \\ d_{21} & 0 & \dots & d_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ d_{r1} & d_{r2} & \dots & 1 \end{bmatrix}_{r \times r} \tag{21}$$

The similar coefficient between m_i and m_j is defined as:

$$s_{ij} = 1 - d_{ij} \tag{22}$$

Table 5: BPAs after disjointing operations

	A	B	C	B,C,D	Θ
m_1	0.72	0.18	0.00	0.00	0.1
m_2	0.72	0.18	0.00	0.00	0.1
m_3	0.72	0.18	0.00	0.00	0.1
m_4	0.00	0.45	0.45	0.1	0.00

Table 6: The combination results of proposed method

$m(A)$	$m(B)$	$m(C)$	$m(B,C,D)$	$m(\Theta)$
0.8796	0.1142	0.0046	0.0008	0.0008

Eq. (22) shows that the smaller the distance, the bigger the similarity between two evidences. So the similarity matrix of all evidence is:

$$S_M = \begin{bmatrix} 0 & s_{12} & \cdots & s_{1r} \\ s_{21} & 0 & \cdots & s_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ s_{r1} & s_{r2} & \cdots & 1 \end{bmatrix}_{r \times r} \quad (23)$$

Sum each line of S_M and the support degree of other evidences to m_i is:

$$spt(m_i) = \sum_{j=1}^r s_{ij} \quad (i, j = 1, 2, \dots, r) \quad (24)$$

Normalize Eq. (24) and get the weight of m_i as:

$$\varepsilon_i = \frac{spt(m_i)}{\sum_{i=1}^r spt(m_i)} \quad (i, j = 1, 2, \dots, r) \quad (25)$$

Then a weighted average method is used to get the new evidences (Murphy, 2000). As there are r evidence sources, there will be $r-1$ times of weighted average process.

NUMERICAL EXAMPLES

The proposed disjointing algorithm is used for example 1 first. The new focal elements are get as $A=[1.0,1.4]$, $B=[1.4,1.5]$, $C=[1.5,1.6]$, $D=[1.6,2.0]$, and $\Theta = \{A, B, C, D\}$. BPAs of the focal elements after disjointing operations are shown in Table 5.

From Table 5, the conflict coefficient is $K = 0.98747$, which means they are high conflicting evidences. Obviously, the value of K is quite different with the result that has no disjointing operation (i.e., $K = 0$). The new value of K is consistent with intuitive judgment. From Eq. (19)~(21), D_M is calculated as:

$$D_M = \begin{bmatrix} 0 & 0 & 0 & 0.6507 \\ 0 & 0 & 0 & 0.6507 \\ 0 & 0 & 0 & 0.6507 \\ 0.6507 & 0.6507 & 0.6507 & 0 \end{bmatrix}$$

Then S_M is:

$$S_M = \begin{bmatrix} 1 & 1 & 1 & 0.3493 \\ 1 & 1 & 1 & 0.3493 \\ 1 & 1 & 1 & 0.3493 \\ 0.3493 & 0.3493 & 0.3493 & 1 \end{bmatrix}$$

From Eq. (24) and (25), weights of four evidence sources are:

$$\varepsilon = [0.2769, 0.2769, 0.2769, 0.1693]$$

By weighted average method, the combined results are obtained, as shown in Table 6.

Contrast Table 2 to 4 with Table 6, it can be seen that the proposed method avoids the contradiction that conflict coefficient is zero while the combination results discard some evidences. $K = 0.98747$ shows that evidences in example 1 are high conflicting. In Table 6, conflicting evidence A ($A=[1.0,1.4]$) is reserved, which avoid to get unsafe or wrong decision when the rule is used in risk evaluation, decision analysis and other occasions. The results are consistent with intuitive judgment.

CONCLUSION

To avoid the contradiction that conflict coefficient is zero while the combination results discard some evidences for interval-valued evidences, a novel combination method is proposed. In the new method, a disjointing algorithm is proposed as a preprocessing and then a weighted average method is used to combine the evidences. The numerical examples show that the proposed method can solve the contradiction and get reasonable combination results.

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