Research Journal of Applied Sciences, Engineering and Technology 6(13): 2451-2458, 2013
DOI:10.19026/rjaset.6.3721
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: December 20, 2012 Accepted: February 01, 2013 Published: August 05, 2013

Research Article

Study on Flow Characteristic of Non-Newtonian fluid in Eccentric Annulus

[1]Li Mingzhong, [1]Wang Ruihe, [1]Wang Chengwen, [1]Fang Qun and [2]XUE Yongbo
[1]School of Petroleum Engineering, China University of Petroleum (Huadong), Qingdao 266580, China
[2]Oil and Gas Cooperation and Development Company, BHDC, 300280, Tianjin Dagang District

Abstract: This study studied the flow characteristic of non-newtonian in eccentric annulus of highly-deviated well. On the basis of dimensionless analysis of motion equations and continuity equation, Hele-Shaw model suitable for fluid flow in the annulus was derived. Combined with H-B rheological model, velocity and stream distribution model were founded and calculated by numerical method. Furthermore, two-dimensional flow characteristic in eccentric annulus was got and the influence of different factors (such as yield stress, pressure gradient or eccentricity) on velocity distribution in condition of laminar flow was analyzed. Width of flow core in the annular is proportional to yield stress and inversely proportional to pressure gradient. In eccentric annulus, eccentricity influences the stream distribution remarkably: with the increment of eccentricity, the contour lines of stream function gradually centralize in the widest annular gap, however distribute the most loosely in the narrowest annular gap. Axial velocity is the largest in the widest gap. The larger eccentricity is, the larger contrast of axial velocity between in the widest gap and in the narrowest gap is. There is the largest azimuthal velocity in an annular gap of a certain azimuthal angle, however which equals to zero in the widest and narrowest annular gap separately. The larger eccentricity is, the more homogeneous azimuthal velocity is. The velocity contrast in the entire annulus can be smoothed by increasing pressure gradient, power law index or decreasing yield stress.

Keywords: Eccentric annulus, H-B fluid, hele-shaw model, stream function, velocity distribution

INTRODUCTION

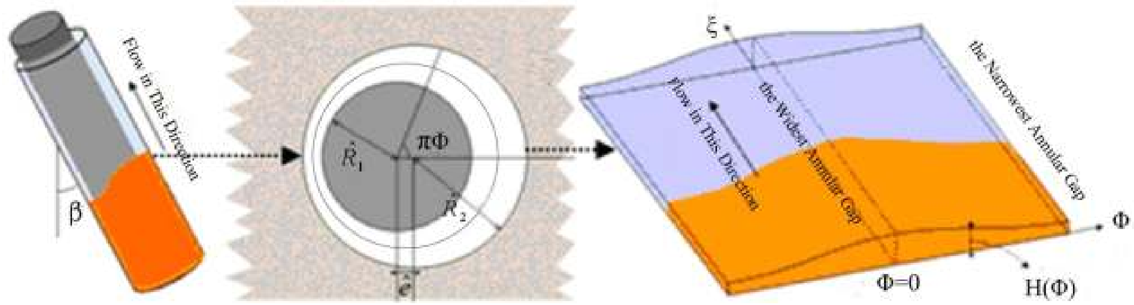
In process of oil wells construction, flow characteristics of non-newtonian fluid in annulus made up with wellbore and drilling pipes or wellbore and casings is the theoretical foundation for drilling fluid hydraulic parameter optimum design and displacement efficiency improvement, which has very important engineering significance (Li *et al*., 2002; Vaugh, 1965). In the annulus of vertical well, fluid gravity is only point to the wellbore axial direction, so researches about fluid only in one-dimensional flow were believable (Wang and Su, 1998; Wang *et al*., 2008). As for highly deviated wells or horizontal wells, fluid flow in both azimuthal and axial direction of the annulus exists simultaneously (Zheng, 1992). However the azimuthal flow was often neglected in related researches up to now. In addition, the non-newtonian property of the fluid in the annulus was usually described by Bingham model or power-law model, which could not reflect rheological property adequately (Redberger and Charles, 1962; Li *et al*., 2004).

Hele-Shaw model is now being used to study two-dimensional flow characteristic in two infinite plates of very short distance (Souhar and Aniss, 2012), which have been put into application in the field of porous media flow (Daripa and Pasa, 2007; Trevelyan *et al*., 2011) and injection molding (Cao and Shen, 2004). In a typical well annulus, a fact that axial length-scale is so much greater than the annular circumference and annular circumference is much greater than annular gap (Pelipenko and Frigaard, 2004), is usually satisfied, which guarantees the eccentric annulus geometry to be mapped to the Hele-Shaw cell geometry. So the Hele-Shaw model will be introduced in our study in order to investigate the fluid flowing characteristics, such as the features about flowing velocity distribution or stream function distribution in the annulus. Moreover the achievement of our research objective needs an accurate description about non-newtonian fluid. In our study H-B fluid model is chosen, which is better than Bingham or power-law model and can describe yield characteristic and shear thinning of non-newtonian fluid (Guo *et al*., 1997). In the end the flow characteristics of H-B fluid in the annulus of highly deviated well can be studied.

FLOW CHARACTERISTIC OF H-B FLUID IN THE ANNULUS

Physical model: In highly-deviated wells or horizontal wells, the eccentric annulus is very common because of

Correspondong Author: Li Mingzhong, School of Petroleum Engineering, China University of Petroleum (Huadong), Qingdao 266580, China



(a) (b) (c)

Fig. 1: The eccentric annulus in wellbore

the gravity force of drilling pipes or casings. Figure 1 shows the basic physical model for our study: the entire annulus is unfolded as a slot geometry in the azimuthal direction from zero to 2π, whose width changes with the azimuthal angle (Bittleston *et al*., 2002), as is shown in Fig.1(c). Several assumptions were made:

- The fluid rheology in the annulus was described by H-B model.
- Flow regime in the annulus satisfied fully developed laminar and flow rate kept constant.
- Fluid physical parameters could not change during the flow in the annulus.

Derivation of the Hele-shaw model suitable for fluid flow in the annulus: Based on motion equation in cylindrical coordinates, characteristic parameters were selected, then annular geometric parameters, fluid properties and fluid injection parameters were non-dimensionalized. The coordinate system (ζ, y, ϕ) was also built, as shown in Fig. 2. Fluid flow in the micro unit, whose width was dø and height was H(ϕ), was chosen as our main research object. After the parameters in the motion equation were substituted by dimensionless parameters and characteristic parameters, the Hele-shaw model suitable for fluid flow in the annulus was derived (Bittleston and Frigaard, 2004; Pelipenko and Frigaard, 2004).

$$\begin{cases} -\dfrac{\partial p}{\partial \phi} + \dfrac{\partial}{\partial y}\tau_{\phi y} + \dfrac{\rho \sin\beta \sin \pi\phi}{St^*} = 0 \\ -\dfrac{\partial p}{\partial \xi} + \dfrac{\partial}{\partial y}\tau_{\xi y} - \dfrac{\rho \cos\beta}{St^*} = 0 \end{cases} \quad (1)$$

St* is expressed as:

$$St^* = \frac{\hat{\tau}^*}{\hat{\rho}^* \hat{g}^* \hat{r}_a^{\ *} \delta^*} \quad (2)$$

where,

$\hat{\tau}^*$ = scale for shear stress

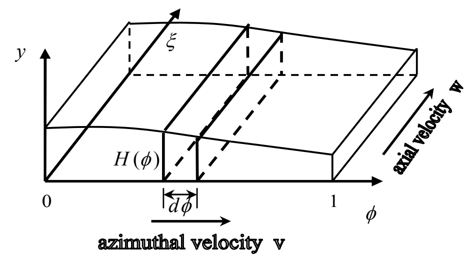


Fig. 2: Foundation of fixed coordinate system

$\hat{\tau}^*$ = max $[\hat{\tau}_{K,Y} + \hat{K}_k(\hat{r}^*)^{nk}$

$\hat{\rho}^*$ = Scale for fluid density

$\hat{\rho}^*$ = Max$[\hat{\rho}_k$ (k=1,2, represent cement slurry and drilling mud separately).

$\hat{g}^*$ = Gravity acceleration

$$\delta^* = (\hat{R}_1 + \hat{R}_2)/(\hat{R}_1 - \hat{R}_2)$$

β is the deviation angle; $\hat{\rho}_k$, $\hat{\tau}_{k,y}$, $\hat{K}_k$ and $\hat{n}_k$ are the density, yield stress, consistency and power law index of fluid k separately; $\hat{r}^*$ is the characteristic scale for the rate of strain; $\hat{R}_k$ is the inner radius of the well hole; $\hat{R}_k$ is the outer radius of the casing; H is the dimensionless half-gap width of the annulus, having H(ϕ) = 1+ecos(π ϕ)(θ and ϕ are separately azimuthal angle of the annulus and dimensionless azimuthal angle, θ = π ϕ, θε($-\pi,\pi$),where ϕ = 0 indicates the widest annular gap, where ϕ = 1 represents the narrowest annular gap); e is the eccentricity; p, τ and ρ_k are dimensionless pressure, deviatoric stress tensor and fluid density.

The vectors for shear stress and pressure gradient were defined as $\bar{\tau}$ and $\bar{G}$. The formula (1) can be simplified as:

$$\frac{\partial}{\partial y}\vec{\tau} = -\vec{G} \quad (3)$$

where,

$$\vec{G} = (G_\phi, G_\xi) = (-\frac{\partial p}{\partial \phi} + \frac{\rho \sin\beta \sin \pi\phi}{St^*}, -\frac{\partial p}{\partial \xi} - \frac{\rho \cos\beta}{St^*})$$

$G = |\vec{G}|$, $\vec{\tau} = (\tau_{\phi y}, \tau_{\xi y})$

Then the continuity equation for incompressible fluid was non-dimensionalized:

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial \xi} = 0 \tag{4}$$

where, u, v and w are dimensionless variables separately for radial velocity, azimuthal velocity and axial velocity. In the micro unit, the average velocity $\bar{u}$, $\bar{u}$,$\bar{v}$, $\bar{W}$ could be calculated by:

$$\bar{u}(\phi, \xi, t) = \frac{1}{H}\int_0^H u dy \quad \bar{v}(\phi,\xi,t) = \frac{1}{H}\int_0^H v dy \quad \bar{w}(\phi,\xi,t) = \frac{1}{H}\int_0^H w dy \tag{5}$$

By substituting (5) into equation (4), we obtained:

$$\frac{\partial}{\partial \phi}[H\bar{v}] + \frac{\partial}{\partial \xi}[H\bar{w}] = 0 \tag{6}$$

Furthermore, the stream function ψ was defined according to the formula (6):

$$\frac{\partial \psi}{\partial \phi} = H\bar{w} \quad \frac{\partial \psi}{\partial \xi} = -H\bar{v} \tag{7}$$

Velocity distribution of H-B fluid in the annulus: H-B rheological model, also called generalized rheological model, or power-law model with yield stress, can reflect yield characteristic and shear thinning of non-newtonian fluid. Non-dimensionalized H-B model for drilling mud and cement slurry and we obtained:

$$\begin{cases} \eta = \kappa \dot{\gamma}^{n-1} + \dfrac{\tau_Y}{\dot{\gamma}} \Leftrightarrow \tau > \tau_Y \\ \dot{\gamma} = 0 \qquad\qquad \Leftrightarrow \tau > \tau_Y \end{cases} \tag{8}$$

where,

$\tau = (\tau_{y\phi}^2 + \tau_{y\delta}^2)1/2$, $\dot{\gamma} = [\partial v/\partial y)2 + (\partial w/\partial y)]^2]^{1/2}$

When H-B fluid flows in the micro unit, there is a domain where velocity gradient is zero, which is known as the flow core. By use of formula (2) and symmetry condition $\tau = 0(y = 0)$, the extent of flow core was $[0, y_0]$ ($y_0 = \tau_Y/G$) in the annulus gap.

In order to get velocity profile in the annulus gap, no-slip condition on wall was needed:

$$\begin{cases} v = 0 \\ w = 0 \end{cases} \quad (y = H) \tag{9}$$

Hele-Shaw model (1), combined with the H-B rheological model (8) and no-slip condition (9), could give the axial velocity and azimuthal velocity distribution in the micro unit when the pressure gradient satisfied the condition $GH > \tau_Y$:

$$v = \begin{cases} (\dfrac{\partial p}{\partial \phi} - \dfrac{\rho \sin\beta \sin\pi\phi}{St^*})[\dfrac{(GH-\tau_Y)^{m+1} - (Gy-\tau_Y)^{m+1}}{G^2\kappa^m(m+1)}] & y \in [y_0, H] \\ (\dfrac{\partial p}{\partial \phi} - \dfrac{\rho \sin\beta \sin\pi\phi}{St^*})[\dfrac{(GH-\tau_Y)^{m+1}}{G^2\kappa^m(m+1)}] & y \in [0, y_0] \end{cases} \tag{10}$$

$$w = \begin{cases} (\dfrac{\partial p}{\partial \xi} + \dfrac{\rho \cos\beta}{St^*})[\dfrac{(GH-\tau_Y)^{m+1} - (Gy-\tau_Y)^{m+1}}{G^2\kappa^m(m+1)}] & y \in [y_0, H] \\ (\dfrac{\partial p}{\partial \xi} + \dfrac{\rho \cos\beta}{St^*})[\dfrac{(GH-\tau_Y)^{m+1}}{G^2\kappa^m(m+1)}] & y \in [0, y_0] \end{cases} \tag{11}$$

Moreover, the average velocity $\bar{v}$ and $\bar{w}$ in the micro unit of azimuthal angle ø were derived by substituting (10) and (11) into (5):

$$\bar{v} = \frac{1}{HG^2(m+1)}(\frac{\partial p}{\partial \phi} - \frac{\rho \sin\beta \sin\pi\phi}{St^*})(\frac{GH-\tau_Y}{\kappa})^m \frac{(GH-\tau_Y)(HG+HGm+\tau_Y)}{G(m+2)} \tag{12}$$

$$\bar{w} = \frac{1}{HG^2(m+1)}(\frac{\partial p}{\partial \xi} + \frac{\rho \cos\beta}{St^*})(\frac{GH-\tau_Y}{\kappa})^m \frac{(GH-\tau_Y)(HG+HGm+\tau_Y)}{G(m+2)} \tag{13}$$

where, m is the inverse of power law index. From (12) and (13), it indicates that the vector of average velocity $(\bar{v}, \bar{w})$ is parallel to the vector $\bar{G}$.
In the condition of $GH \le \tau_Y$, we got v = w = 0.

Stream function formulation of H-B fluid in the annulus: In order to analyze flow characteristic of H-B fluid in the entire annular, the solution of stream function ψ is needed (Frigaard and Pelipenko, 2003; Frigaard and Ngwa, 2010). The modules of stream function gradient is $|\nabla\psi| = H(\bar{v}^2 + \bar{w}^2)^{1/2}$:

$$|\nabla \psi| = \begin{cases} \dfrac{(GH-\tau_Y)[HG(m+1)+\tau_Y]}{G^2(m+1)(m+2)}(\dfrac{GH-\tau_Y}{\kappa})^m & HG > \tau_Y \\ 0 & HG \le \tau_Y \end{cases} \tag{14}$$

Substituting (14) back into (12) and (13) when $GH>\tau_Y$, we had:

$$-\frac{\partial \psi}{\partial \xi} \cdot \frac{1}{|\nabla \psi|} = \frac{G_\phi}{G} \quad \frac{\partial \psi}{\partial \phi} \cdot \frac{1}{|\nabla \psi|} = \frac{G_\xi}{G} \tag{15}$$

Meanwhile, we defined parameter χ as $\chi = G - \tau_Y/H$, which meant the drive pressure gradient left after the yield stress of fluid was overcome. Substituting into (15) for G in terms of x:

$$\frac{[\chi(|\nabla\psi|)+\tau_Y/H]}{|\nabla\psi|}\frac{\partial\psi}{\partial\xi}=\frac{\partial p}{\partial\phi}-\frac{\rho\sin\beta\sin\pi\phi}{St^*} \tag{16}$$

$$\frac{[\chi(|\nabla\psi|)+\tau_Y/H]}{|\nabla\psi|}\frac{\partial\psi}{\partial\phi}=-\frac{\partial p}{\partial\xi}-\frac{\rho\cos\beta}{St^*} \tag{17}$$

Differentiating Eq. (16) with respect to ζ and differentiating Eq. (17) with respect to ø and adding the two second-order partial differential equations, then we concluded:

$$\nabla\cdot\vec{S}=-f \tag{18}$$

(18) was the stream function formulation[12-17].
where,

$$\begin{cases}\vec{S}=\dfrac{[\chi(|\nabla\psi|)+\tau_Y/H]}{|\nabla\psi|}\nabla\psi & \chi>0\\ |\nabla\psi|=0 & \chi\le 0\end{cases} \tag{19}$$

$$f=\nabla\cdot\left(\frac{\rho\cos\beta}{St^*},\frac{\rho\sin\beta\sin\pi\phi}{St^*}\right) \tag{20}$$

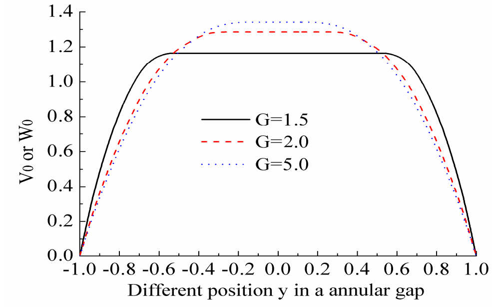
ANALYSIS AND DISCUSSION

Analysis of velocity distribution in the annulus of azimuthal angle ø: In this section, we introduced parameters v_0 and w_0. In the annulus gap where azimuthal angle equals to ϕ, the ratio between axial velocity w and the average axial velocity $\overline{w}$ in the gap is defined as w_0, having $w_0 = w/\overline{w}$; In the same way, v_0 is defined $v_0 = v/\overline{v}$. v_0 and w_0. can be used to characterize the velocity profile of H-B fluid in the annulus gap. w_0 and v_0 were derived from (10)~(13):

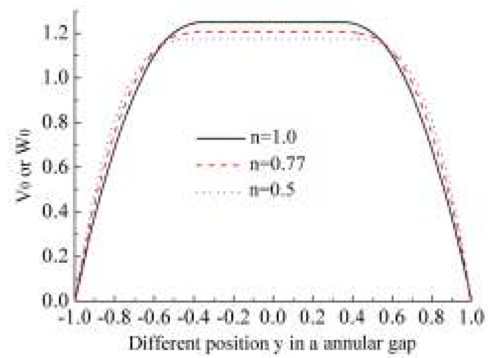
$$v_0=w_0=\begin{cases}\left[1-\left(\dfrac{y/H-\tau_Y/GH\cdot}{1-\tau_Y/GH}\right)^{m+1}\right]\cdot\dfrac{m+2}{1+m+\tau_Y/GH} & y\in[y_0,H]\\ \dfrac{m+2}{1+m+\tau_Y/GH} & y\in[0,y_0]\end{cases} \tag{21}$$

Here we studied the influence of factors, such as pressure gradient, yield stress, power law index and so on, on velocity profile in the annulus gap by controlling variable method. The basic datum were chosen as τ_Y = 0.8, m = 1/0.77, H = 1, G = 2. For example, substituting pressure gradient 1.5, 2, 5 and the other basic datum into (21), the pressure gradient's influence on velocity profile in the annulus was got, as shown in Fig. 3(a). The influence of the other factors are shown in Fig. 3(b)~(d).

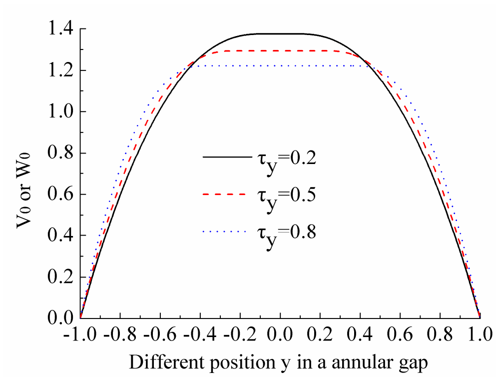
From Fig. 3, some conclusions were drawn. As the pressure gradient decreases and yield stress increases, the velocity profile of H-B fluid in the annulus becomes more applanate and flow core gets wider. The increment of power law index leads the velocity profile in the annulus to get longer, however does not affect the width of flow core. The increment of annular gap width makes the velocity of H-B fluid increase. To sum up,



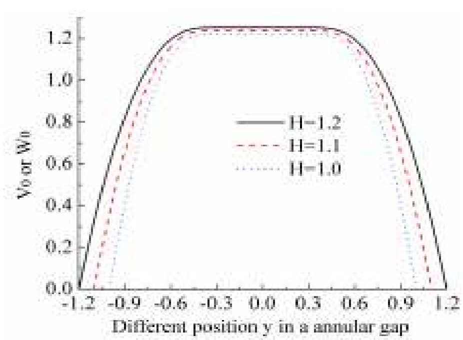
(a) Pressure Gradient



(b) Yield Stress

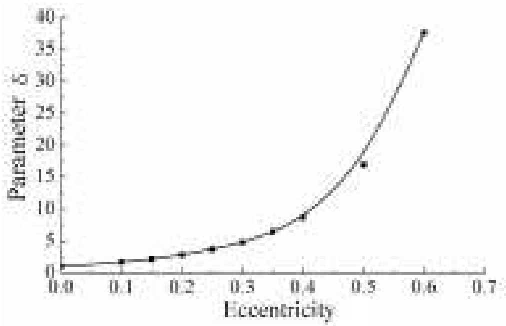


(c) Power Law Index

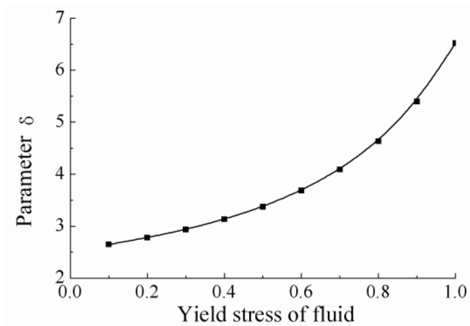


(d) Width of Annular Gap

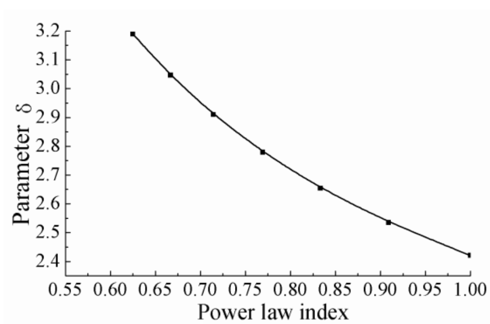
Fig. 3: Influence of different factors on v^0 and w^0



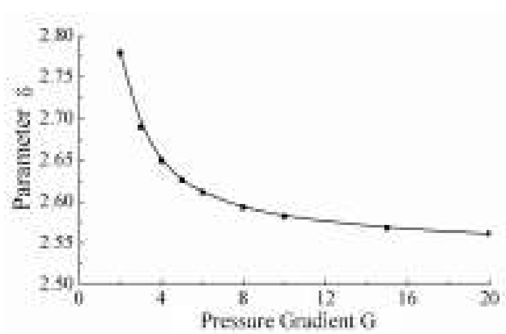
(a) Eccentricity



(b) Yield Stress



(c) Power Law Index



(d) Pressure gradient

Fig. 4: Influence of different factors on δ

the width of flow core in the annulus is proportional to yield stress, inversely proportional to pressure gradient and has no relationship with power law index and consistency.

Contrast of velocity between in the widest gap and in the narrowest gap: Calculated the resultant velocity $|\bar{\mu}|$ in the annulus of an azimuthal angle:

$$|\vec{u}| = (\bar{v}^2 + \bar{w}^2)^{1/2} = \frac{(GH-\tau_Y)[HG(m+1)+\tau_Y]}{HG^2(m+1)(m+2)}\left(\frac{GH-\tau_Y}{\kappa}\right)^m \quad (22)$$

Parameter δ was defined as:

$$\delta = \frac{\{|\vec{u}|(H_{\max})\}}{\{|\vec{u}|(H_{mix})\}} \quad (23)$$

where, $\{|\bar{\mu}|(H_{max})\}$ is the resultant velocity in the widest annular gap; $|\bar{\mu}|(H_{min})\}$ is the resultant velocity in the narrowest annular gap.

Furthermore, we have:

$$\delta = \frac{H_{\min}}{H_{\max}} \cdot \left(\frac{GH_{\max} - \tau_Y}{GH_{\min} - \tau_Y}\right)^{m+1} \cdot \frac{GH_{\max} \cdot (m+1) + \tau_Y}{GH_{\min} \cdot (m+1) + \tau_Y} \quad (24)$$

δ can be used to evaluate the nonhomogeneity of fluid velocity in eccentric annular. The more δ is, the more nonhomogeneous the velocity is in the annulus. δ was calculated by formula (24). The research approach was still controlling variable method. The basic datum were $\tau_Y = 0.8$, G = 2, m = 1/0.77, e = 0.2. Results are shown in Fig. 4.

Some conclusions were obtained from the Fig. 4. Casing eccentricity affects the velocity distribution severely, the larger eccentricity is, the more contrast of velocity between in the widest annular gap and in the narrowest annular gap is. Increasing pressure gradient and power law index or decreasing yield stress can reduce velocity contrast in the annulus.

Characteristic of stream function distribution in the annulus:

Numerical solution of the steam function formulation: The numerical solution of stream function formulation (18) could help us analyze stream function distribution in the eccentric annulus and calculate velocity in every annular gap. If there was only one kind of fluid in the annulus and the fluid was incompressible, the (18) could be simplified as:

$$\begin{cases} \nabla \cdot [r \dfrac{\chi(|\nabla \psi|) + \tau_Y / H}{|\nabla \psi|} \nabla \psi] = 0 & \chi > 0 \\ |\nabla \psi| = 0 & \chi \le 0 \end{cases} \quad (25)$$

Here the flow region was bounded in the are a $(\phi, \zeta) = (0, 1) \times (0, Z)$. The symmetry of annular geometry determines the symmetry of stream function in the annulus, so at dimensionless azimuthal angle $\phi = 0$ and $\phi = 1$, the azimuthal velocity satisfies:

$$\bar{v}(0,\xi,t) = 0 \quad \bar{v}(1,\xi,t) = 0 \tag{26}$$

From formula (26), stream functions in the widest and narrowest annular gap have:

$$\psi(0,\xi,t) = 0 \quad \psi(1,\xi,t) = Q(t) \tag{27}$$

where, Q(t) is the dimensionless flow rate: $Q(t) = \hat{Q}(\hat{t})/\hat{Q}^*$. If the flow rate keep unchanged, $Q(t) = 1$.

We assume that the flow is only in the axial direction at the entrance and outlet of the annulus (Bittleston *et al*., 2002), the conditions following can be got:

$$\psi(\phi,0,t) = \psi_0(\phi,t) \quad \psi(\phi,Z,t) = \psi_Z(\phi,t) \tag{28}$$

Unfolded the stream function formulation (25) further for x > 0:

$$\frac{\chi(|\nabla\psi|)+\tau_Y/H}{|\nabla\psi|}\cdot\frac{\partial^2\psi}{\partial\phi^2}+\left[\frac{1}{|\nabla\psi|}\frac{\partial G}{\partial\phi}-\frac{G}{(|\nabla\psi|)^2}\frac{\partial(|\nabla\psi|)}{\partial\phi}\right]\cdot\frac{\partial\psi}{\partial\phi}+\frac{\chi(|\nabla\psi|)+\tau_Y/H}{|\nabla\psi|}\cdot\frac{\partial^2\psi}{\partial\xi^2}=0 \tag{29}$$

where,

$$\frac{\partial(|\nabla\psi|)}{\partial\phi}=\frac{H(m+1)(m+2)(HG-\tau_Y)^m}{G(m+1)(m+2)\kappa^m}\left(H\frac{\partial G}{\partial\phi}+G\frac{\partial H}{\partial\phi}\right)-\frac{2(HG-\tau_Y)^{m+1}(mHG+HG+\tau_Y)}{G^3(m+1)(m+2)\kappa^m}\cdot\frac{\partial G}{\partial\phi} \tag{30}$$

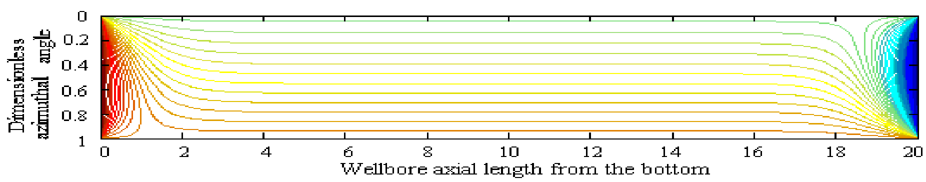
$$\frac{\partial H}{\partial\phi}=\frac{\partial}{\partial\phi}[1+e\cdot\cos(\pi\phi)]=-\pi\cdot e\cdot\sin(\pi\phi) \tag{31}$$

$$\frac{\partial G}{\partial\phi}=\left[\left(-\frac{\partial p}{\partial\phi}+\frac{\rho\sin\beta\sin\pi\phi}{St^*}\right)^2+\left(-\frac{\partial p}{\partial\xi}-\frac{\rho\cos\beta}{St^*}\right)^2\right]^{-0.5}\left(-\frac{\partial p}{\partial\phi}+\frac{\rho\sin\beta\sin\pi\phi}{St^*}\right)\cdot\frac{\rho\sin\beta}{St^*}\cdot\pi\cdot\cos\pi\phi \tag{32}$$

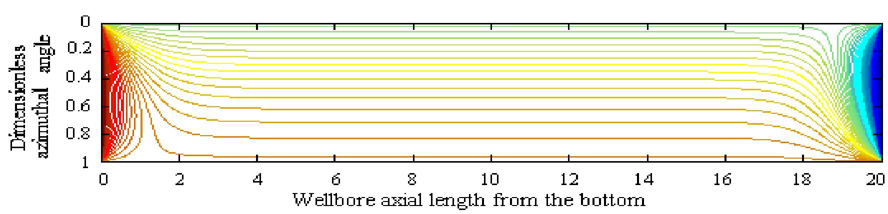
In order to settle the Eq. (29), finite difference method was introduced. The discrete methods about the differential terms were:

$$\frac{\partial^2\psi}{\partial\phi^2}=\frac{\psi_{i+1,j}-2\psi_{i,j}+\psi_{i-1,j}}{\Delta\phi^2}$$
$$\frac{\partial^2\psi}{\partial\xi^2}=\frac{\psi_{i,j+1}-2\psi_{i,j}+\psi_{i,j-1}}{\Delta\xi^2} \quad \frac{\partial\psi}{\partial\phi}=\frac{\psi_{i,j}-\psi_{i-1,j}}{\Delta\phi} \tag{33}$$

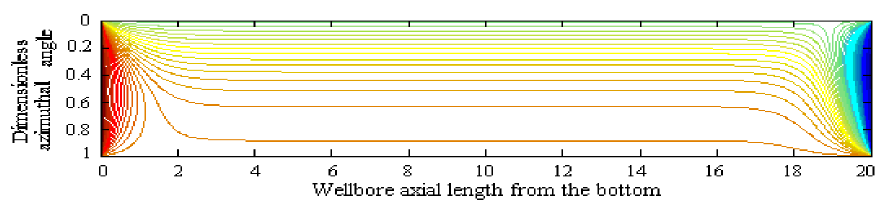
By the discrete methods above, the differential Eq. (29) was transformed into difference equation and a



(a) e = 0



(b) e = 0.2



(e) e = 0.4

Fig. 5: Effect of eccentricity on stream function distribution in wellbore annular

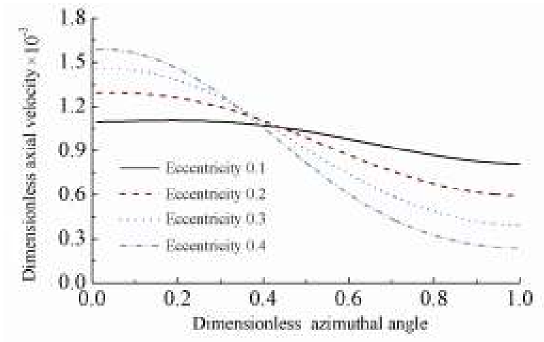


Fig. 6: Effect of eccentricity on axial velocity in the annulus

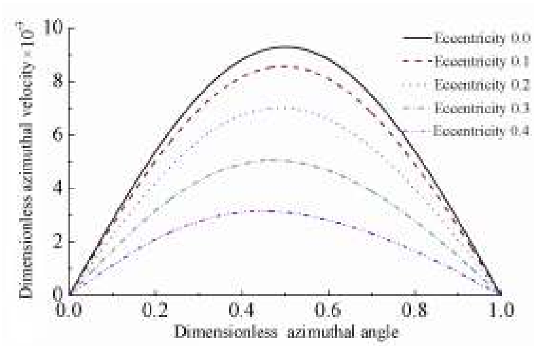


Fig. 7: Effect of eccentricity on azimuthal velocity in the annulus

calculation program was given in software Matlab. Through partial derivatives of the stream function with respect to ϕ and ζ separately, the axial velocity and azimuthal velocity could be got.

$$\overline{w}(i,j)=\frac{\psi_{i,j}-\psi_{i-1,j}}{\Delta\phi\cdot H(i)}\quad \overline{v}(i,j)=-\frac{\psi_{i,j+1}-\psi_{i,j}}{\Delta\xi\cdot H(i)} \tag{34}$$

Influence of eccentricity on stream function distribution: For the purpose of the discussion about influence of eccentricity on stream function distribution, some basic datum were given and substituted into the differential Eq. (29): $\tau_Y = 0.2$, $k = 0.8$, $m = 1.3$, $St^* = 0.15$, $\partial p/\partial\phi = -0.5$, $\partial p/\partial\zeta = -0.5$, $\partial p/\partial\zeta = -10$, $p = 1$, $\beta = 85°$. The boundary conditions were $\psi(0, \zeta, t)$, $\psi(ø, 0, t) = 2$ and $\psi(\phi, Z, t) = -2$. The eccentricity was 0, 0.2 and 0.4 separately. After the numerical solution about the Eq. (28), we got the stream function distribution and velocity feature in the entire annulus, as shown in Fig. 5, 6 and 7.

When the eccentricity equals to zero in the annulus, the stream function is uniform distribution and the axial velocity is all the same. But as the eccentricity increases, the contour lines of stream function gradually centralize to the widest gap and distribute the most loosely in the narrowest gap. This characteristic of stream function distribution indicates that the axial velocity in the widest gap is the largest and the larger the eccentricity is, the larger axial velocity difference between in the widest gap and in the narrowest gap is. As for azimuthal velocity, its distribution in the annulus is just like a parabola, which equals to zero respectively in the widest gap and in the narrowest gap and is the largest in an annular gap of a certain azimuthal angle. The larger eccentricity is, the more homogeneous azimuthal velocity in the annulus is.

CONCLUSION

We realized the fact that fluid in the annulus of highly deviated wells or horizontal wells does not flow in the axial direction only. Based on the Hele-Shaw model suitable for the flow in the annulus, the velocity distribution was analyzed and stream function formulation was founded. The conclusions are as follows:

- The width of flow core in the annulus is proportional to yield stress and inversely proportional to pressure gradient, has no relationship with power law index and consistency. The increment of power law index leads the velocity profile in the annulus to get longer and the increment of annular gap width makes the velocity of H-B fluid increase.
- Velocity contrast between in the widest gap and in the narrowest gap in eccentric annulus can be reduced by increasing pressure gradient and power law index or decreasing yield stress.
- Eccentricity influences the stream distribution in the annulus obviously. As the eccentricity increases, the contour lines of stream function gradually centralize to the widest gap and distribute the most loosely in the narrowest gap.
- Axial velocity is the largest in the widest gap of eccentric annulus and the larger eccentricity is, the larger axial velocity contrast between in the widest gap and in the narrowest gap is. Azimuthal velocity is zero respectively in the widest and narrowest gap and is the largest in an annular gap of a certain azimuthal angle. The larger eccentricity is, the more homogeneous azimuthal velocity is in the annulus.

ACKNOWLEDGMENT

The study is supported by program for Changjiang Scholars and Innovative Research Team in University (NO.RT1086) and supported by fund: National Natural Science Foundation of China under grant number 51174226.

REFERENCES

Bittleston, S.H., J. Ferguson and I.A. Frigaard, 2002. Mud removal and cement placement during primary cementing of an oil well: Laminar non-Newtonian displacements in an eccentric annular Hele-Shaw cell. J. Eng. Math., 43: 229-253.

Bittleston, S.H. and I.A. Frigaard, 2004. Mud removal and cement placement during primary cementing of an oil well: Steady-state displacements. J. Eng. Math., 48: 1-26.

Cao, W. and C.Y. Shen, 2004. Flow simulation for injection molding based on solid model. Chinese J. Comput. Mech., 21(1): 115-119.

Daripa, P. and G. Pasa, 2007. Stabilizing effect of diffusion in enhanced oil recovery and three-layer Hele-Shaw flows with viscosity gradient. Transp. Porous Med., 70: 11-23.

Frigaard, I.A. and S. Pelipenko, 2003. Effective and Ineffective Strategies for Mud Removal and Cement Slurry Design. SPE 80999.

Frigaard, I.A. and Ngwa., 2010. Slumping flows in narrow eccentric annuli: Design of chemical packers and cementing of subsurface gas pipelines. Transp. Porous Med., 83: 29-53.

Guo, X.Y., C.J. Liu, *et al*., 1997. Research on non-newtonian liquid rheological model. Natural Gas Ind., 17(4): 43-49.

Li, Z.M., Y. Wang, *et al*., 2002. The law of velocity distribution of Bingham Fluid's flowing in the encircle pipe. Acta Petrolei Sinica, 23(2): 87-91.

Li, Z.M., P. Zhang, *et al*., 2004. Study of velocity and temperature profiles for the annulus pipe flow of H-B fluid. J. Hydrodynamics A, 19(1): 31-37.

Pelipenko, S. and I.A. Frigaard, 2004. Mud removal and cement placement during primary cementing of an oil well: Steady-state displacements. J. Eng. Math, 48: 1-26.

Redberger, P.J. and M.E. Charles, 1962. Axial laminar flow in a circular pipe, containing a fixed eccentric core. Cdn. J. Chem. Eng., 40(4): 148-151.

Souhar, K. and S. Aniss, 2012. Effect of Coriolis force on the thermosolutal convection threshold in a rotating annular Hele-Shaw cell. Heat Mass Transfer, 48: 175-182.

Trevelyan, P.M., C. Almarcha and A. Dewit., 2011. Buoyancy-driven instabilities of miscible two-layer stratifications in porous media and Hele-Shaw cells. J. Fluid Mech., 670(10): 38-65.

Vaugh, R.D., 1965. Axial laminar flow of non-Newtonian fluids in narrow eccentric annuli. SPE J., 5(4): 277-280.

Wang, H.G. and Y. Su, 1998. Flow of Robertson-Stiff Fluid through an eccentric annulus. Appl. Math. Mech., 19(10): 931-939.

Wang, Y.P., Y.D. Li, *et al*., 2008. Study of velocity distribution of H-B fluid in the concentrical annulus. Drilling Fluid Completion Fluid, 25(2): 34-36.

Zheng, Y.G., 1992. Laminar flow of Bingham fluid in inclined eccentric annulus. J. Southwestern Petroleum Institute, 14(8): 43-53.