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Research Article

Decay of MHD Turbulence before the Final Period for Four-point Correlation in a Rotating System

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Abstract: The aim of this study is to determine decay of magnetic field fluctuations in MHD turbulence for four-point correlation in a rotating system before the final period. Two, three and four point correlation equations have been obtained and the set of correlation equations is made determinate by neglecting the quintuple correlations in comparison to the third and fourth order correlation terms. The correlation equations are converted to spectral form by taking their Fourier-transforms. Finally, integrating the energy spectrum over all wave numbers. The energy decay of magnetic field fluctuations for four-point correlations in a rotating system is obtained and is shown graphically in the text.

Keywords: Correlations, deissler's method, MHD turbulence, rotating system

INTRODUCTION

Magneto-Hydrodynamics is the science which deals with the motion of highly conducting fluids in the presence of a magnetic field. The Coriolis force and centrifugal force must be supposed to act on the fluid. The coriolis and centrifugal force due to rotation plays an important role in a rotating system of turbulent flow. Deissler (1958, 1960) developed a theory "Decay of homogeneous turbulence before the final period." By considering Deissler's theory, Sarker and Kishore (1991) studied the decay of MHD turbulence before the final period. Funada *et al*. (1978) considered the effect of Coriolis force on turbulent motion in presence of strong magnetic field. Islam and Sarker (2001b) studied the decay of dusty fluid MHD turbulence before the final period in a rotating system. Sarker and Islam (2001) also considered the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Islam and Sarker (2001a) also extended their previous problem for first order reactant. Kishore and Golsefid (1988) discussed the effect of Coriolis force on acceleration covariance in turbulent flow with rotational symmetry. Loeffler and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Chandrasekhar (1951) studied the invariant theory of isotropic turbulence in magneto-hydrodynamics. Corrsin (1951) considered the spectrum of isotropic temperature fluctuations in isotropic turbulence. Bkar *et al.* (2012) studied the decay of energy of MHD turbulence for four-point correlation. Bkar *et al*. (Year) also studied the first-order reactant in homogeneou turbulence prior to the ultimate phase of decay for four-point correlation in presence of dust particles.Azad *et al.* (2012) discussed the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles.

It is noted that all above cases, the researcher had considered three-point correlations and through their study they obtained the energy decay law prior to the ultimate phase.

In the present study, we have studied the decay of MHD turbulence before the final period for four-point correlation in a rotating system. The energy decay of MHD turbulence depends on the variation of the magnitude of coriolis parameters in the magnetic field and causes important role between rotating and non-rotating system. The effects of rotation in magnetic field fluctuation of MHD turbulence are graphically discussed. It is observed that energy decays increases with the decreases of rotation and maximum at the point where the rotation i.e., coriolis force is zero.

Four point correlation and equations: To find the four point correlation equation, we take the momentum equation of MHD turbulence at the point p and the induction equation of magnetic field fluctuation at $p', p'' and\ p'''$ as:

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} - 2\varepsilon_{\mathrm{pkl}}\Omega_p u_l \tag{1}$$

$$\frac{\partial h_i'}{\partial t} + u_k' \frac{\partial h_i'}{\partial x_k'} - h_k' \frac{\partial u_i'}{\partial x_k'} = \frac{\nu}{p_M} \frac{\partial^2 h_i'}{\partial x_k' \partial x_k'} - 2\varepsilon_{qki}\Omega_q u_i' \tag{2}$$

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$$\frac{\partial h''_j}{\partial t} + u''_k \frac{\partial h''_j}{\partial x''_k} - h''_k \frac{\partial u''_j}{\partial x''_k} = \frac{\nu}{p_M} \frac{\partial^2 h''_j}{\partial x''_k \partial x''_k} - 2\varepsilon_{rkj}\Omega_r u''_j \tag{3}$$

$$\frac{\partial h'''_m}{\partial t} + u'''_k \frac{\partial h'''_m}{\partial x'''_k} - h'''_k \frac{\partial u'''_m}{\partial x'''_k} = \frac{\nu}{p_M} \frac{\partial^2 h'''_m}{\partial x'''_k \partial x'''_k} - 2\varepsilon_{skm}\Omega_s u'''_m \tag{4}$$

where,

$\omega = P/\rho + \frac{1}{2}|\bar{h}|^2$ = The total MHD pressure
$\rho(x,t)$ = The hydrodynamic pressure
ρ = The fluid density
$P_M = \nu/\lambda$ = The Magnetic Prandtl number
ν = The kinematics viscosity
λ = The magnetic diffusivity
$h_i(x, t)$ = The magnetic field fluctuation
$u_k(x, t)$ = The turbulent velocity
t = The time
x_k = The space co-ordinate and repeated subscripts are summed from 1 to 3

Multiplying Eq. (1) by $h'_i h''_j h'''_m$ (2) by $u_l h''_j h'''_m$ (3) by $u_l h'_i h'''_m$ (4) by $u_l h'_i h''_j$ and adding the four equations, we than taking the space or time averages or ensemble average, both of the averages gives the same representation .Space or time averages denoted by $(\overline{\dots\dots})$ and ensemble average denoted by <………>. We get:

$$\frac{\partial}{\partial t}(\overline{u_l h'_i h''_j h'''_m}) + \frac{\partial}{\partial x_k}(\overline{u_l u_k h'_i h''_j h'''_m}) - \frac{\partial}{\partial x_k}(\overline{h_k h_l h'_i h''_j h'''_m}) +$$

$$\frac{\partial}{\partial x'_k}(\overline{u_l u_k h'_i h''_j h'''_m}) - \frac{\partial}{\partial x'_k}(\overline{u_l u'_i h'_k h''_j h'''_m}) + \frac{\partial}{\partial x''_k}(\overline{u_l u''_k h'_i h''_j h'''_m}) -$$

$$\frac{\partial}{\partial x''_k}(\overline{u_l u''_j h'_i h''_k h'''_m}) + \frac{\partial}{\partial x'''_k}(\overline{u_l u'''_k h'_i h''_j h'''_m}) - \frac{\partial}{\partial x'''_k}(\overline{u_l u''_j h'_i h''_j h'''_m}) =$$

$$-\frac{\partial}{\partial x_l}(\overline{w h'_i h''_j h'''_m}) + \frac{\partial^2}{\partial x_k \partial x_k}(\overline{u_l h'_i h''_j h'''_m}) + \frac{\nu}{p_M}[\frac{\partial^2}{\partial x'_k \partial x'_k}(\overline{u_l h'_i h''_j h'''_m}) +$$

$$\frac{\partial^2}{\partial x''_k \partial x''_k}(\overline{u_l h'_i h''_j h'''_m}) + \frac{\partial^2}{\partial x'''_k \partial x'''_k}(\overline{u_l h'_i h''_j h'''_m})]$$

$$+ 2[\varepsilon_{pkl}\Omega_p\left(\overline{u_l h'_i h''_j h'''_m}\right) + \varepsilon_{qki}\Omega_q\left(\overline{u_l u'_i h''_j h'''_m}\right)$$

$$+ \varepsilon_{rkj}\Omega_r\left(\overline{u_l u''_j h'_i h'''_m}\right) + \varepsilon_{skm}\Omega_s\left(\overline{u_l u'''_m h'_i h''_j}\right)] \tag{5}$$

By using:

$$\frac{\partial}{\partial x''_k} = \frac{\partial}{\partial r'_k}, \ \frac{\partial}{\partial x'_k} = \frac{\partial}{\partial r'_k}, \ \frac{\partial}{\partial x_k} = -(\frac{\partial}{\partial r'_k} + \frac{\partial}{\partial r'_k} + \frac{\partial}{\partial r''_k}) \tag{6}$$

into Eq. (5) and then following nine dimensional Fourier transforms:

$$\left\langle u_l h'_i(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{7}$$

$$\left\langle u_l u'_k h'_i(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi'_k(\bar{k}) \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{8}$$

$$\left\langle u_l u'_i h'_k(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi'_i(\bar{k}) \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{9}$$

$$\left\langle u_l u''_k(\bar{r}') h'_i(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi''_k(\bar{k}) \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{10}$$

$$\left\langle u_l u''_j(\bar{r}') h'_i(\bar{r}) h''_k(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi''_j(\bar{k}) \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{11}$$

$$\left\langle u_l u_k h'_i(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \phi_l \phi_k \gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{12}$$

$$\left\langle w h'_i(\bar{r}) h''_j(\bar{r}') h'''_m(\bar{r}'') \right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\langle \delta\gamma'_i(\bar{k}) \gamma''_j(\bar{k}') \gamma'''_m(\bar{k}'') \right\rangle \exp[i(\bar{k}.\bar{r} + \bar{k}'.\bar{r}' + \bar{k}''.\bar{r}'')] d\bar{k} d\bar{k}' d\bar{k}'' \tag{13}$$

and interchange of point's p/ and p etc, in the subscripts with the facts:

$$\overline{u_l u'''_k h'_i h''_j h'''_m} = \overline{u_l u'_k h'_i h''_j h'''_m}; \ \overline{u_l u''_k h'_i h''_j h'''_m} = \overline{u_l u'_k h'_i h''_j h'''_m};$$
$$\overline{u_l u'''_m h'_i h''_j h'''_m} = \overline{u_l u'_i h'_i h'_k h''_j h'''_m}; \ \overline{u_l u''_j h'_i h''_k h'''_m} = \overline{u_l u'_i h'_i h'_k h''_j h'''_m};$$

and taking contraction of the indices i and j we obtain the spectral equation is:

$$\frac{\partial}{\partial t}(\overline{\phi_l \gamma'_i \gamma''_i \gamma'''_m}) + \frac{\nu}{p_M}[(1 + p_M)(k^2 + k'^2 + k''^2) + 2p_M(kk' + k'k'' + kk'')](\overline{\phi_l \gamma'_i \gamma''_i \gamma'''_m})$$

$$+ 2(\varepsilon_{pkl}\Omega_p + \varepsilon_{qki}\Omega_q + \varepsilon_{rkj}\Omega_r + \varepsilon_{skm}\Omega_s)(\overline{\phi_l \gamma'_i \gamma''_i \gamma'''_m}) = i(k_k + k'_k + k''_k)(\overline{\phi_l \phi_k \gamma'_i \gamma''_i \gamma'''_m})$$

$$- i(k_k + k'_k + k''_k)(\overline{\phi_l \gamma_k \gamma'_i \gamma''_i \gamma'''_m}) - i(k_k + k'_k + k''_k)(\overline{\phi_l \gamma'_k \gamma'_i \gamma''_i \gamma'''_m})$$

$$+ i(k_k + k'_k + k''_k)(\overline{\phi_l \phi'_i \gamma'_k \gamma''_i \gamma'''_m}) + i(k_k + k'_k + k''_k)(\overline{\delta\gamma'_i \gamma''_i \gamma'''_m}). \tag{14}$$

If we take the derivative with respect to x_l of the momentum Eq. (1) at p, we have:

$$-\frac{\partial^2 w}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_l}(u_l u_k - h_l h_k) \tag{15}$$

Multiplying Eq. (15) by $h'_i h''_j h'''_m$, taking time averages and writing the equation in terms of the independent variables $\vec{r}, \vec{r}', \overrightarrow{r''}$ we have:

$$-(\overline{\delta\gamma'_i \gamma''_j \gamma'''_m}) = \frac{(K_l K_k + K_l K'_k + K_l K''_k + K'_l K_k + K'_l K'_k + K'_l K''_k + K''_l K_k + K''_l K_k + K''_l K'_k + K''_l K''_k)}{K_l K_l + K'_l K'_l + K''_l K''_l + 2K_l K'_l + 2K'_l K''_l + 2K_l K''_l}$$

$$\overline{(\phi_l \phi_k \gamma'_i \gamma''_j \gamma'''_m} - \overline{\gamma_l \gamma_k \gamma'_i \gamma''_j \gamma'''_m)} \tag{16}$$

Eq. (16) can be used to eliminate $\left(\overline{\delta\gamma'_i \gamma''_j \gamma'''_m}\right)$ from Eq. (14). Equation (14) and (16) are the spectral Eq.

corresponding to the four–point correlation equation. The spectral equations corresponding to the three-point correlation equations by contraction of the indices i and j are:

$$\frac{\partial}{\partial t}\overline{(\phi_l\beta_i'\beta_i'')}+\frac{\nu}{p_M}[(1+P_M)(K^2+K'^2)+2p_M KK']\ \overline{(\phi_l\beta_i'\beta_i'')} = \mathrm{i}\,(K_k+K_k')\ \overline{(\phi_l\phi_k\beta_i'\beta_i'')} - \mathrm{i}(K_k+K_k')\ \overline{(\beta_l\beta_k\beta_i'\beta_i'')} - \mathrm{i}\,(K_k+K_k')\ \overline{(\phi_l\phi_k'\beta_i'\beta_i'')} + \mathrm{i}(K_k+K_k')\ \overline{(\phi_l\phi_i'\beta_i'\beta_i'')} + \mathrm{i}(k_l+k_l')\overline{\gamma\beta_i'\beta_i''} \tag{17}$$

$$\text{and-}(\gamma\ \overline{\beta_i'\beta_j''}) = \frac{(K_lK_k+K_l'K_k+K_lk_k'+K_l'K_k')}{(K_l^2+K_l'^2+2K_lK_l')}\ \overline{(\phi_l\phi_k\beta_i'\beta_i''} - \overline{\beta_l\beta_k\beta_i'\beta_j'')} \tag{18}$$

here the spectral tensors are defined by:

$$\left\langle u_l h_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\beta_i'(\bar k)\beta_j''(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{19}$$

$$\left\langle u_l u_k'(\bar r)h_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\phi_k'(\bar k)\beta_i(\bar k)\beta_j'(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{20}$$

$$\left\langle u_l u_i'(\bar r)h_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\phi_i'(\bar k)\beta_k'(\bar k)\beta_j'(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{21}$$

$$\left\langle u_l h_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l\phi_k'(\bar k)\beta_i'(\bar k)\beta_j'(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{22}$$

$$\left\langle u_l h_k h_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_l\beta_k'(\bar k)\beta_i'(\bar k)\beta_j'(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{23}$$

$$\left\langle wh_i'(\bar r)h_j''(\bar{r'})\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma\beta_i'(\bar k)\beta_j'(\bar{k'})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'})\right]d\bar k d\bar{k'} \tag{24}$$

A relation between $\phi_l\phi_k'\beta_i'\beta_j''and\ \phi_l\gamma_i'\gamma_j''\gamma_m'''$ can be obtained by letting $\vec r$`` = 0 in Eq. (7) and comparing the result with Eq. (20):

$$\left\langle \phi_l\phi_k'(\bar k)\beta_i'(\bar k)\beta_j'(\bar{k'})\right\rangle = \int_{-\infty}^{\infty}\left\langle \phi_l\gamma_i'(\bar k)\gamma_j''(\bar{k'})\gamma_m'''(\bar{k''})\right\rangle \exp\left[i(\bar k.\bar r+\bar{k'}.\bar{r'}+\bar{k''}.\bar{r''})\right]d\bar k d\bar{k'}d\bar{k''} \tag{25}$$

The spectral equation corresponding to the two-point correlation equation taking contraction of the indices is:

$$\frac{\partial}{\partial t}\left\langle \varphi_i\varphi_i'(\bar k)\right\rangle+\frac{2\nu}{p_M}k^2\left\langle \varphi_i\varphi_i'(\bar k)\right\rangle = 2ik_k[\left\langle \alpha_i\varphi_k\varphi_i'(\bar k)\right\rangle - \left\langle \alpha_k\varphi_i\varphi_i'(-\bar k)\right\rangle] \tag{26}$$

where, $\varphi_i\varphi_i'$ and $\alpha_i\phi_k\phi_i'$ are defined by:

$$\left\langle h_i h_i'(\bar r)\right\rangle = \int_{-\infty}^{\infty}\left\langle \varphi_i\varphi_i'(\bar k)\right\rangle \exp(i\bar k.\bar r)dk \tag{27}$$

and

$$\left\langle h_i h_k h_i'(\bar r)\right\rangle = \int_{-\infty}^{\infty}\left\langle \alpha_i\varphi_k\varphi_i'(\bar k)\right\rangle \exp(i\bar k.\bar r)dk \tag{28}$$

The relation between $\alpha_i\varphi_k\varphi_j'(\overrightarrow{k})$ and $\varphi_l\beta_i'\beta_j''$ is obtained by letting $\vec r$ = 0 in Eq. (19) and comparing the result with Eq. (24), then:

$$\left\langle \alpha_i\varphi_k\varphi_i'(\bar k)\right\rangle = \int_{-\infty}^{\infty}\phi_l\beta_i'(\bar k)\beta_i''(\bar{k'})dk' \tag{29}$$

Solution neglecting quintuple correlations: As it stands the set of linear Eq. (14), (17), (18), (19), (21), (22) and (29) is indeterminate as it contains more unknowns than equations. Thus neglecting all the terms on the right side of Eq. (14), the equation can be integrated between to t_1 and t to give:

$$\left\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle = \left\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle_1 \exp[\frac{-\nu}{p_M}(1+p_M)(k^2+k'^2+k''^2+2kk'+2k'k''+2kk') - 2(\varepsilon_{pkl}\Omega_p+\varepsilon_{qki}\Omega_q+\varepsilon_{rkj}\Omega_r+\varepsilon_{skm}\Omega_s)](t-t_1) \tag{30}$$

where, $\left\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle_1$ is the value of $\left\langle \phi_l\gamma_i'\gamma_j''\gamma_m'''\right\rangle$ at t = t_1 that is stationary value for small values of k, $k'andk''$ when the quintuple correlations are negligible. Substituting of Eq. (18), (25), (30) in Eq. (17) we get:

$$\frac{\partial}{\partial t}\overline{(k_k\phi_l\beta_i'\beta_i'')}+\frac{\nu}{p_M}[(1+p_M)(k^2+k'^2)+2p_Mkk']\overline{(k_k\phi_l\beta_i'\beta_i'')}$$

$$[a]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M(kk'+k'k''+kk')\}]dk''$$

$$\exp[2(\varepsilon_{pkl}\Omega_p+\varepsilon_{qki}\Omega_q+\varepsilon_{rkj}\Omega_r+\varepsilon_{skm}\Omega_s)(t-t_1)]$$

$$+[b]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M(kk'-kk'')\}]dk''$$

$$\exp[2(\varepsilon_{pkl}\Omega_p+\varepsilon_{qki}\Omega_q+\varepsilon_{rkj}\Omega_r+\varepsilon_{skm}\Omega_s)(t-t_1)]$$

$$+[c]_1\int_{-\infty}^{\infty}\exp[\frac{-\nu}{p_M}(t-t_1)\{(1+p_M)(k^2+k'^2+k''^2)+2p_M(kk'-k'k'')\}]dk''$$

$$\exp[2(\varepsilon_{pkl}\Omega_p+\varepsilon_{qki}\Omega_q+\varepsilon_{rkj}\Omega_r+\varepsilon_{skm}\Omega_s)(t-t_1)] \tag{31}$$

At t_1 ,γ'^{s} have been assumed independent of $\overrightarrow{k''}$; that assumption is not, made for other times. This is one of several assumptions made concerning the initial conditions, although continuity equation satisfied the conditions. The complete specification of initial turbulence is difficult; the assumptions for the initial conditions made here in are partially on the basis of simplicity. Substituting $dk'' = dk_1''dk_2''dk_3''$ and integrating with respect to k_1'', k_2'' and k_3'', we get:

$$\frac{\partial}{\partial t}(\overline{k_k\phi_l\beta_i'\beta_i''}) + \frac{\nu}{p_M}\left[(1+p_M)(k^2+k'^2)+2p_M kk'\right]$$

$$\overline{(k_k\phi_l\beta_i'\beta_i'')} = \frac{\sqrt{\pi p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)]}}$$

$$[a_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{\frac{(1+2p_M)(k^2+k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)^2}\right\}\right]$$

$$\exp\{2(\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\}$$

$$+\frac{\sqrt{\pi p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)]}}$$

$$[b_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{\frac{(1+2p_M)(k^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}+k'^2\right\}\right]$$

$$\exp\{(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\} \quad +$$

$$\frac{\sqrt{\pi p_M}}{\sqrt{[\nu(t-t_1)(1+p_M)]}}$$

$$[c_1]\exp\left[-\frac{\nu(t-t_1)(1+p_M)}{p_M}\left\{k^2+\frac{(1+2p_M)(k'^2)}{(1+p_M)^2}+\frac{2p_M kk'}{(1+p_M)}\right\}\right]\exp$$

$$\{(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\} \tag{32}$$

Integration of Eq. (32) with respect to time and in order to simplify calculations, we will assume that $[\alpha]_1 = 0$; That is we assume that a function sufficiently general to represent the initial conditions can obtained by considering only the terms involving $[b]_1$ and $[c]_1$ The substituting of Eq. (29) in Eq. (26) and setting: $H = 2\pi k^2\varphi_i\varphi_i'$ result in

$$\frac{\partial H}{\partial t} + \frac{2\nu k^2}{p_M}H = G \tag{33}$$

where,

$$G = k^2\int_{-\infty}^{\infty} 2\pi i[\ \overline{k_k\phi_l\beta_i'\beta_i''(\vec{k},\vec{k}')}\ \ \overline{k_k\phi_l\beta_i'\beta_i''(-\vec{k},-\overrightarrow{k'})}\ ]_0 .$$

$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2)+2p_M kk'\}]dk'$$

$$+k^2\int_{-\infty}^{\infty}\frac{2\pi^{\frac{5}{2}}i}{\nu}[b(\vec{k}.\vec{k}') - b(-\vec{k}.-\overrightarrow{k'})]_1$$

$$\exp\{-(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\}$$

$$\{-\ \omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2}+\frac{2P_m kk'}{1+p_M}+k'^2\right)] \ +k\exp$$

$$[-\omega^{-2}\ (1+p_M)(k^2+k'^2)+2p_M kk']$$

$$\int_0^{\frac{\omega k}{2}}\exp(x^2)dx\}dk'$$

$$+\ k^2\int_{-\infty}^{\infty}\frac{2\pi^{\frac{5}{2}}i}{\nu}[c(\vec{k}.\vec{k}') - c(-\vec{k}.-\overrightarrow{k'})]_1$$

$$\exp\{-(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\}$$

$$\{-\ \omega^{-1}\exp[-\omega^2\left(k^2+\frac{2P_m kk'}{1+p_M}+\frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$

$$+k`\exp[-\omega^2(1+p_M)(k^2+k'^2)+2p_M kk')].$$

$$\int_0^{\frac{\omega k'}{2}}\exp(x^2)dx\}dk' \tag{34}$$

where H is the magnetic energy spectrum function, which represents contributions from various wave numbers (or eddy sizes) to the energy and G is the energy transfer function, which is responsible for the transfer of energy between wave numbers .In order to make further calculations, an assumption must be made for the forms of the bracketed quantities with the subscripts 0 and 1 in Eq. (37) which depends on the initial conditions:

$$(2\pi)^2[\ \overline{k_k\phi_l\beta_i'\beta_i''}(\vec{k},\vec{k}')\text{-}\ \overline{k_k\phi_i\beta_i'\beta_i''}(-\vec{k},-\overrightarrow{k'})\ ]_0 = -\xi_0(k^2k'^4 - k^4k'^2) \tag{35}$$

where, ξ_0 is a constant depending on the initial conditions for the other bracketed quantities in Eq. (34), we get:

$$\frac{4\pi^{\frac{7}{2}}i}{\nu}[b(\vec{k}.\vec{k}') - b(-\vec{k}.-\overrightarrow{k'})]_1 = \frac{4\pi^{\frac{7}{2}}i}{\nu}[c(\vec{k}.\vec{k}') - c(-\vec{k}.-\overrightarrow{k'})]_1 = -2\,\xi_1(k^4k'^6 - k^6k'^4) \tag{36}$$

Remembering,

$d\overrightarrow{k'} = -2\pi\overrightarrow{k'^2}d(\cos\theta)dk'$,

$kk' = kk'\cos\theta, \theta$ *is the angle between* $\vec{k}$ *and* $\vec{k'}$ and carrying out the integration with respect to θ, we get:

$$G = \int_0^\infty [\frac{\xi_0(k^2k'^4 - k^4k'^2)kk'}{\nu(t-t_0)}\{$$
$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2) - 2p_M kk'\}] -$$
$$\exp[-\frac{\nu}{p_M}(t-t_0)\{(1+p_M)(k^2+k'^2) + 2p_M kk'\}]\} +$$
$$\frac{\xi_1(k^4k'^6 - k^6k'^4)kk'}{\nu(t-t_0)}$$
$$\exp\{(-2\varepsilon_{pkl}\Omega_p - 2\varepsilon_{qki}\Omega_q - 2\varepsilon_{rkj}\Omega_r - 2\varepsilon_{skm}\Omega_s)(t-t_1)\}$$
$$\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2} - \frac{2P_M kk'}{1+p_M} + k'^2\right)] -$$
$$\omega^{-1}\exp[-\omega^2\left(\frac{(1+2p_M)k^2}{(1+p_M)^2} + \frac{2P_M kk'}{1+p_M} + k'^2\right)] +$$
$$\exp[-\omega^2\left(k^2 - \frac{2P_M kk'}{1+p_M} + \frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)] -$$
$$\omega^{-1}\exp[-\omega^2\left(k^2 + \frac{2P_M kk'}{1+p_M} + \frac{(1+2p_M)k'^2}{(1+p_M)^2}\right)]$$
$$+ \{k\exp[-\omega^2((1+p_M)(k^2+k'^2) - 2p_M kk')]$$
$$-k\exp[-\omega^2((1+p_M)(k^2+k'^2) + 2p_M kk')]\}\int_0^{\frac{\omega k}{2}}\exp(x^2)dx$$
$$+\{k'\exp[-\omega^2((1+p_M)(k^2+k'^2) - 2p_M kk')]$$
$$-k'\exp[-\omega^2((1+p_M)(k^2+k'^2) + 2p_M kk')]\}$$
$$\int_0^{\frac{\omega k'}{2}}\exp(x^2)dx)]\,dk' \tag{37}$$

where, $\omega = \left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}$. Integrating Eq. (37) with respect to k`we have:

$$G = G_\beta + G_\gamma$$
$$\exp\{-(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s)(t-t_1)\} \tag{38}$$

where,

$$G_\beta = -\frac{\pi^{\frac{1}{2}}\xi_0 p_M^{\frac{5}{2}}}{\nu^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+p_M)^{\frac{5}{2}}}\exp\left\{-\frac{\nu(t-t_0)(1+2p_M)k^2}{p_M(1+p_M)}\right\} \tag{39}$$
$$\left[\frac{15p_Mk^4}{4\nu^2(t-t_0)^2(1+p_M)} + \left\{\frac{5p_M^2}{(1+p_M)^2\nu(t-t_0)} - \frac{3}{2\nu(t-t_0)}\right\}k^6 + \frac{p_M}{1+p_M}\left\{\frac{p_M^2}{(1+p_M)^2} - 1\right\}k^8\right]$$

and

$$G_\gamma = G_{\gamma_1} + G_{\gamma_2} + G_{\gamma_3} + G_{\gamma_4} \tag{40}$$

where,

$$G_{\gamma_1} = \frac{\xi_1\pi^{\frac{1}{2}}p_M^5}{8\nu^2(t-t_1)^2(1+p_M)^5}\exp$$
$$\left(\frac{-\nu(t-t_1)(1+2p_M-p_M^2)}{p_M(1+p_M)}\right)k^2 [\frac{90p_Mk^6}{\nu^4(t-t_1)^4(1+p_M)} + 3$$
$$\left\{\frac{4p_M}{\nu^2(t-t_1)^2(1+p_M)} + \frac{2p^2_M}{\nu^3(t-t_1)^3(1+p_M)^2} - \frac{1}{\nu^3(t-t_1)^3}\right\}k^8 +$$
$$\left\{\frac{64p^2_M}{\nu(t-t_1)(1+p_M)^2} + \frac{10p^3_M}{\nu^2(t-t_1)^2(1+p_M)^3} - \frac{40}{\nu(t-t_1)}\right\}k^{10}$$
$$+8\left\{\left(\frac{p_M}{1+p_M}\right)^2 - \left(\frac{p_M}{1+p_M}\right)\right\}k^{12}] \tag{41}$$

$$G_{\gamma_2} = \frac{\xi_1\pi^{\frac{1}{2}}p_M^5(1+p_M)^4}{8\nu^2(t-t_1)^2(1+2p_M)^{\frac{9}{2}}}\exp$$
$$\left(\frac{-\nu(t-t_1)(1+p_M)(1+2p_M-p_M^2)}{p_M(1+p_M)}\right)k^2$$
$$[\frac{90p_M(1+p_m)}{\nu^4(t-t_1)^4(1+2p_M)}k^6 +$$
$$\left\{\frac{120p_M(1+p_m)}{\nu^2(t-t_1)^2(1+2p_M)} + \frac{2p^2_M(1+p_m)^2}{\nu^3(t-t_1)^3(1+2p_M)^2} - \frac{1}{\nu^3(t-t_1)^3}\right\}k^8 +$$
$$\left\{\frac{64p^2_M(1+p_m)^2}{\nu(t-t_1)(1+2p_M)^2} - \frac{40}{\nu(t-t_1)} + \frac{10p^3_M(1+p_M)^3}{\nu^2(t-t_1)^2(1+2p_M)^3}\right\}k^{10}$$
$$+\left\{\frac{8p^3_M(1+p_M)^3}{(1+2p_M)^3} - \frac{p_M(1+p_M)}{(1+2p_M)}\right\}k^{12}] \tag{42}$$

$$G_{\gamma_3} = \frac{\xi_1\pi^{\frac{1}{2}}p_M^{\frac{9}{2}}}{8\nu^{\frac{3}{2}}(t-t_1)^{\frac{3}{2}}(1+p_M)^8}\exp\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2$$
$$[\frac{90p_M}{\nu^4(t-t_1)^4(1+p_M)^2}k^7 +$$
$$\left\{\frac{120p_M}{\nu^2(t-t_1)^2} + \frac{60p^2_M}{\nu^3(t-t_1)^3(1+p_M)^2} - \frac{30}{\nu^3(t-t_1)^3}\right\}k^9 +$$
$$\left\{\frac{64p^2_M}{\nu(t-t_1)} + \frac{10p^3_M}{\nu^2(t-t_1)^2(1+p_M)^2} - \frac{40(1+p_M)^2}{\nu(t-t_1)}\right\}k^{11} +$$
$$\{p^2_M - p_M(1+p_M)^2\}k^{13}]\int_0^{\frac{\omega_1}{2}}\exp(y^2)dy \tag{43}$$

where,

$$\omega_1 = \left(\frac{\nu(t-t_1)(1+p_M)}{p_M}\right)^{\frac{1}{2}}k$$

$$G_{\gamma_4} = \frac{\xi_1 \pi^{\frac{1}{2}} p_M^{\frac{15}{2}}}{2^8 \nu (t-t_1)(1+p_M)^{\frac{29}{2}}} \exp\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right) k^2 [$$

$$\frac{7560(1+p_M)^3}{\nu^4 (t-t_1)^4 p_M^2} k^6 + \left\{\frac{20160(1+p_M)^5}{\nu^3 (t-t_1)^3 p_M} - \frac{4233600(1+p_M)^7}{\nu^3 (t-t_1)^3 p_M^3}\right\} k^8 +$$

$$\left\{\frac{12096(1+p_M)^5}{\nu^2 (t-t_1)^2} - \frac{3360(1+p_M)^7}{\nu^2 (t-t_1)^2 p_M^2}\right\} k^{10} + \left\{\frac{2304(1+p_M)^5 p_M}{\nu (t-t_1)} - \frac{1344(1+p_M)^9}{p_M^2}\right\} k^{12} +$$

$$\left\{128(1+p_M)^5 p^2_M - 128(1+p_M)^7\right\} k^{14} + \ldots\ldots\ldots\ldots] \quad (44)$$

The integral expression in Eq. (39) , the quantity G_β represents the transfer function arising owing to consideration of magnetic field at three point correlation equation; G_γ arises from consideration of the four-point equation. Integration of Eq. (39) over all wave numbers shows that:

$$\int_0^\infty G . d\vec{k} = 0 \quad (45)$$

Indicating that the expression for G satisfies the conditions of continuity and homogeneity, physically, it was to be expected, since G is a measure of transfer of energy and the numbers must be zero. From (34):
We get:

$$H = \exp\left[-\frac{2\nu k^2 (t-t_0)}{p_M}\right] \int G \exp\left[-\frac{2\nu k^2 (t-t_0)}{p_M}\right] dt + J(k) \exp\left[-\frac{2\nu k^2 (t-t_0)}{p_M}\right]$$

$J(k) = N_0 k^2 / \pi$, is a constant of integration and can be obtained as by Corrsin (1951)
Therefore we obtained:

$$H = \frac{N_0 k^2}{\pi} \exp\left[\frac{-2\nu k^2 (t-t_0)}{p_M}\right] + \exp\left[\frac{-2\nu k^2 (t-t_0)}{p_M}\right] \int (G_\beta + G_{\gamma_1} + G_{\gamma_2} + G_{\gamma_3} + G_{\gamma_4}) \exp\left[\frac{-2\nu k^2 (t-t_0)}{p_M}\right] dt \quad (46)$$

where,

$$G = G_\beta + G_{\gamma_1} + G_{\gamma_2} + G_{\gamma_3} + G_{\gamma_4} \quad (47)$$

After integration Eq. (46) becomes:

$$H = \frac{N_0 k^2}{\pi} \exp\left[-\frac{2\nu k^2 (t-t_0)}{p_M}\right] + H_\beta + [H_{\gamma_1} + H_{\gamma_2} + H_{\gamma_3} + H_{\gamma_4}]$$

$$\exp\left\{-\left(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s\right)(t-t_1)\right\} \quad (48)$$

where,

$$H_\beta = \frac{\xi_0 \pi^{\frac{1}{2}} p_M^{\frac{5}{2}}}{8\nu^{\frac{3}{2}} (1+p_M)^{\frac{7}{2}}} \exp\left(\frac{-\nu(t-t_0)(1+2p_M)}{p_M (1+p_M)}\right) k^2$$

$$+ [\frac{3p_M k^4}{2\nu^2 (t-t_0)^{\frac{5}{2}}} + \left(\frac{(7p^2_M - 6p_M)}{3\nu(1+p_M)(t-t_0)^{\frac{3}{2}}}\right) k^6 - \left(\frac{(3p^2_M - 2p_M + 3)}{3(1+p^2_M)(t-t_0)^{\frac{1}{2}}}\right) k^8 +$$

$$+ \left(\frac{8\nu^{\frac{1}{2}} (3p^2_M - 2p_M + 3)}{3(1+p_M)^{\frac{5}{2}} p^{\frac{1}{2}}_M}\right) k^9 F(\omega)]$$

$$\mathrm{F}(\omega) = \exp(-\omega^2) \int_0^\omega \exp(x^2) dx, \omega = \left[\frac{\nu(t-t_0)}{p_M (1+p_M)}\right]^{\frac{1}{2}} k$$

$$H_{\gamma_1} = -\frac{\xi_1 \pi^{1/2} p_M^{\ 5}}{8\nu^2 (1+p_M)^5} \exp\left(\frac{-\nu(t-t_1)(1+2p_M - p^2_M)}{p_M (1+p_M)}\right) k^2$$

$$[\left(\frac{18p_M}{\nu^4 (1+p_M)(t-t_1)^5}\right) k^6 + \left(\frac{15 - 6p_M + 21p^2_M}{4\nu^3 (1+p_M)^2 (t-t_1)^4} + \frac{4p_M}{\nu^2 (1+p_M)(t-t_1)^3}\right) k^8$$

$$+ \left(\frac{15 - 6p_M + 36p^2_M - 6p^3_M + 61p^4_M}{12\nu^2 p_M (1+p_M)^3 (t-t_1)^3} + \frac{14p^2_M - 40p_M - 18}{\nu(1+p_M)^2 (t-t_1)^2}\right) k^{10}$$

$$+ \left(\frac{(1+p_M)^2 (75 - 30p_M + 180p^2_M - 30p^3_M + 305p^4_M)}{120\nu p^2_M (1+p_M)^4 (t-t_1)^2}\right.$$

$$+ \left(\frac{14p^4_M - 56p^3_M - 12p^2_M - 40p_M - 18}{p_M (1+p_M)^3 (t-t_1)}\right) k^{12}$$

$$+ \left(\frac{(1+p_M)^2 (75 - 3p_M + 90p^2_M - 30p^3_M + 21p^4_M)}{120p^3_M (1+p_M)^5 (t-t_1)}\right) k^{14}$$

$$+ \left(\frac{\nu(1+p^2_M)(14p^4_M - 56p^3_M - 12p^2_M - 40p_M - 18) k^{14}}{p^2_M (1+p_M)^4 (t-t_1)}\right.$$

$$\left. + \frac{\nu(1+p_M)^3 (75 - 3p_M + 90p^2_M - 30p^3_M + 21p^4_M) k^{16}}{120p^4_M (1+p_M)^6}\right)] \exp(-\omega_2) Ei(\omega_2)$$

$$Ei(\omega_2) = \int \exp[(\frac{\nu(1+p^2_M) t k^2}{p_M (1+p_M)}) / (t-t_1)] dt$$

$$H_{\gamma_2} = -\frac{\xi_1 \pi^{1/2} p^5_M (1+p_M)^4}{8\nu^2 (1+2p_M)^{9/2}} \exp$$

$$\left(\frac{-\nu(t-t_1)(1+2p_M - p_M^{\ 2})}{p_M (1+p_M)}\right) k^2$$

$$[\{18p_M (1+p_M) / \nu^4 (1+2p_M)(t-t_1)^5\} k^6 +$$

$$\{(17+32p_M-2p^2{}_M+4p^3{}_M+20p^4{}_M)/(4\nu^3(1+2p_M)^2(t-t_1)^4)$$
$$+120p_M(1+p_M)/3\nu^2(1+2p_M)(t-t_1)^3\}k^8$$

$$+\{[(17+49p_M+13p^2{}_M+13p^3{}_M+98p^4{}_M+134p^5{}_M+104p^6{}_M$$
$$+60p^7{}_M)/4\nu^3(1+2p_M)^2(t-t_1)^4]+[(52p^4{}_M+64p^3{}_M-48p^2{}_M$$
$$-40p_M)/\nu(1+2p_M)^2(t-t_1)^2]\}k^{10}$$

$$+\{[(1+p_M-p^2{}_M+p^3{}_M)(17+49p_M+13p^2{}_M-13p^3{}_M+98p^4{}_M+134p^5{}_M$$
$$+104p^6{}_M+60p^7{}_M)/24\nu p^2{}_M(1+2p_M)^4(t-t_1)^2]+[(-40p_M-89p^2{}_M+51p^3{}_M$$
$$+124p^4{}_M-40p^5{}_M+36p^6{}_M+60p^7{}_M)/p_M(1+2p_M)^3(t-t_1)]\}k^{12}$$
$$+\{[(1+p_M-p^2{}_M+p^3{}_M)^2(17+49p_M+13p^2{}_M-13p^3{}_M+98p^4{}_M+134p^5{}_M$$
$$+104p^6{}_M+60p^7{}_M)/24p^2{}_M(1+2p_M)^5(t-t_1)]\}k^{14}$$

$$-\{[\nu(1+p_M-p^2{}_M+p^3{}_M)(-40p_M-89p^2{}_M+51p^3{}_M+124p^4{}_M-40p^5{}_M+36p^6{}_M$$

$$+60p^7{}_M)/p^2{}_M(1+2p_M)^4]k^{14}+[\nu(1+p_M-p^2{}_M+p^3{}_M)^3(17+49p_M+13p^2{}_M-13p^3{}_M$$

$$+98p^4{}_M+134p^5{}_M+104p^6{}_M+60p^7{}_M)k^{16}/24p^4{}_M(1+2p_M)^6].\exp(\omega_3)Ei(\omega_3)$$

$$Ei(\omega_3)=\int\exp[\{(-\nu(t-t_1)(1+2p_M-p^2{}_M)tk^2)/(p_M(1+2p_M))\}/(t-t_1)]dt$$

and, $\omega_3=[\{-\nu(t-t_1)(1+2p_M-p^2{}_M)t\}/\{p_M(1+2p_M)\}]k^2$

$$H_{\gamma_3}=-\frac{\xi_1\pi^{1/2}p^4{}_M}{16\nu(1+p_M)^{15/2}}\exp$$

$$\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2[\frac{45p_M}{2\nu^4(1+p_M)^2(t-t_1)^4}k^8+$$
$$\left\{\frac{(20p^2{}_M-70p_M-5)}{2\nu^3(1+p_M)^2(t-t_1)^3}+\frac{60p_M}{\nu^2(t-t_1)^2}\right\}k^{10}+$$

$$\left\{\frac{(20p^4{}_M-40p^3{}_M+160p^2{}_M-60p_M-5)}{4\nu^2p_M(1+p_M)^2(t-t_1)^2}+\frac{(24p^2{}_M-200p_M+20)}{\nu(t-t_1)}\right\}k^{12}+$$

$$\left\{\frac{(1-2p_M)(20p^4{}_M-40p^3{}_M+160p^2{}_M-60p_M-5)}{4\nu p^2{}_M(1+p_M)^2(t-t_1)}\right\}k^{14}$$

$$-\{((20-240p_M+424p^2{}_M-48p^3{}_M)/p_M)k^{14}+((1-2p_M)^2(20p^4{}_M-40p^3{}_M$$

$$+160p^2{}_M-60p_M-5)/4p^3{}_M(1+p_M)^2)k^{16}\}\exp(-\omega_4)Ei(\omega_4)]$$
$\omega_4=\{\nu(1-2p_M)t/p_M\}k^2$ and
$$Ei(\omega_4)=\int\exp[\{\nu(1-2p_M)t/p_M\}/(t-t_1)]dt$$

$$H_{\gamma_4}=-\frac{\xi_1\pi^{1/2}p^{9/2}{}_M}{2^8\nu(1+p_M)^{11/2}}\exp\left(\frac{-\nu(t-t_1)(1+2p_M)}{p_M}\right)k^2[$$
$$\left\{\frac{1890p_M}{\nu^4(1+p_M)^6(t-t_1)^4}\right\}k^6+$$

$$+\{(-423170-16938180p_M-25381440p^2{}_M-16894080p^3{}_M-4213440p^4{}_M)/\nu^3(1+p_M)^6(t-t_1)^3\}k^8$$
$$+\{(-2115855-4237380p_M+4245780p^2{}_M+16927680p^3{}_M+14783328p^4{}_M$$
$$+4218816p^5{}_M+4368^6p_M)/\nu^2(1+p_M)^6(t-t_1)^2\}k^{10}$$

$$+\{(-2115855-5670p_M+12720540p^2{}_M+8436120p^3{}_M-19072032p^4{}_M$$
$$-25347840p^6{}_M-4128p^7{}_M+2304p^8{}_M)/\nu(1+p_M)^6p^2{}_M(t-t_1)\}k^{12}$$

$$-\{((-2115855+4226040p_M+12731880p^2{}_M-17004960p^3{}_M-35944272p^4{}_M$$

$$+12796224p^5{}_M+42264592p^6{}_M+16857280p^7{}_M+9920p^8{}_M-4864p^9{}_M)k^{14}$$

$$+1344p_M)k^{12}/(1+p_M)^6p^3{}_M\}\exp(\omega_3)Ei(\omega_3)]$$
$$\omega_5=\exp\{\nu(1-2p_M)t/p_M\}k^2,$$
$$Ei(\omega_5)=\int\exp[\{\nu(1-2p_M)tk^2/p_M\}/(t-t_1)]dt$$

From Eq. (48) we get:

$$H=H_1+H_2$$
$$\exp\{-(2\varepsilon_{pkl}\Omega_p+2\varepsilon_{qki}\Omega_q+2\varepsilon_{rkj}\Omega_r+2\varepsilon_{skm}\Omega_s)(t-t_1)\}\qquad(49)$$

$H_1=\frac{N_0k^2}{\pi}\exp\left[-\frac{2\nu k^2(t-t_0)}{p_M}\right]+H_\beta$ and $H_2=(H_{\gamma_1}+H_{\gamma_2}+H_{\gamma_3}+H_{\gamma_4})$

In Eq. (49) H_1 and H_2 magnetic energy spectrum arising from consideration of the three and four–point correlation equations respectively. Eq. (49) can be integrated over all wave numbers to give the total magnetic turbulent energy. That is:

$$\frac{\overline{h_ih_i'}}{2}=\int_0^\infty Hdk\qquad(50)$$

Now, $\int_0^\infty H_1dk=\frac{N_0p^{3/2}{}_M\nu^{-3/2}(t-t_0)^{-3/2}}{8\sqrt{2\pi}}+$
$$\xi_0Q\nu^{-6}(t-t_0)^{-5}$$
$$\int_0^\infty H_2dk=\xi_1[R\nu^{-17/2}(t-t_1)^{-15/2}+S\nu^{-19/2}(t-t_1)^{-17/2}$$
$$\exp\{-(2\varepsilon_{pkl}\Omega_p+2\varepsilon_{qki}\Omega_q+2\varepsilon_{rkj}\Omega_r+2\varepsilon_{skm}\Omega_s)(t-t_1)\}$$

where, $R=Q_2+Q_4+Q_6+Q_7$, $S=Q_1+Q_3+Q_5$ and Q's values are:

$$Q=\frac{\pi.p^6M}{(1+P_M)(1+2P_M)^{5/2}}$$
$$\left\{\frac{9}{16}+\frac{5P_M(7P_M\_6)}{(1+2P_M)}-\frac{35P_M(3p^2{}_M-2p_M+3)}{8(1+2p_M)^2}+\frac{8p_M(3p^2{}_M-2p_M+3)}{3.2^6.(1+2p_M)^3}+\ldots\right\}$$

$$Q_1=-\frac{\pi.p^6M}{(1+P_M)^{5/2}(1+2P_M-p^2{}_M)^{7/2}}$$
$$\left\{\frac{15.9}{2^6}+\frac{15.7(15-6p_M+21p^2{}_M)}{2^{10}(1+2P_M-p^2{}_M)}+\frac{15.7.3(15-6p_M+36p^2{}_M-6p^3{}_M+61p^4{}_M)}{2^{11}(1+2p_M-p^2M)^2}\right.$$
$$\frac{11.9.7(1+P^2{}_M)(75-30p_M+180p^2{}_M-30p^3{}_M+305p^4{}_M)}{2^{13}(1+2p_M-p^2M)^3}+$$
$$\left.\frac{13.11.9.7(1+P^2{}_M)^2(75-3p_M+90p^2{}_M-30p^3{}_M+15p^4{}_M)}{2^{14}(1+2p_M-p^2M)^4}-\ldots\right.$$

$$Q_2 = -\frac{\pi . p^{21/2}{}_M}{(1+p_M)^{3/2}(1+2p_M - p^2{}_M)^{9/2}}\left[\frac{15.7}{2^6} + \right.$$

$$\frac{15.9.7(14p^2{}_M - 18 - 40p_M)}{2^9(1+2p_M - p^2{}_M)} +$$

$$\left.\frac{15.11.9.7(14p^4{}_M - 56p^3{}_M - 12p^2{}_M - 40p_M - 18)}{2^{10}(1+2p_M - p^2{}_M)^2} - \ldots\right]$$

$$Q_3 = -\frac{\pi . p^{19/2}{}_M (1+p_M)^{1/2}}{(1+2p_M)^2(1+2p_M - p^2{}_M)^{7/2}} \cdot \left[\frac{9.15}{2^6} + \right.$$

$$\frac{15.7(17+32p_M - 2p^2{}_M + 4p^3{}_M + 20p^4{}_M)}{2^{10}(1+p_M)^2(1+2p_M - p^2{}_M)}$$

$$+$$

$$\frac{9.7.5(17+49p_M + 13p^2{}_M - 13p^3{}_M + 98p^4{}_M + 134p^5{}_M + 104p^6{}_M + 60p^7{}_M)}{2^{11}(1+p_M)^3(1+2p_M - p^2{}_M)^2}$$

$$+\left(\frac{(11.9.7.5(1+p_M - p^2{}_M + p^3{}_M)(17+49p_M + 13p^2{}_M - 13p^3{}_M + 98p^4{}_M + 134p^5{}_M + 104p^6{}_M + 60p^7{}_M)}{2^{13}(1+p_M)^4(1+2p_M - p^2{}_M)^3}\right.$$

$$\left.+\left(\frac{(13.11.9.7.5(1+p_M - p^2{}_M + p^3{}_M)^2(17+49p_M + 13p^2{}_M - 13p^3{}_M + 98p^4{}_M + 134p^5{}_M + 104p^6{}_M + 60p^7{}_M)}{2^{14}(1+p_M)^5(1+2p_M - p^2{}_M)^4}\right) - \ldots\right]$$

$$Q_4 = -\frac{\pi . p^{21/2}M}{(1+P_M)^{1/2}(1+2P_M)(1+2P_M - p^2M)^{9/2}}\left[\frac{25.7.3}{2^5} + \right.$$

$$\frac{15.9.7(-40p_M - 48P^2{}_M + 64p^3m + 52p^4{}_M) +}{2^9(1+p_M)^2(1+2p_m - p^2m)}$$

$$\left.\frac{15.11.9.7(-40p_M - 89p^2{}_M + 51p^3{}_M + 124p^4M - 40p^5{}_M + 36p^6{}_M + 60p^7{}_M)}{2^{10}(1+p_M)^3(1+2p_M - p^2{}_M)^2}\ldots\right]$$

$$Q_5 = -\frac{\pi . p^{19/2}M}{(1+P_M)^{19/2}(1+2P_M)^{9/2}}$$

$$\left\{\begin{aligned}&\frac{45.7.53}{2^{10}} + \frac{9.7.5.3(20p^2{}_M - 70p_M - 5)}{2^{11}(1+2P_M)} + \frac{11.9.7.5.3(20p^4{}_M - 40p^3{}_M + 160p^2{}_M - 60p_M - 5)}{2^{13}(1+2p_M - p^2M)^2} + \\ &\frac{13.11.9.7.5.3(1-2p_M)(20p^4{}_M - 40p^3{}_M + 160p^2{}_M - 60p_M - 5)}{2^{14}(1+2p_M)^3} - \ldots\end{aligned}\right\}$$

$$Q_6 = -\frac{\pi . p^{21/2}M}{(1+P_M)^{15/2}(1+2P_M)^{11/2}}$$

$$\left\{\frac{15.9.7.5.3}{2^8} + \frac{11.9.7.5.3(24p^2{}_M - 200p_M + 20)}{2^{11}(1+2P_M)} - \ldots\right\}$$

$$Q_7 = -\frac{\pi . p^9{}_M}{(1+p_M)^{23/2}(1+2p_M)^{7/2}}$$

$$\left[\left\{\frac{9.7.5.3}{2^{11}} - \frac{7.5.3(4231710 + 16938180p_M + 25381440p^2{}_M + 1689480p^3{}_M + 4213440p^4{}_M}{2^{13}(1+2p_M)}\right\}\right.$$

$$\left.-\left\{\frac{\begin{aligned}&(9.7.5.3(2115855 + 4237380p_M - 4245780p^2{}_M - 16927680p^3{}_M - 14783328p^4{}_M \\ &- 4218816p^5{}_M - 43680p^6{}_M)\end{aligned}}{2^{14}(1+2p_M)^2}\right\}\ldots\right]$$

Therefore from Eq. (50) we get:

$$\frac{\overline{h_i h_i'}}{2} = \frac{N_0 p^{3/2}{}_M \nu^{-3/2}(t-t_0)^{-3/2}}{8\sqrt{2\pi}} + \xi_0 Q \nu^{-6}(t-t_0)^{-5} + \xi_1[R\nu^{-17/2}(t-t_1)^{-15/2} + S\nu^{-19/2}(t-t_1)^{-17/2}]$$

$$\exp\left\{-\left(2\varepsilon_{pkl}\Omega_p + 2\varepsilon_{qki}\Omega_q + 2\varepsilon_{rkj}\Omega_r + 2\varepsilon_{skm}\Omega_s\right)(t-t_1)\right\} \quad (51)$$

Thus, the decay of MHD turbulence for four-point correlation in a rotating system may be written as:

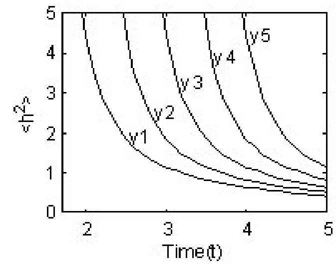

Fig. 1: Energy decay curves for $f = 0.75$

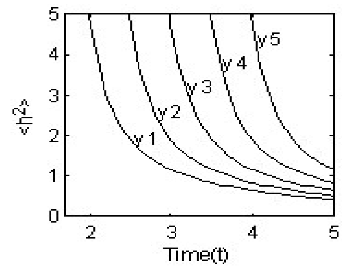

Fig. 2: Energy decay curves for f = 0.50

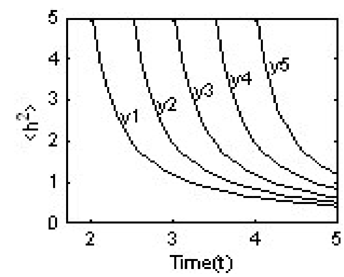

Fig. 3: Energy decay curves for $f = 0.15$

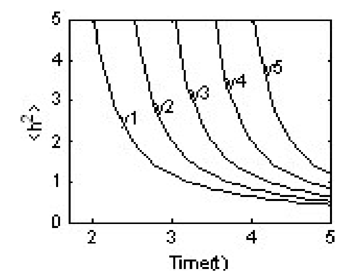

Fig. 4: Energy decay curves for f = 0

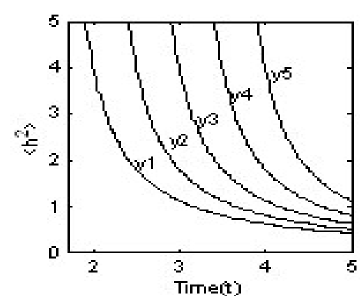

Fig. 5: Energy decay curves for $f = 1.5$

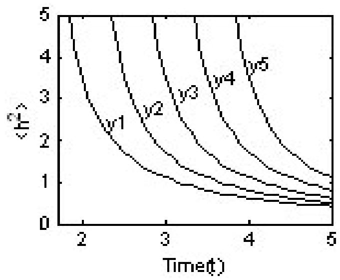

Fig. 6: Decay curves of Eq. (54)

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + [C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}] \exp\{-f(t-t_1)\} \quad (52)$$

where f is Coriolis force parameter. If all omegas are equal to zero in Eq. (52) then, we get:

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} + [C(t-t_1)^{-15/2} + D(t-t_1)^{-17/2}] \quad (53)$$

This is the decay of MHD turbulence for four-point correlation in non-rotating system. Which is same as [12]. If ξ1= 0, then:

$$\langle h^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \quad (54)$$

This is the decay of MHD turbulence for three-point correlation. This is completely same with the result obtained by Sarker and Kishore (1991).

RESULTS AND DISCUSSION

Here $y1$, $y2$, $y3$, $y4$ and $y5$ are solutions of Eq. (52) in a rotating system at $t_0 = t_1 = 0.5$, 1 , 1.5 , 2, 2.5 corresponding with Coriolis force 0.75, 0.50 ,0.15, 0 and 1.50, which indicated in the Fig. 1 to 5 respectively. Also $y1$, $y2$, $y3$, $y4$ and $y5$ are solutions of Eq. (54) at $t_0 = t_1 = 0.5$, 1, 1.5, 2, 2.5 which indicated in Fig. 6.

From Fig. 1 to 5, we observe that energy decay curves successively increased for decreasing the values of the Coriolis force and maximum at the point where the Coriolis force is equal to zero. For three-point correlation energy decay very slowly. If the quadruple and quintuple correlations were not neglected, than more terms in negative higher power of $(t - t_1)$ would be added to the Eq. (52) and for large times the last terms in the Eq. (52), becomes negligible, leaving the -3/2 power decay law for the final period. Decay of energy of MHD turbulence for both rotating and non-rotating system with different values of Coriolis force is graphically shown in Fig. 1 to 6. We conclude that in the magnetic field fluctuation the energy decay increases with the decreases of rotation and maximum at the point where the rotation (i.e., coriolis force) is zero.

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