Research Article
Improvement of Inventory Control for Defective Goods Supply Chain with Imperfect Quality of Commodity Components in Uncertain State

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Abstract: In this study, we proposed a mathematical model for four-level defective goods supply chain with imperfect quality of commodity components in an uncertain state to maximize profit of supply chain. It is assumed that the inspection of incoming parts in suppliers is randomly done and incomplete. This lead some of the manufactured products will not be properly manufactured because of defective parts and are considered as defective goods and in most cases, the defective products can be repaired by replacing with the good parts. The defective parts will be collected and then returned to the suppliers for repairing. Out proposed model considers defective parts problem by optimizing the costs of production, maintenance, shipping, reworking on the defective goods and parts, shortage in retailers due to the production of defective goods and cost of capital incurred by the companies. The model can anticipate the active suppliers/manufacturers/distributors and the quantity of parts and goods that must be exchanged between them. Our proposed model is novel and we used MINOS solver and LINGO software to solve the problem. The results ascertained the correctness and fine function of the proposed model.

Keywords: Defective goods, inventory control, LINGO, MINOS, supply chain, uncertain model

INTRODUCTION

One of important advantage in today's competitive global markets is reduction of defective products by manufacturers. The fractions of defective parts during production have significant direct relationship with the suppliers. In recent years, Supply Chain Management (SCM) (Guo and Tang, 2008; Sana, 2011; Lee and Kim, 2000; Tzeng *et al*., 2006) are the fields which cover from strategic issues to operational models. In the past two decades, the problem in supply chain management has drawn a great attention from manufacturers and organizations due to its optimal effects on their operations. Subsequently, the manufacturers need to find alternative ways to ensure the continuous provision of their products and utilize appropriate strategies to meet the market demands on time.

In addition, there are factors such as imperfect quality and defective raw materials that can affect the efficiency of productivity (Sarker *et al*., 2008; Roy *et al*., 2009; Mondal *et al*., 2009). So the manufacturers need a dedicated facility to perform the task of reworking these defective goods. Previous studies show that the selection of an appropriate supplier is an important role in reducing costs (Golmohammadi and Mellat-Parast, 2012; Basnet and Leung, 2005; Aissaoui *et al*., 2007). However, when there are uncertain states

occur such as market fluctuation, natural disasters, currency drops and sudden demand and return, a robust modelling involved stochastic approach is an essential need especially when the location between the suppliers, warehouse and manufacturers are at far distance.

Recent study by Amin and Zhang (2013) on mathematical model for uncertain demand and return of a closed-loop Supply Chain Network (SCN) was employed to multiple plants, collection centers, demand markets and products. The result of their study shows the proposed statistical model can handle uncertain parameters.

Also, the effects of uncertainty product portfolios demand and prices on the optimal closed-loop supply chain planning have been taken in account by Amaro and Barbosa-Póvoa (2009). They used GAMS/CPLEX for solving their model. The results of their study provided great details on the supply chain partners production, transportation and inventory.

In a recent study, Ramezani *et al*. (2013) introduced a new stochastic multi-objective model of forward/reverse logistic network under uncertainty conditions with the objectives of maximization of profit, responsiveness and minimization of defective parts from suppliers. The stochastic model involved three levels in forward and two levels in reverse including suppliers, plants, distribution centers,

collection centers and disposal centers, respectively.

Other researchers also examined uncertainty model
In such uncertain state of SCN, the inventory control models that oversee concepts of warehouses and imperfect products after manufacturing is also needed (Chung *et al*., 2009). For example, literatures by Jacobs *et al*. (2005) and Lee and Kim (2000) proposed a hybrid method for solving the optimization problems of production-distribution planning in SCN. Other research by Eroglu and Ozdemir (2007) has developed a model of effective and returned goods as shortages but it only can solve numeral examples which showed that defective and useless goods can decrease the overall profits. Another model is the Economic Production Quantity (EPQ) model for defective goods (Jamal *et al*., 2004), however, the EPQ model does not consider the reworking function of defective goods.

Furthermore, if the raw materials are very expensive and have limited source, the reworking task can save so much cost. Several methods have been proposed for the improvement of efficiency and they showed that the correlation of simultaneous application of Total Quality Management (TQM), SCM and Just in Time (JIT) can enhance the effectiveness of the companies ((Kannan and Tan, 2005).

In this study, a new mathematical model for four-level defective goods supply chain by considering imperfect quality of commodity components and optimization of costs is presented. The related cost which affects the total profits includes the cost of production, maintenance, shipping, reworking on the

(Zeballos *et al*., 2012; Lin *et al*., 2011; Li and Liu, 2012).
defective goods and parts, shortage in retailers due to the production of defective goods and cost of capital incurred in uncertain state to maximize profit of supply chain network.

MODELING

Supply chain network: The aim of the proposed model is to maximize the profit of the defective goods in SCN in which will associate with the cost of capital incurred by the companies. The considered supply chain model is divided into four-levels as shown in Fig. 1. The first level (Level 1) is suppliers of parts/raw materials that convey the parts to next level. Here, most of the inspections of parts are done based on random selection and not complete. The second level (Level 2) is manufacturers which receive multiple types of incoming parts from suppliers. Here, the parts are assembled in a manufacturing line to produce the goods and send them to the next level. It is assumed that in the manufacturing process, some of the products may not be properly manufactured because of defective parts and considered as defective goods. In most cases, the defective products can be repaired by replacing with good parts and the defective parts are returned to the suppliers for repair.

The goods produced are received by distributors (Level 3) which will keep the goods in their warehouse before sending them to various retailers. In the
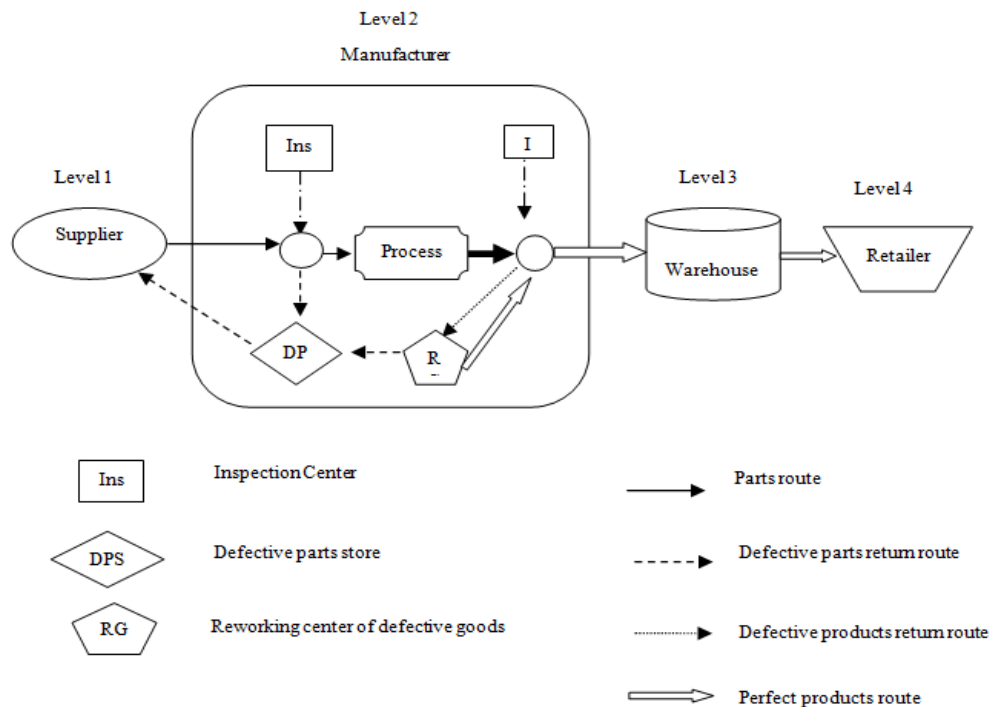

Fig. 1: Structure of supply chain network studied in this research

warehouse, the duration of storage time will affect the rent. So, it is important to schedule the appropriate time to process each time period so that these cost can be minimized.

The fourth level (Level 4) comprises of the retailers. In this study, the proposed model is adapted to maximize profit by optimize costs including production, maintenance, shipping, reworking on the defective goods and shortage in retailers due to the production of defective goods and cost of capital incurred by the companies.

Mathematical model: The proposed SCN is formulated as a mathematical model. Here, we set-up the assumptions, limitations, sets, parameters, decision variables, objective function and constraints in uncertain states.

Assumptions and limitations of the model: The following assumptions are made to develop the mathematical model:

- The amounts of demands were given to the manufacturers for products and to the suppliers for parts at the beginning of the period.
- Duration of each period is equal to the total of production time and rework time in that period.
- Shortage of products in retailers is allowed.
- Shortage of parts in manufacturers is not allowed.
- The model is designed for multi-supplier, multi-manufacturer, multi-warehouse, multi-retailer, multi-product and multi-component.
- The inspection of the products is perfect. However, inspection of the parts is random.
- The reworking of imperfect quality products and parts are allowed.
- Locations of plants, warehouses, retailers and suppliers are fixed
- Production capacities of suppliers and plants, storage of warehouses and retailers are known.
- The amounts of parameters except capacity parameters are uncertainty.

We also considered limitations for our model:

- The capacities of manufacturer's production, warehouse and retailers are limited.
- The storage capacities for each perfect product are limited.
- Store capacities and storage allocated capacities are limited for defective goods.
- For each period, the customers demand should be provided.
- The production and reworking times are limited.

Sets:
I = Set of Manufacturing and Remanufacturing plants (1... i ... I)

J = Set of Warehouses (1 ... j ... J)
L = Set of Products (1 ... l ... L)
T = Set of Periods (1 ... t ... T)
K = Set of Retailers (1 ... k ... K)

Parameters of the proposed model:
M = Set of Suppliers (1 ... m ... M)
N = Set of Parts (1 ... n ... N)
S = Set of Scenarios (1 ... s ... S)
γ_{ilt}^s = Rework cost of per defective goods l by manufacturer i during period under scenario s
hw_{jlt}^s = Holding cost of product l in distributor j during period t under scenario s
hd_{ilt}^s = Holding cost of product l in defective goods store inside the manufacturer i during period t under scenario s
θ_{ilt}^s = Time required to production of per goods l by manufacturer i during period t under scenario s
$T\theta_t$ = Total of production time during period t under scenario s
μ_{ilt}^s = Time of reworking required for goods l by manufacturer i during t under scenario s
μ_{ilt}^s = Total of production time during period t under scenario s
$T\mu_t$ = The total of reworking time during period t under scenario s
Pc_{ilt}^s = Production cost of per item by manufacturer i during period t under scenario s
tcw_{ijlt}^s = Shipping cost each product l from manufacturer i to distributor j during period t under scenario
tcr_{jklt}^s = Shipping cost each product l from distributor j to retailer k during period t under Scenario s
tcd_{ilt}^s = Shipping cost each defective goods l inside manufacturer I during period t (from production process to defective goods store and inverse) under scenario s
β_{klt}^s = Shortage cost for each product l in retailer k during period t under scenario s
ps_{ilt}^s = Selling price per unit of perfect product l in factory i during period t under scenario s
p_t^s = Probability of Occurrence scenario s in period t
p_{ilt}^s = Percentage of defective goods l produced by manufacturer i during period t under scenario s
pcp_{nmilt}^s = The cost of buying each part n from supplier m by factory i for producing goods l in period t under scenario s
ε_{nmill}^s = Inspection fee for each part n produced by supplier m and product l by factory i in period t under scenario s

d_{klt}^s = Product demand l by retailer k in period t under scenario s

τ_{nmilt}^s = Shipping Cost of each part n from supplier m to factory I for producing product l in period t under scenario s

υ_{nmilt}^s = Shipping cost of defective parts returned n from factory i to supplier m before and after application in product l during period t under scenario s

Ψ_{nmilt}^s = Reworking cost of defective parts returned n from factory i to supplier m before and after application in product l during period t under scenario s

ir_t^s = Cost of capital incurred by the companies in period t under scenario s

dpa_{nmilt}^s = Amount of parts ordered n by factory I to supplier m for product l during period t under scenario s

npa_{nmilt}^s = The parts required n for manufacturing product l in factory i that produce by supplier m in period t under scenario s

ppa_{nmilt}^s = Percentage of defective parts n before application in product l in factory i produced by supplier m in period t under scenario s

$nrpa_{nmilt}^s$ = The parts required n for repairing defective product l in factory i that produce by supplier m in period t under scenario s

Variables of the model:

$profit_{max}$ = Maximum profit of supply chain network

$bdfpa_{nmilt}^s$ = Amounts of defective parts n before application in product l in factory i produced by supplier m in period t under scenario s

$tdefpa_{nmilt}^s$ = Total of defective parts that be produce by supplier m for product l in factory I in period t under scenario s

$trpa_{nmilt}^s$ = Total of need parts n for repair product in factory I that be produce by supplier m in period t under scenario s

$ntcpa_{nmilt}^s$ = Total of required perfect parts n for producing all of the perfect product l by factory I produced by supplier m in period t under scenario s

tpa_{nmilt}^s = Total of parts n that sent from supplier m to factory I for produce goods l in period t under scenario s

X_{ijlt}^s = Amount of product l transported from factory i to distributor j during period t under scenario s

Y_{jklt}^s = Amount of product l transported from distributor j to retailer k during period t under scenario s

Q_{ilt}^s = Economic production quantity of product l by factory i during period t under scenario s

Def_{ilt}^s = Amount of defective goods l produced by factory i during period t under scenario s

Co_{ilt}^s = Amount of perfect products l produced by factory i during t before reworking under scenario s

B_{klt}^s = Amount of the shortage of product l in retailer k during period t under scenario s

Objective function:

Profit maximum = Probability of good occurrence scenarios × [Total revenue - ((Total costs/ $(1+ir_t^s)^t$))]

$$profit_{max} = \sum_{l=1}^{L}\sum_{t=1}^{T}\sum_{s=1}^{S}(1-p^s).\left[\left(\sum_{i=1}^{I}ps_{ilt}^s.Q_{ilt}^s - \begin{array}{l}\sum_{i=1}^{I}pc_{ilt}^s.Q_{ilt}^s + \\ \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}pcp_{nmilt}^s.tpa_{nmilt}^s + \\ \sum_{j=1}^{J}hw_{jlt}^s.\left(\sum_{t'=1}^{t'\in T}\sum_{i=1}^{I}X_{ijlt}^s - \sum_{t'=1}^{t'\in T}\sum_{k=1}^{K}Y_{jklt}^s\right) + \\ \sum_{i=1}^{I}hd_{ilt}^s.Def_{ilt}^s + \\ \sum_{i=1}^{I}\gamma_{ilt}^s.Def_{ilt}^s + \\ \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\varepsilon_{nmilt}^s.tpa_{nmilt}^s + \\ \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\tau_{nmilt}^s.tpa_{nmilt}^s + \\ \sum_{i=1}^{I}\sum_{j=1}^{J}tcw_{ijlt}^s.X_{ijlt}^s + \\ \sum_{j=1}^{J}\sum_{k=1}^{K}tcr_{jklt}^s.Y_{jklt}^s + \\ \sum_{i=1}^{I}tcd_{ilt}^s.Def_{ilt}^s + \\ \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\upsilon_{nmilt}^s.tdefpa_{nmilt}^s + \\ \sum_{n=1}^{N}\sum_{m=1}^{M}\sum_{i=1}^{I}\psi_{nmilt}^s.tdefpa_{nmilt}^s \\ \sum_{k=1}^{K}\beta_{klt}^s.B_{klt}^s\end{array}\right)\middle/(1+ir_t^s)^t\right]$$

(1)

Equation (1) shows the objective function, the aim is to maximize profit by minimizing the total costs of supply chain. It includes:

- Production cost of goods
- Cost of parts buying
- Cost of maintenance in the distributors and defective goods stores
- Cost of defective goods reworking
- Inspection cost of parts
- Shipping cost of parts from suppliers to manufacturers
- Cost of logistic from manufacturers to distributers
- Cost of logistic from distributers to retailers
- Cost of logistic from manufacturers to defective goods stores and vice versa
- Shipping cost of parts from manufacturers to suppliers

- Cost of defective parts reworking
- Cost of shortage in retailers because of defective goods production
- Cost of capital incurred by the companies

CONSTRAINTS

$$\sum_{n=1}^{N}\sum_{m=1}^{M} npa_{nmilt}^{s}.Q_{ilt}^{s} = \sum_{n=1}^{N}\sum_{m=1}^{M} ntcpa_{nmillt}^{s} \qquad \forall i,l,t,s \tag{2}$$

Equation (2) required Parts for manufacturing of products:

$$dpa_{nmilt}^{s} = ntcpa_{nmilt}^{s} + trpa_{nmilt}^{s} + bdfpa_{nmilt}^{s} \qquad \forall n,m,i,l,t,s \tag{3}$$

Equation (3) determinates the ordered parts:

$$dpa_{nmilt}^{s} \le tpa_{nmilt}^{s} \qquad \forall n,m,i,l,t,s \tag{4}$$

Equation (4) Supply of all parts are ordered by suppliers:

$$bdfpa_{nmilt}^{s} = ppa_{nmilt}^{s}.tpa_{nmilt}^{s} \qquad \forall n,m,i,l,t,s \tag{5}$$

Equation (5) shows the amounts of defective parts at the factories before using them:

$$\sum_{n=1}^{N}\sum_{m=1}^{M} nrpa_{nmilt}^{s}.def_{ilt}^{s} = \sum_{n=1}^{N}\sum_{m=1}^{M} trpa_{nmilt}^{s} \qquad \forall i,l,t,s \tag{6}$$

Equation (6) investigates the amounts of required parts for reworking:

$$tdefpa_{nmilt}^{s} = bdfpa_{nmilt}^{s} + trpa_{nmilt}^{s} \qquad \forall n,m,i,l,t,s \tag{7}$$

Equation (7) shows all of the defective parts:

$$tpa_{nmilt}^{s} \le V_{nmilt}^{s} \qquad \forall n,m,i,l,t,s \tag{8}$$

Equation (8) shows the limitation of parts production by suppliers:

$$\sum_{s=1}^{S} Q_{ilt}^{s} \le S_{ilt} \qquad \forall i,l,t \tag{9}$$

Equation (9) shows the restriction of the EPQ capacity in manufacturers:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{s=1}^{S} X_{ijlt}^{s} \le W_{jt} \qquad \forall j,t \tag{10}$$

$$\sum_{i=1}^{I}\sum_{s=1}^{S} X_{ijlt}^{s} \le W_{jlt} \qquad \forall j,l,t \tag{11}$$

Equation (10) and (11) consider the delivery capacity constraint for distributors:

$$\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{s=1}^{S} Y_{jklt}^{s} \le W'_{kt} \qquad \forall k,t \tag{12}$$

$$\sum_{j=1}^{J}\sum_{s=1}^{S} Y_{jklt}^{s} \le W'_{klt} \qquad \forall k,l,t \tag{13}$$

Equation (12) and (13) describe capacity constraint for retailers:

$$Def_{ilt}^{s} = p_{ilt}^{s}.Q_{ilt}^{s} \qquad \forall i,l,t,s \tag{14}$$

Equation (14) shows the amount of defective goods:

$$Co_{ilt}^{s} = \left(1 - p_{ilt}^{s}\right).Q_{ilt}^{s} \qquad \forall i,l,t,s \tag{15}$$

The amount of perfect goods before reworking describes by Eq. (15):

$$Q_{ilt}^{s} = Co_{ilt}^{s} + Def_{ilt}^{s} \qquad \forall i,l,t,s \tag{16}$$

The total of produced products by manufacturers in each period are shown with Eq. (16):

$$\sum_{k=1}^{K} d_{klt}^{s} \le \sum_{i=1}^{I} Q_{ilt}^{s} \qquad \forall l,t,s \tag{17}$$

Equation (17) assures that the total demands to each manufacturer in per period do not exceed the production capacity of those manufacturers; also, the all demands are covered by perfect products:

$$\sum_{k=1}^{K} B_{klt}^{s} = \sum_{k=1}^{K} d_{klt}^{s} - \sum_{i=1}^{I} Co_{ilt}^{s} \qquad \forall l,t,s \tag{18}$$

$$\sum_{i=1}^{I} Co_{ilt}^{s} \le \sum_{k=1}^{K} d_{klt}^{s} \qquad \forall l,t,s \tag{19}$$

Constrains (18) and (19) show the amount of shortage in retailer due to production of defective products:

$$\sum_{i=1}^{I} X_{ijlt}^{s} = \sum_{k=1}^{K} Y_{jklt}^{s} \qquad \forall j,l,t,s \tag{20}$$

Equation (20) investigates the final inventory balance for per warehouse and it shows that the inventory for each warehouse is zero at the end:

$$\sum_{j=1}^{J} Y_{jklt}^{s} = d_{klt}^{s} \qquad \forall k,l,t,s \tag{21}$$

The inventory of retailers is not more than demands; it shows by Eq. (21):

$$\sum_{k=1}^{K}\sum_{t'=1}^{t'\neq T} Y_{jklt'}^{s} \leq \sum_{i=1}^{I}\sum_{t'=1}^{t'\neq T} X_{ijlt'}^{s} \qquad \forall \quad j,l,s,t \neq T \quad (22)$$

Limitation (22) explain that the amount of goods shipped by each warehouse to the retailers in per period do not exceed the inventory of that warehouse:

$$\sum_{j=1}^{J} X_{ijlt}^{s} \leq Q_{ilt}^{s} \qquad \forall \quad i,l,t,s \qquad (23)$$

Equation (23) shows the amount of transported goods from the manufacturers to the distributors is less than or equal to the total production:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{s=1}^{S} \theta_{ilt}^{s}.Q_{ilt}^{s} \leq T\theta_{t} \qquad \forall \quad t \qquad (24)$$

Constraint (24) represents the limitation of available times of production facilities:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{s=1}^{S} \mu_{ilt}^{s}.Def_{ilt}^{s} \leq T\mu_{t} \qquad \forall \quad t \qquad (25)$$

Equation (25) shows the limitation of available times for reworking:

$$T\theta_{t} + T\mu_{t} = T_{t} \qquad \forall \quad t \qquad (26)$$

Equation (26) explains the required time for production and reworking in each period is equals with the length of that particular production period:

$$Q_{ilt}^{s}, X_{ijlt}^{s}, Y_{jklt}^{s}, B_{klt}^{s}, Def_{ilt}^{s}, Co_{ilt}^{s}, profit_{max}, bdfpa_{nmilt}^{s}, tdefpa_{nmilt}^{s}, tpa_{nmilt}^{s} \geq 0$$
$$\forall \quad n,m,i,j,k,l,t,s$$
$$(27)$$

Where the amount of production, delivery to warehouses and retailers, shortage in retailers, defective goods, perfect goods, profit of the supply chain network, defective parts before of process in factory, total defective parts and total parts are non-negative values are shown in Eq. (27).

Our proposed model is novel and we used MINOS solver and LINGO software to solve the problem. For solving the mentioned model, we assumed the amounts of parameters according to Table 1.

RESULTS AND DISCUSSION

We proposed a mathematical model of four-level defective goods supply chain by considering imperfect quality of commodity components in uncertain state to maximize profit of supply chain network. The

Table 1: Parameters and their values

Parameters	Value
γ_{ilt}^{s}	Uniform (100, 300)
hw_{jlt}^{s}	Uniform (5, 7)
hd_{ilt}^{s}	Uniform (4, 6)
θ_{ilt}^{s}	Uniform (9, 13)
μ_{ilt}^{s}	Uniform (5, 7)
pc_{ilt}^{s}	Uniform (2000, 4000)
tcw_{ijlt}^{s}	Uniform (10, 15)
tcr_{jklt}^{s}	Uniform (7, 9)
tcd_{ilt}^{s}	Uniform (4, 7)
β_{klt}^{s}	Uniform (70, 90)
ps_{ilt}^{s}	Uniform (10000, 17000)
npa_{nmilt}^{s}	Uniform (3, 10)
ppa_{nmilt}^{s}	Uniform (0, 0.03)
$nrpa_{nmilt}^{s}$	Uniform (1, 3)
p_{ilt}^{s}	Uniform (0, 0.1)
p_{t}^{s}	Uniform (0, 0.04)
pcp_{nmilt}^{s}	Uniform (30, 45)
ε_{nmilt}^{s}	Uniform (1, 3)
d_{klt}^{s}	Uniform (7000, 10000)
τ_{nmilt}^{s}	Uniform (5, 9)
U_{nmilt}^{s}	Uniform (3, 6)
ψ_{nmilt}^{s}	Uniform (3, 5)

Table 2: Results of seven sample problems by different dimensions

Sample problems	Dimensions	MINOS	LINGO
	n, m, i, j, k, l, t, s	profit$_{max}$	Profit$_{max}$
1	1, 1, 1, 1, 1, 1, 1, 1	6216515	6216515
2	2, 1, 1, 1, 1, 2, 1, 1	1.129012E+7	1.129012E+7
3	2, 2, 1, 1, 1, 1, 1, 1	4502152	4502152
4	2, 2, 2, 1, 1, 1, 2, 1	9004304	9004304
5	2, 3, 1, 1, 1, 2, 2, 1	1.343697E+7	1.343697E+7
6	2, 2, 2, 2, 2, 2, 2, 1	3.601721E+7	-
7	4, 2, 1, 1, 2, 2, 1, 1	2.795026E+8	-

manufacturers ordered to suppliers the required parts quantity (dPa_{nmilt}^{s}) to produce of ordered products by their customers (d_{klt}^{s}). It is assumed that the inspection of parts in suppliers is randomly and not complete. So, all parts are inspected before entering to the production process in manufacturers ($bdfpa_{nmilt}^{s}$). Also, during the production process some of the products not properly manufactured because of damaged parts in the production process in manufacturers which considered as defective goods (Def_{ilt}^{s}). In most cases, the defective products can be repaired by replace the correct parts ($trpa_{nmilt}^{s}$). The defective parts due to inspection and production process of goods in factories will be collected and returned to the suppliers for repairing ($tdefpa_{nmilt}^{s} = bdfpa_{nmilt}^{s} + trpa_{nmilt}^{s}$).

The results for seven sample problems with different parameters are shown in Table 2. The first column is number of sample problems, second column dimensions; third and forth columns are output of MINOS solver and LINGO software to find the profit of supply chain network. The sample problems 6 and 7 because of large size dimension could not be solved by LINGO. Table 3 reports the values of the decision variables of the sample problem 7 solved by MINOS. The results show the validation and correctness of the model.

Table 3: Sample problem 7 solved by MINOS

Q_{ilt}^s	$ntcpa_{nmilt}^s$	$bdfpa_{nmilt}^s$	$trpa_{nmilt}^s$	tpa_{nmilt}^s	Def_{ilt}^s	Co_{ilt}^s	B_{klt}^s	X_{ijlt}^s	Y_{jklt}^s
Q_{111}^1 = 16992	$ntcpa_{11111}^1$ = 1.2261E+5	$bdfpa_{11111}^1$ = 234	$trpa_{11111}^1$ = 4	tpa_{11111}^1 = 1.2284E+5	Def_{111}^1 = 47	Co_{111}^1 = 16945	B_{111}^1 = 0	X_{1111}^1 = 16993	Y_{1111}^1 = 9790
Q_{121}^1 = 16906	$ntcpa_{11121}^1$ = 1.0593E+5	$bdfpa_{11121}^1$ = 298	$trpa_{11121}^1$ = 709	tpa_{11121}^1 = 1.0694E+5	Def_{121}^1 = 315	Co_{121}^1 = 16591	B_{121}^1 = 315	X_{121}^1 = 16906	Y_{1121}^1 = 9689
	$ntcpa_{12111}^1$ = 1.0082E+5	$bdfpa_{12111}^1$ = 2691	$trpa_{12111}^1$ = 0	tpa_{12111}^1 = 1.0351E+5			B_{211}^1 = 47		Y_{1211}^1 = 7203
	$ntcpa_{12121}^1$ = 0	$bdfpa_{12121}^1$ = 3	$trpa_{12121}^1$ = 124	tpa_{12121}^1 = 127			B_{221}^1 = 0		Y_{1221}^1 = 7217
	$ntcpa_{21111}^1$ = 1.1697E+5	$bdfpa_{21111}^1$ = 2928	$trpa_{21111}^1$ = 0	tpa_{21111}^1 = 1.1990E+5					
	$ntcpa_{21121}^1$ = 46859	$bdfpa_{21121}^1$ = 513	$trpa_{21121}^1$ = 718	tpa_{21121}^1 = 48091					
	$ntcpa_{22111}^1$ = 0	$bdfpa_{22111}^1$ = 0	$ntrpa_{22111}^1$ = 0	tpa_{22111}^1 = 0					
	$ntcpa_{22121}^1$ = 1.0041E+5	$bdfpa_{22121}^1$ = 1420	$trpa_{22121}^1$ = 559	$ntpa_{22121}^1$ = 1.0239E+5					
	$ntcpa_{31111}^1$ = 25767	$bdfpa_{31111}^1$ = 446	$trpa_{31111}^1$ = 4	tpa_{31111}^1 = 26209					
	$ntcpa_{31121}^1$ = 1.3329E+5	$bdfpa_{31121}^1$ = 564	$trpa_{31121}^1$ = 752	tpa_{31121}^1 = 1.3461E+5					
	$ntcpa_{32111}^1$ = 1.4367E+5	$bdfpa_{32111}^1$ = 1051	$trpa_{32111}^1$ = 0	tpa_{32111}^1 = 1.4472E+5					
	$ntcpa_{32121}^1$ = 1.0724E+5	$bdfpa_{32121}^1$ = 567	$trpa_{32121}^1$ = 742	tpa_{32121}^1 = 1.0855E+5					
	$ntcpa_{41111}^1$ = 1.2280E+5	$bdfpa_{41111}^1$ = 2009	$trpa_{41111}^1$ = 0	tpa_{41111}^1 = 1.2481E+5					
	$ntcpa_{41121}^1$ = 1.3800E+5	$bddpa_{41121}^1$ = 1313	$trpa_{41121}^1$ = 644	tpa_{41121}^1 = 0.3996E+5					
	$ntcpa_{42111}^1$ = 1.2597E+5	$bdfpa_{42111}^1$ = 3368	$trpa_{42111}^1$ = 704	tpa_{42111}^1 = 1.3004E+5					
	$ntcpa_{42121}^1$ = 1.1093E+5	$bdfpa_{42121}^1$ = 2810	$trpa_{42121}^1$ = 636	tpa_{42121}^1 = 1.1437E+5					

CONCLUSION

In this study, a mathematical model for four-level defective goods supply chain network with imperfect quality of commodity components in uncertain state to maximize profit of supply chain has been presented. The proposed model is adapted to maximize profit by optimizing the costs including production, maintenance, shipping, reworking on the defective goods/parts, shortage in retailers due to the production of defective goods and cost of capital incurred by the companies. The uniqueness of the model is that ability to forecast the active suppliers/manufacturers/distributors and the quantity of parts/goods that must be exchanged between them.

Finally, the model determined the amounts of produced goods and components and the results show the correctness and fine function of the model.

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