

## Research Article

### Influence of the Heredity Kernels on the Prediction of the Life Time of Fiberglass Composites

<sup>1</sup>E.T. Olodo, <sup>2</sup>E.C. Adjovi and <sup>3</sup>F. Niang

<sup>1</sup>Institut Universitaire de Technologie (IUT)/Université d'Abomey-Calavi (UAC), Benin

<sup>2</sup>Ecole Polytechnique d'Abomey-Calavi (EPAC)/Université d'Abomey-Calavi (UAC), Benin

<sup>3</sup>University Institute of Technology - University of Thies, Senegal

**Abstract:** One of the major problems encountered in the prediction of the hereditary viscoelastic behavior of polymeric composites, is the determination of the heredity kernels. This issue comes down to the identification of the model characterizing the viscoelastic properties of these materials. The purpose of this work is to propose a model for prediction of viscoelastic nonlinear behavior of a composite laminate matrix polyester KACT-B by the study and analysis of the heredity kernels and their influence on the life time of this material. The identification of this model required the experimental determination at room temperature, of the viscoelastic parameters of the heredity kernels by a macroscopic approach. These data provide predictive tools for the establishment of the life time under static complex solicitation and yield strength in the long term for this type of material. Sufficiently interesting in terms of concept and methodology, experimental tests in this study, performed on polymeric fiberglass composite, are still few but develop at a pace that we can look in the near future access suitable for purposes of design data.

**Keywords:** Heredity kernels, life time, viscoelastic parameters

## INTRODUCTION

According to the nature of resins in their manufacture, the polymeric composites frequently present linear or nonlinear viscoelastic properties. In particular, for some fiberglass polymeric composites, the behavior becomes nonlinear before reaching strains corresponding to normal use constraints (Schapery, 2000; Zhang and Xiang, 1992; Brinson and Tuttle, 1986). The principle of superposition of Boltzmann being no more applicable in this case, models taking into account of this phenomenon should be used (Pobedria, 2007). On the other hand, factors such as temperature and the history of the constraints will considerably affect the life time under solicitation (Sullivan, 1990; Augl, 1987; Suvorova, 2010).

The question of the determination of the influence functions of nonlinear viscoelastic models of polymeric fiberglass composites is only partially addressed in the technical literature and few models are proposed. These models are unfortunately not robust. One of the major difficulties encountered in the establishment of these models lies in the experimental identification of viscoelastic parameters of the material taking strictly into account the viscoelastic nonlinear behavior thereof. On the other hand, one of the current problems in the

study of the composites is the prediction of their holding in the long term, necessary for the design of structures based on these materials. This theme based approaches are based on models of damage and hereditary creep in which assessment of the long-term behavior models must establish as integral operators (Schapery, 1997; Dimitrienko and Yu, 1989). In these conditions, one of the critical issues is the determination of the type of influence functions or kernels of operators and their parameters. The experimental data on these issues are very few much. The objective of this study is to study and analyze the possible influence functions through the experimental determination of their parameters in order to assess their impact on the life time of fiberglass composites.

## MATERIALS AND METHODS

The object of the study material is an orthotropic laminate glass/polyester composite orthogonal 2D (Fig. 1). Anisotropic nonlinear viscoelastic models are based mostly on approaches damaged functions of the tensors of damage. At the macroscopic scale, these approaches are similar Goldenblat-Kopnov functional anisotropic damage  $\phi(t)$  long term criterion:

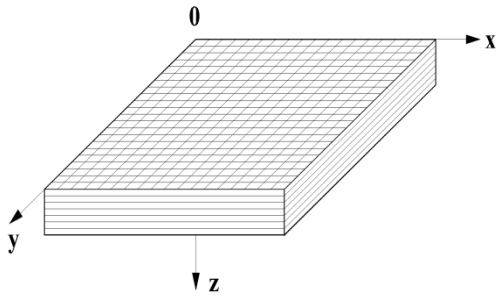


Fig. 1: Laminate glass/polyester composite

$$\Phi(t) = \int_0^t \left[ \int_0^{\zeta} \int_0^{\zeta} \omega_{ikmn}(t-\xi_1, t-\xi_2) \sigma_{ik}(\xi_1) \sigma_{mn}(t-\xi_2) d\xi_1 d\xi_2 + \omega_{ik}(t-\xi) \sigma_{ik}(\xi) d\xi \right] \quad (1)$$

$\frac{1}{2} \leq 1.$

This non-linear model is based on a coupling Mises-Hill plastic potential for anisotropic materials and A.A. Ilyushin kinetic approach,

where,

- t = Is the time after the loading of the material
- ζ = Including time in the interval 0 ≤ ζ ≤ t
- σ<sub>ik</sub>(ζ), σ<sub>mn</sub>(ζ) = Functionals setting program loading of the material; Argument (t-ζ) assumes their invariance in loading
- ω<sub>ik</sub> and ω<sub>ikmn</sub> = The heredity kernels of the model proposed in the present study

The expression (1) assumes that the first full operator including the linear components of the stress tensor is linear pulse input σ<sub>ik</sub>(ξ) dξ and corresponding heredity kernel ω<sub>ik</sub>(t-ξ), in the total value of the damage φ(t).

Similarly, the second term of (1) is the joint influence of the two previous pulses σ<sub>ik</sub>(ξ<sub>1</sub>) dξ<sub>1</sub>, σ<sub>mn</sub>(ξ<sub>2</sub>) dξ<sub>2</sub> and the kernel ω<sub>ikmn</sub>(t-ξ<sub>1</sub>, t-ξ<sub>2</sub>) on the total value of the damage.

The third approximation, in principle, is operated in a similar manner on three previous time intervals and so on. Therefore the global supply of pulses in the functional φ(t) would be the sum of the input linear, quadratic, cubic, etc.

The expression (1) is limited to linear and quadratic terms because this model assumes that tensor heredity kernel should respectively be the second and fourth order that preserves the invariance of the model to the transformation groups.

Accordingly, the heredity kernels must meet the following conditions:

$$\omega_{ik}(t, \xi) = \omega_{ik}(t - \xi) \quad (2)$$

$$\omega_{ikmn}(t, \xi_1, \xi_2) = \omega_{ikmn}(t - \xi_1, t - \xi_2) \frac{\partial \omega_{ik}(t, \xi)}{\partial \xi} > 0 \quad (3)$$

$$\frac{\partial^2 \omega_{ikmn}(t, \xi_1, \xi_2)}{\partial \xi_1 \partial \xi_2} > 0$$

The first condition means that the model must keep its invariance to the origin of the time.

The second condition means that with the increasing of ζ, the influence of the stresses occurring in the final moments also increases on the rate of damage to the material. That is why the determination of the heredity kernels is critical to the assessment of long term of the material held.

The shape of the heredity kernel of the model must be specific to each material. Especially for the polymeric composites reinforced fiberglass a functional relationship must be established with the static strength tensor (Pobedria, 1983) to be of the form:

$$\begin{aligned} \omega_{ik}(t - \xi) &= \Pi_{ik} f_1(t - \xi) \\ \omega_{ikmn}(t - \xi_1, t - \xi_2) &= \Pi_{ikmn} f_2(t - \xi_1, t - \xi_2) \end{aligned} \quad (4)$$

Π<sub>ik</sub> and Π<sub>ikmn</sub> are static resistance respectively the second and fourth order tensors; the functions f<sub>1</sub> and f<sub>2</sub> must be determined by tests of resistance in the long term.

For the composite laminate object of the study, we will consider the Abelian kernel:

$$\begin{aligned} \omega_{ik}(t - \xi) &= \frac{a(1 - \beta) \Pi_{ik}}{(t - \xi)^\beta} \\ \omega_{ikmn}(t - \xi_1, t - \xi_2) &= \frac{a^2(1 - \beta)^2 \Pi_{ikmn}}{(t - \xi_1)^\beta (t - \xi_2)^\beta} \end{aligned} \quad (5)$$

where, β and a are parameters to be determined by uniaxial testing in the long term;

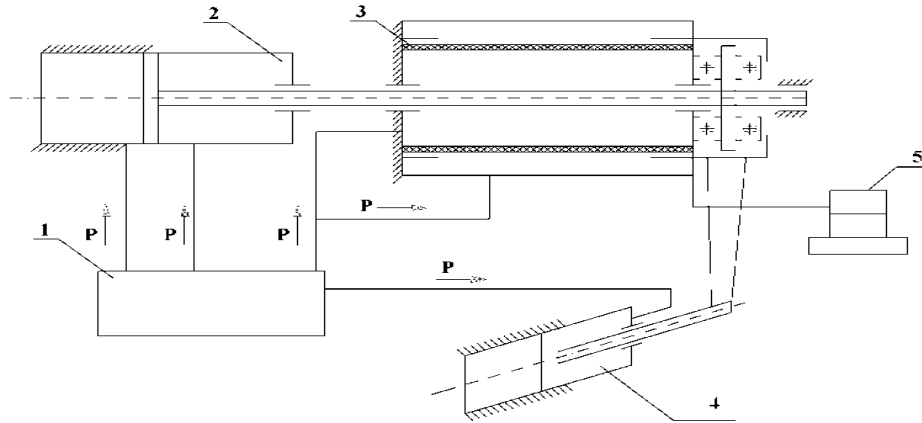
- a = Expressed in [time]<sup>β</sup>
- β = Without unit

We will adopt a loading of the form:

$$\sigma_{ik}(\xi) = \sigma_{ik(0)} \cdot \xi^r$$

For a static loading . r = 0, so:

$$\begin{aligned} \sigma_{ik}(\xi) &= \sigma_{ik(0)} \\ \sigma_{mn}(\xi) &= \sigma_{mn(0)} \end{aligned}$$



Legend: 1, 2, 4-loading system of the test piece; 3-test tube; 5 -electronic lock and data logger

Fig. 2: Experimental long term testing device on glass/polyester composite

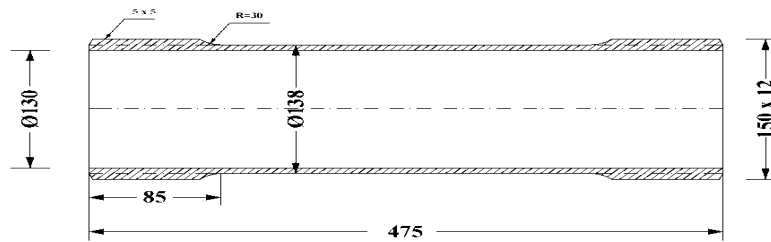


Fig. 3: Tubular tube in glass/polyester composite

Expression (1) then becomes:

$$\Phi(t) = \int_0^t \frac{a(1-\beta)\prod_{ik}\sigma_{ik(0)}}{(t-\xi)^\beta} d\xi + \left[ \int_0^t \int_0^t \frac{a^2(1-\beta)^2 \prod_{ikmn}\sigma_{ik(0)}\sigma_{mn(0)}}{(t-\xi_1)^\beta(t-\xi_2)^\beta} d\xi_1 d\xi_2 \right]^{0.5} \leq 1 \quad (6)$$

or

$$a(\prod_{ik}\sigma_{ik(0)} + \sqrt{\prod_{ikmn}\sigma_{ik(0)}\sigma_{mn(0)}})t^{1-\beta} \leq 1 \quad (7)$$

The limit state,  $t$  becomes the time of failure  $t_*$  of the material, so:

$$t_* = \left[ \frac{a(\prod_{ik}\sigma_{ik(0)} + \sqrt{\prod_{ikmn}\sigma_{ik(0)}\sigma_{mn(0)}})}{\sigma_b} \right]^{\frac{1}{\beta-1}} \quad (8)$$

For a uniaxial sollicitation, the expression (8) will be:

$$t_* = \left( \frac{a\sigma_{bi*}}{\sigma_b} \right)^{\frac{1}{\beta-1}} \quad (9)$$

and the limit of long term strength of the material as an expression:

$$\sigma_{bi*} = \frac{\sigma_b}{a t_*^{1-\beta}} \quad (10)$$

$\sigma_b$  is the limit of resistance of the material in uniaxial sollicitation.

The expression (8) represents the life time of the material under static sollicitation.

In this expression the part in parentheses is known, the determination of the parameters  $a$  and  $\beta$  of the heredity kernels is necessary for the prediction of the life time for this type of loading.

## RESULTS AND DISCUSSION

**Description of the test:** The parameters  $a$  and  $\beta$  are determined by long term strength in uniaxial compression tests, using an experimental device used for testing of deformation. This device is equipped with an electronic clock to set the time corresponding to a given level of stress including the time of rupture of the test piece (Fig. 2). The tests are performed on tubular composite tubes with fiberglass reinforced polyester matrix (Fig. 3): reinforcements meaning string are arranged along the longitudinal axis while reinforcements meaning frame are arranged on the circular side of the tube, all drowned in a polymer polyester matrix.

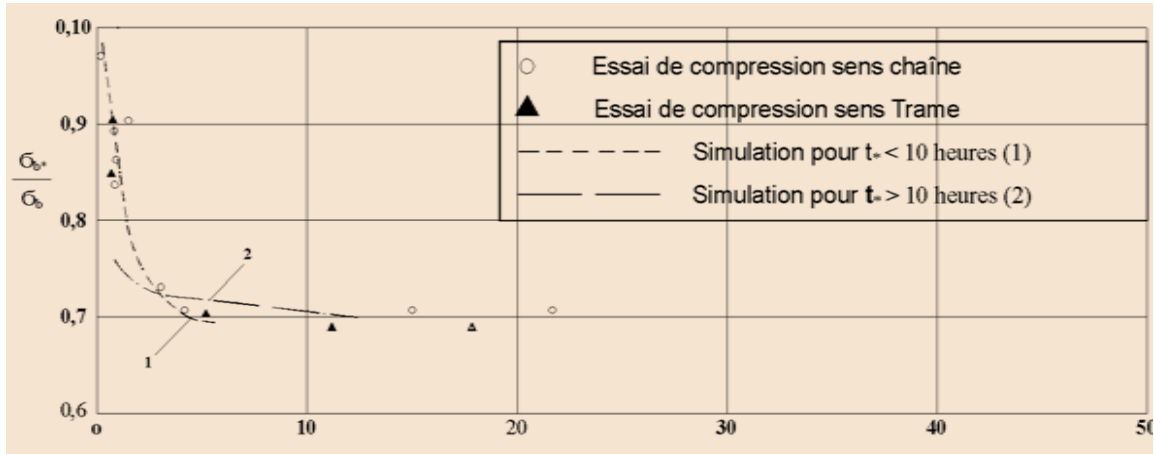


Fig. 4: Experimental points and Abelian kernel approximation curves

Table 1: Values of parameters  $a$  and  $\beta$

Break time inter vals	* $t < 10$ h		* $t > 10$ h	
	$\beta$	$a, (h^{\beta-1})$	$\beta$	$a, (h^{\beta-1})$
Values	0.63	1.02	0.98	1.36

**Determination of the parameters  $\beta$  and  $a$ :** From the value limit of resistance  $\sigma_b$  of the material (the value determined by the breakdown of the material testing), it determines the loading values for the test in the long term.

In our case, these values were taken respectively  $0.9 \sigma_b$ ;  $0.8 \sigma_b$ ;  $0.7 \sigma_b$  and  $0.6 \sigma_b$ . For the object of the study material.  $\sigma_b = 98$  MPa.

The expression (9) requires that testing should occur to both temporal intervals:

- A first campaign of tests for a range of small life-spans \* $t < 10$  h
- A second campaign of tests performed for a domain of major life-spans \* $t > 10$  h.

For these tests, we use the constraint standardized report  $\sigma_{bi} / \sigma_b$ :

The numerical values of the parameters  $a$  and  $\beta$  determined using experimental points in Fig. 4 and expression (9) are given in Table 1.

Figure 4 shows the results of tests including the experimental points and curves approximation of resistance in the long term for the two areas of longevity.

**Explicit forms of heredity kernels and determination of the life time and yield long term strength:** The values of the thus determined parameters provide explicit forms of heredity kernels.

Thus, for the areas of major longevities we have:

$$\omega_{ik}(t - \xi) = \frac{0.0272 \Pi_{ik}}{(t - \xi)^\beta} \tag{11}$$

$$\omega_{ikmn}(t - \xi_1, t - \xi_2) = \frac{7 \cdot 10^{-4} \Pi_{ikmn}}{(t - \xi_1)^\beta (t - \xi_2)^\beta}$$

and in this case, the life time will be:

$$t_* = \left[ 1.36 (\Pi_{ik} \sigma_{ik(0)} + \sqrt{\Pi_{ikmn} \sigma_{ik(0)} \sigma_{mn(0)}}) \right]^{-50} \tag{12}$$

where, the long-term constraint limit:

$$\sigma_{bi*} = \frac{\sigma_b}{1.36 t_*^{0.02}} \tag{13}$$

In the case of small longevities, we have:

$$\omega_{ik}(t - \xi) = \frac{0.444 \Pi_{ik}}{(t - \xi)^\beta} \tag{14}$$

$$\omega_{ikmn}(t - \xi_1, t - \xi_2) = \frac{0.1971 \Pi_{ikmn}}{(t - \xi_1)^\beta (t - \xi_2)^\beta}$$

And

$$t_* = \left[ 1.2 (\Pi_{ik} \sigma_{ik(0)} + \sqrt{\Pi_{ikmn} \sigma_{ik(0)} \sigma_{mn(0)}}) \right]^{-2.7} \tag{15}$$

The long term constraint limit will be:

$$\sigma_{bi*} = \frac{\sigma_b}{1.2 t_*^{0.37}} \tag{16}$$

Curve (1) obtained in the field of small longevities gives a better approximation than the curve (2), showing that the Abelian kernel which is a power law gives more satisfactory results for small periods of loading. However many works related to long term tests on polymeric composites show that in the case of long-term loading tests, there is an exponential relationship between the long-term strength and the duration of the loading of the material. Therefore the heredity kernel of the model must have an exponential form in the case of large longevities.

Indeed, consider the exponential kernels of the form:

$$\begin{aligned} \omega_{ik} &= b \prod_{ik} e^{-\alpha(t-\xi)} \\ \omega_{ikmn} &= b^2 \prod_{ikmn} e^{-\alpha(2t-\xi_1-\xi_2)} \end{aligned} \quad (17)$$

By integrating them in (1) with the same law of loading, we would obtain:

$$t_* = -\frac{1}{\alpha} \ln\left(1 - \frac{\alpha}{b} \frac{\sigma_b}{\sigma_{b_*}}\right) \quad (18)$$

This is the expression of the life time of the material using an exponential kernel. The long term constraint limit would be:

$$\sigma_{b_*} = \frac{\alpha \sigma_{b_i}}{b(1 - e^{-\alpha t_*})} \quad (19)$$

This last expression shows that, when  $t \rightarrow \infty$ ,  $\sigma_{b_i^*}$  tends towards a limit value non-zero. This is a considerable advantage because it means that one can make predictions of the behavior of the material for large domains of longevity. On the other hand expression (10) shows that when  $t \rightarrow \infty$ ,  $\sigma_{b_i^*}$  tends towards zero. One may infer that the Abelian type kernels are better suited for small longevity predictions as shown Fig. 4 experimental results.

The values of the parameters depend on the fields of longevity of the material. Asymptotic approximation curves boundaries define the values limits of the constraints to be taken into account for each type of loading.

On the other hand, the time needed for the rupture usually appears on the stress-rupture digraphs whose use is obvious. If one knows the stress, one can read the life time. If we design for a certain period of life, we can read the constraint not to exceed while sizing.

Failure of many glass/polyester composites can intervene earlier than expected and for low levels of stress. Therefore when it is a design of structures based on these materials, we must be sure that:

- Yield stress in the long term for the life in the design is acceptable.
- The time of breach loading provided by the design exceeds the duration of intended use, so the material life.

The results of this study show that for the prediction of the life of the polymeric fiberglass composite particular importance should be given on the establishment of the heredity kernels of the predictive

model. This requires a correct experimental identification of the parameters of the kernels taking into account the expected operating life.

## CONCLUSION

In this study we can retain:

- We present the terms of tests with a long-term behavior that generates data required for the calculations of design (engineering) and life prediction, information unavailable in the technical literature.
- It is preceded to the study and analysis of the heredity kernels. We have shown that for the purpose of the study material, these kernels are the product of static tensors of different orders by only the time-dependent functions.
- A nonlinear viscoelastic model is identified by the experimental determination of the viscoelastic parameters allowing establishing explicit forms of heredity kernels.
- It is highlighted the influence of these parameters therefore the heredity kernels on the prediction of the composite life. In particular, we have shown that Abelian kernels respond well to predictions from small periods of loading tests while for long loading times tests, exponential kernels are better suited for the prediction of the long term behavior of the material.
- It is proposed formulas for the determination of the life time of the polymeric composites, obtained for Abelian and exponential kernels.
- The expressions for the calculation of the limit of resistance in the long term of this material are obtained.

This proposed prediction method gives the possibility to establish the behavior of the material for long periods of operation and may be useful in engineering calculation for the assessment of the resistance in the long term of these materials.

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