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Research Article

A Damage Diagnosis Method for Bridge Damage Based on Improved R-ARX Model

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Abstract: In this study, we focused mainly on establishing a new sensitive method to identify location of damages in Service Bridge. The improved method of combination of Autoregressive models (AR) and autoregressive models with exogenous inputs (ARX) was applied to identify the damage in Service Bridge. Firstly, an improved AR-ARX model was established for damage identification of bridge on service. Secondly, the detailed steps of identifying location of damages are explained using proposed damage diagnosis method. Finally, the improved AR-ARX model was used to identify location of damage in a two-span steel beam. The results of calculated showed the damage diagnosis method based on improved AR-ARX model that considered the ambient excitation is much more powerful and accurate than AR-ARX method. So the proposed method can apply to bridge damage detection with few acceleration sensors. This study laid the foundation for the bridge identification.

Keywords: Ambient excitation, AR-ARX model, damage diagnosis method, improved, service bridge

INTRODUCTION

In the search for more sensitive damage features that can capture more information from the measured vibration time history responses of structures, researchers have proposed using different types of time series models. Time series models rely on the fact that the value of the measured response of the structure at any time can be predicted based on a linear combination of its values at a previous time and some random errors. The analogy behind using time series models for vibration-based damage identification of structures lies in the fact that if a time series model is fitted to the vibration responses of the bridge, the obtained coefficients and properties of the model can capture the dynamic characteristics of the structure. Then, any deviation from the obtained model can be a sign of change or damage in the structure; however, different researchers have extracted different types of damage features based on time series models in order to capture the deviations in the time series models.

Sohn *et al*. (2001a, b) used a combination of AR-ARX models to extract damage features for damage detection in a patrol boat. Sohn *et al*. (2001a, b) used a similar approach to identify damages in an 8 DOF mass-spring system. Fasel *et al*. (2002) used a combination of Autoregressive models (AR) and Autoregressive models with exogenous inputs (ARX) in order to extract damage features. Sohn *et al*. (2005) showed the robustness of EVS in detecting a nonlinear damage introduced into a linear system through a study for damage detection in an 8 DOF spring-mass structural system. Omenzetter and Brownjohn (2006) used both vector and univariate seasonal Autoregressive Integrate Moving Average (ARIMA) model for damage identifications in a bridge. Overbey *et al*. (2007) used a state-space-based prediction error technique for damage detection in an aluminum frame with bolted joints. Haroon and Adams (2007) used ARX models to identify damage in mechanical systems by capturing the nonlinear nature of the damages. A very similar approach has been used by Zhang (2007) to identify the location of damage in a numerically simulated three span beam. The beam was defined as a combination of frame elements and zero-length elements to connect frame elements together and the damages were introduced by reducing the stiffness at the zero length node elements. Carden and Brownjohn (2007) used Autoregressive Moving Average (ARMA) models for structural health monitoring of several different structures. Gul and Catbas (2009) used the deviations in the coefficients of AR models for damage identification on two different laboratory scale structures. Next they used Mahalanobis distances to statistically measure the deviation of the damage features from the undamaged condition of the structure.

There is considerable amount of research conducted to identify locations of damage based on vibration-base techniques. Almost all the reviewed

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vibration-based techniques use either simple modal characteristics such as modal frequencies or modal shapes, some modally related indices such as flexibility matrix or some sort of model parameter updating. All these methods go through a process of data condensation. This means that the measured vibration data on the structure is used to derive a condensed set of information like natural frequencies, mode shapes or mode shape curvatures; however, a useful portion of the data could be lost during the data condensation process. The other problem of these methods is that usually a limited number of vibration modes are used in the reviewed methods in order to identify damages in the structures.

In the search for more sensitive damage features that can capture more information from the measured vibration time history responses of structures, researchers have proposed using time series models. This research focuses mainly on developing a new damage feature to identify location of damages. The research starts with modifying a damage diagnosis method which was previously developed in the literature based on combination of AR-ARX time series models for identifying existence of damage and uses it to identify location of damage in a two-span steel beam.

IMPROVED AR-ARX MODELS

Autoregressive (AR) and Autoregressive Moving Average (ARMA) models: AR models are actually regression models that can predict the present observations in the structural response as sum of two uncorrelated parts, one is dependent on the previous observations in the structural response and the other one can be a series of uncorrelated sequences. The simplest such models are:

$$X_t = \varphi X_{t-1} + e_t \tag{1}$$

where,

X_t = Observation in the structural response at time t

φ = Model constant

e_t = A sequence of uncorrelated variables or the prediction error of the model

The other simple time series model is when an observation can be related as sum of a series of uncorrelated sequences. The simplest such model can be represented as:

$$X_t = \theta e_{t-1} + e_t \tag{2}$$

where,

θ = Model constant

This model is called Moving Average (MA) model. In order to model responses of a complex dynamical system, higher order Autoregressive moving average, ARMA (p, q), models can be used:

$$X_t = \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + e_t \tag{3}$$

where,

X_t = Observation in the structural response at time t

φ_i = ith predictor coefficients for the Autoregressive part of the model

θ_j = jth predictor coefficients for the Moving Average part of the model

e_t = Residual error of the model prediction at time t

P,q = Orders of Autoregressive and Moving average terms of the model, respectively

AR and MA model can be used to represent the same dynamical model interchangeably. X_t Can be a stationary response of the structure only if φ_1<1. Therefore, if the process has started at infinite past, the above equation can be written as:

$$X_t = \frac{\varphi_0}{1-\varphi_1} + \sum \varphi_1^j e_{t-j} \tag{4}$$

If the number of modes excited in the dynamic response of the structure is n/2, then φ_1 can be written in term of natural frequencies of the structure:

$$\varphi_1 = (-1)^{m+1} \sum_{i_1, i_2 \cdots i_m = 1}^{n} \lambda_{i_1} \lambda_{i_2} \lambda_{i_3} \cdots \lambda_{i_m} \tag{5}$$

where,

λ_i = i^{th} pole of the system

Each pair of poles is related to the natural frequency, ω_j and damping ratio, ζ_j, of a mode through the following equation, where $i = 2j$:

$$\lambda_i, \lambda_{i-1} = e^{\Delta(-\zeta_j \omega_j \pm \omega_j \sqrt{\zeta_j^2 - 1})} \tag{6}$$

where,

λ_{i-1} = Complex conjugate of λ_i

Δ = The sampling time interval

Therefore, if only modal properties of the structure are required, the vibration response can be modeled by just the AR part of the model. As a result, using AR models instead of ARMA models has the advantage of less trial and error in estimating the model parameters. Another advantage of using AR models rather than ARMA models is the fact that a linear least-square method can be used for estimating AR parameters while a nonlinear least-square technique is required for estimating parameters of an ARMA model.

The method that can be used to identify the proper orders for the times series models are the autocorrelation and partial autocorrelation functions. Autocorrelation function can be defined as:

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \frac{Cov(X_t, X_{t+h})}{\sqrt{Var(X_t)}\sqrt{Var(X_{t+h})}} \quad (7)$$

where,

$\gamma(h)$ = Auto covariance function of an observation with itself in a time shift h

$Cov(X_t, X_{t+h})$ = Covariance of an observation with itself in a time shift h

$Var(X_t)$ = Variance of the observation

Partial autocorrelation function, $\Phi(h)$, is also defined as:

$$\Phi(h) = \frac{Cov(X_t - \hat{X}_t, X_{t+h} - \hat{X}_{t+h})}{\sqrt{Var(X_t - \hat{X}_t)}\sqrt{Var(X_{t+h} - \hat{X}_{t+h})}} \quad (8)$$

where,

$\hat{X}_t$ = The best linear unbiased predictor of X_t.

Autoregressive with exogenous (ARX) inputs models: Autoregressive with exogenous (ARX) model can be represented as the following:

$$X_t = \sum_{i=1}^{a} \alpha_i X_{t-i} + \sum_{j=1}^{b} \beta_j e_{t-j} + \varepsilon_t \quad (9)$$

where,

α_i = Predictor coefficients for the AR part of the model at time lag *i*

β_j = Predictor coefficients for the exogenous part of the model at time lag *j*

ε_t = Residual error of the model prediction at time *t*

e_{t-j} = External input to the model at time *t-j*; α,b Orders of Autoregressive and exogenous terms of the model, respectively

It can be shown that the ARX model is a linear approximation of an ARMA model for a stationary time series. So it is suggested that orders of the AR part and exogenous part in the ARX model are chosen so that:

$$a + b \leq p \quad (10)$$

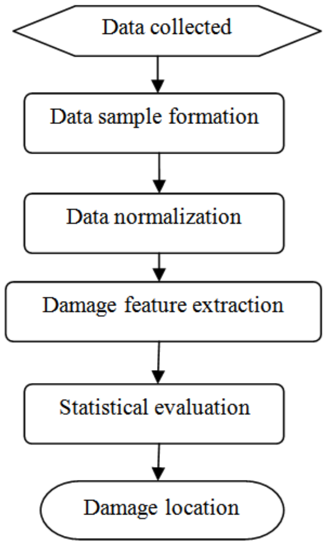

Fig. 1: Steps of damage diagnosis approach

where,

a = Order of the AR part of the ARX model

b = Order of the exogenous part of the ARX model

p = Order of the AR model in Eq. (3)

Estimating parameters of an ARX model can be treated as a linear regression problem using least-square techniques. A model will be regressed on the observation values at previous times and some external inputs, which can be taken from the error term in the AR model, to predict the value of the observation at a present time.

DAMAGE DIAGNOSIS METHOD

These damage diagnosis approaches based on AR-ARX models are proposed in four steps are shown in Fig. 1:

- Data sample formation
- Data normalization
- Damage feature extraction
- Statistical evaluation

During these four steps, the vibration responses of the structure obtained at its healthy and damage conditions will be indirectly compared to each other by extracting sensitive damage features. The damage features will be obtained by application of AR and ARX models to the measured vibration response of the structure and probability values will be used to statistically measure the amount of variations in the extracted damage features from healthy to the damage condition of the structure.

IMPLEMENTING THE DAMAGE DIAGNOSIS METHODS

One 330 s acceleration time histories measured at different damage conditions of the beam were used to evaluate the ability of the proposed damage diagnosis method to identify damage locations on the beam. The geometry of the beam is shown as Fig. 2. The vibration responses measured at different damage conditions of the beam were indirectly compared to the vibration responses measured at its healthy condition. All the vibration responses used for this comparison were measured under the same random loading excitations. The vibration responses of the beam were used to extract damage features and to evaluate the probability of occurring damage at different sensor locations along the length of the beam.

Generation of these smaller data samples in the Reference, Healthy and Damage data sets are shown in Fig. 3 and 4. One of the reasons for segmenting the vibration response of the beam to smaller data samples was to form data samples which represent the response of the beam under varying loading conditions. A better match can be found for the data samples in the Healthy and Damaged data sets in the pool of Reference data samples that represent similar operational conditions of the beam.

The average correlation values calculated for between the AR model coefficients of the Healthy and Damage data samples and the AR model coefficients of the matched Reference data samples are presented in Table 1. The matched data samples in different data sets can potentially represent the responses of the beam that are obtained under similar loading conditions.

The average values of these calculated damage features are presented in Table 2 for different sensor locations at different conditions of the beam. As can be seen, the average damage features for different sensors in the healthy condition beam are closer to 1, as expected. As the damage grows, the average damage features grow. The larger the damage features are, the greater the likelihood is that damage is located at that sensor location.

Probability densities of the transformed damage features are plotted in Fig. 5 for different sensors at damage conditions D11 to D15. Therefore, although the existence of damages can be detected by observing the deviations in the probability density plots of the damage features with respect to the healthy condition of the beam, it is difficult to quantify the extent of damages by measuring the amount of deviations. In order to identify the damage location, the amount of deviation in the probability density plots should be compared between different sensors.

Fig. 2: Geometry of the beam

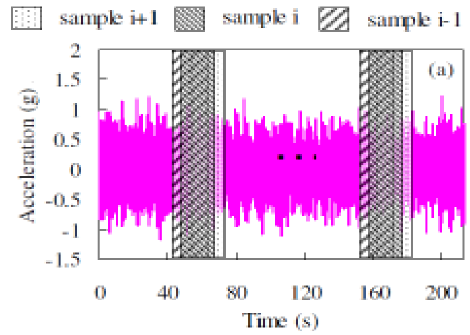

Fig. 3: Smaller data samples generated in reference data sets

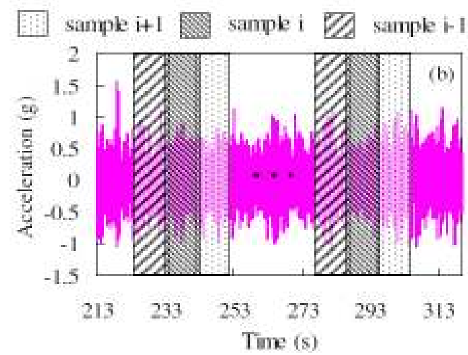

Fig. 4: Smaller data samples generated in healthy and damage data sets

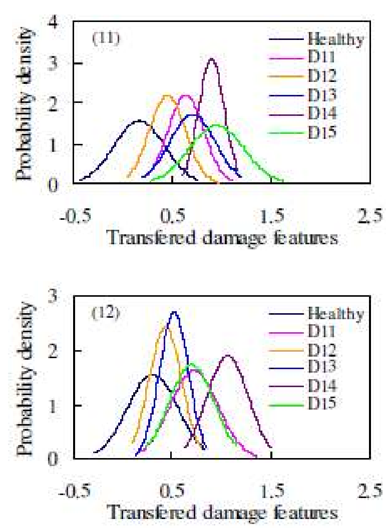

Fig. 5: Probability density plots of the transferred damage features for sensors1 1 to 15 for different damage conditions at the damage location

Table 1: Average correlation values between the coefficients of the AR models fitted to the matched data samples in the healthy and damage data sets and the reference data sets

Sensor	Healthy	D11	D12	D13	D14	D15	D21	D22	D23	D24
1	0.993	0.952	0.967	0.952	0.937	0.944	0.946	0.95	0.927	0.958
2	0.992	0.914	0.962	0.916	0.901	0.903	0.901	0.903	0.921	0.936
3	0.99	0.959	0.965	0.958	0.908	0.915	0.917	0.921	0.946	0.974
4	0.991	0.951	0.976	0.952	0.944	0.938	0.938	0.947	0.937	0.973
5	0.991	0.967	0.964	0.968	0.931	0.928	0.929	0.925	0.942	0.983
6	0.995	0.975	0.967	0.975	0.967	0.954	0.954	0.965	0.954	0.987
7	0.995	0.985	0.971	0.986	0.956	0.943	0.942	0.946	0.948	0.985
8	0.992	0.965	0.965	0.965	0.951	0.937	0.939	0.944	0.941	0.976
9	0.992	0.972	0.973	0.976	0.957	0.911	0.916	0.953	0.927	0.977
10	0.985	0.966	0.97	0.967	0.938	0.926	0.928	0.939	0.925	0.956
11	0.982	0.944	0.968	0.945	0.908	0.897	0.899	0.914	0.913	0.968
12	0.987	0.971	0.977	0.973	0.924	0.928	0.928	0.921	0.905	0.946
13	0.981	0.958	0.969	0.958	0.945	0.925	0.925	0.934	0.923	0.953
14	0.986	0.976	0.977	0.976	0.935	0.928	0.927	0.922	0.918	0.965
15	0.984	0.969	0.976	0.969	0.938	0.939	0.941	0.944	0.938	0.953

Table 2: Average damage features extracted from different sensors at different conditions of the beam

Sensor	Healthy	D11	D12	D13	D14	D15	D21	D22	D23	D24
1	1.23	1.46	1.53	1.76	1.94	1.17	1.49	2.21	2.36	1.78
2	1.19	1.35	1.45	2.84	2.75	1.94	2.31	3.24	3.76	2.08
3	1.43	2.17	1.76	2.38	3.37	2.15	2.95	4.37	4.88	3.48
4	1.21	1.98	1.54	1.98	3.90	2.06	3.15	4.82	5.94	3.68
5	1.23	5.55	4.06	5.56	8.05	6.49	7.12	10.88	13.66	9.09
6	1.36	1.83	1.55	1.71	2.34	2.61	3.55	4,16	4.95	2.88
7	1.21	2.03	1.68	1.75	2.65	2.43	2.89	5.53	4.70	2.96
8	1.21	2.04	1.53	1.69	2.43	1.86	2.16	4,17	3.56	2.57
9	1.21	2.59	1.79	2.55	4.37	2.26	2.99	6.27	6.75	4.33
10	1.23	1.81	1.46	1.64	4.51	2.69	4.03	6.66	8.15	5.54
11	1.19	2.23	1.86	2.13	4.55	3.17	4.64	8.41	9.75	7.76
12	1.47	2.65	1.98	2.26	4.49	3.69	5.73	7.68	8.36	5.76
13	1.18	1.79	1.43	1.89	2.88	1.65	2.09	3.88	4.32	3.43
14	1.16	1.73	1.54	1.64	3.30	2.97	3.15	6.55	4.27	3.25
15	1.22	2.25	1.57	1.96	4.45	2.05	3.16	5.64	6.77	4.37

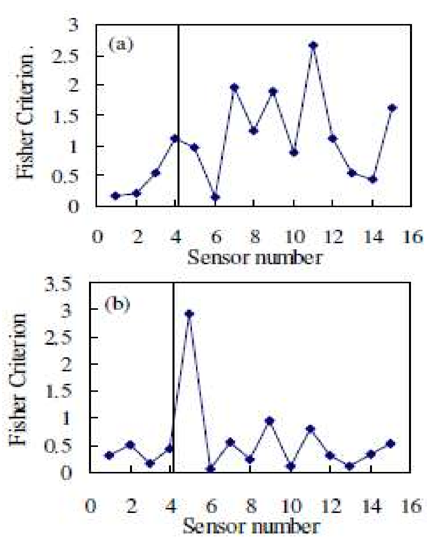

Fig. 6: Fisher criterion calculated for different sensors at (a) D11, (b) D12 damage conditions

The sensor which corresponds to the highest value of the calculated Fisher criterion is potentially a sensor located close to the physical damage location. The calculated Fisher criterion values in Fig. 6 are related to the conditions where damage was only induced at the damage location.

CONCLUSION

This research focuses mainly on developing a new damage feature to identify location of damages. The research established a damage diagnosis method based on combination of AR-ARX time series models for identifying existence of damage and uses it to identify location of damage in a two-span steel beam. The results of experiment proposed damage diagnosis method is able to locate the general damage region in bridge girders.

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