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Research Article

Lyapunov Theory-Based Robust Control

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Abstract: In modern science technology and many motion process of engineering fields, such as neural network activity, movement of missile and spacecraft, control of robot and so on, there are many phenomena which are suddenly changed when their movement is disturbed in some time. The mathematical model of the instantaneous phenomena which we referred is impulsive differential equation. More and more control experts and mathematicians pay close attention to impulsive system, because the study of impulsive has the wide actual background and the value of the application. The robust control problems of the uncertain linear impulsive delay system are studied. Based on the stability of the impulsive delay system, we take advantage of the stability theory of Lyapunov, Lyapunov function and the technology of linear matrix inequality to design a robust feedback controller in order to eliminate the impact of pulse and time delay on the stability of the system. The controller can be got by the way of solving linear matrix inequality by using MATLAB toolbox. And some sufficient conditions of the robust exponential stability are proposed so that the system contains robust feedback control.

Keywords: Exponential stability, linear matrix inequality, Lyapunov function, robust feedback controller, robust stability

INTRODUCTION

The robust problem is a widespread problem in the control system. In the control system, the feedback control is the most basic control (Guan *et al*., 2001; Stamova and Stamov, 2001). The principle of the feedback control is that the control device is applied to the controlling role in the controlled object, which is taken from the charged amount of feedback information used to continually correct the deviation between the amount charged and the input in order to achieve the controlled object task (Silva and Pereira, 2002; Akhmet, 2003; Dashkovskyiy *et al*., 2012; Chen and Zheng, 2011).Usually, the added feedback control system has two types (Chen and Xu, 2012):one is used as the input and the other is the disturbance. The useful input determines the variation of the system charged amount, which will disturb the stability of the system.

The stable analysis and design based on the pulse control system of Lapunov functions have more results in theory and it has been widely used in the control of the chaotic system and the secure communication system. Using synchronous error feedback approach constructors the suitable Lyapunov function for the synchronous analysis of the chaotic systems to give out a sufficient condition for a class of delay chaotic system pulse synchronous exponential stability (Chen and Xu, 2012; Cheng *et al*., 2012; Yang and Xu, 2007a). Using the Lyapunov function set up several pulse stabilization criteria applied to the Lorenz system after the pulse feedback control and the controlled Lorenz system solution converges to an equilibrium point (Yang and Xu, 2007b; Liu, 2004; Liu and Hill, 2007; Liu *et al*., 2007). The robust stability of the Hopfield impulsive neural networks was studied based on Lyapunov functions to get the sufficient conditions of the robust stability and robust asymptotic stability of the pulse Hopfield neural network, on the basis, design the implement pulse controller to calm Hopfield neural network.

This study is mainly based on the stability theory of Lyapunov to select appropriate Lyaounov function combined with linear matrix inequalities and design the control system of the robust feedback control in order to eliminate the impact of the time delay and the pulse of the system, which makes the system to achieve robust exponential stability.

LITERATURE REVIEW

The stability is the important characteristics of the system and it is a necessary condition of the system to work properly and it describes the initial condition whether the system equations have convergence. In the classical control theory, algebraic criterion, Nyquist criterion, logarithmic criterion, root locus criterion is established to determine the stability of linear time-invariant systems but does not apply to non-linear time-varying systems. In the analysis of the stability of certain nonlinear systems, Lyapunov theory effectively

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solves the problem which cannot be solved by other methods. Lyapunov theory on the establishment of a series based on the concept of stability are two ways to determine the stability of the system (Liu and Hill, 2007a): one method is the use of a linear system differential equations to determine the stability of the system called the first method or the indirect method of Lyapunov; another method is to take advantage of the experience and skills to construct Lyapunov functions and then use the Lyapunov functions to determine the stability of the system known as the second method or the direct method of Lyapunov. Due to the indirect method require the solution of a linear system differential equations, solving the system differential equations is not an easy task, so the indirect method is greatly restricted the application. The direct method does not require the solution of the system differential equations, which determines the stability of the system brought great convenience and access to a wide range of applications and in the various branches of modern control theory, optimal control, adaptive control, non-linear system control and it can continue to get the application and development (Wu *et al*., 2010).
The stability of Lyapunov sense:
Set system equation:

$$\bar{x} = f(x,t) \quad (1)$$

where,

x : An n-dimensional state vector and explicit time variable t
f (x, F) : A linear or non-linear

The n-dimensional function of the steady or the time-varying and assume that the equation is x (t, x_0, t_0), x_0 t_0 respectively are the initial state vector and the initial moment and the initial condition x_0 meets x (t, x_0, t_0) = x_0.

Balanced state: Lyapunov stability for a state of equilibrium, for all t satisfying:

$$\bar{x}_e = f(x_e,t) = 0$$

State x_e is referred to as a state of equilibrium.

Stability of Lyapunov significance: Set the initial state of the system located in the related state and x_e is the center of the sphere radius δ of the domain of the closed ball S (δ), i.e., $\|x_0-xe\| \le \delta$, t = t_0.

If the system equation x (t, x_0, t_0) is in t→∞ process, which is located in the center of the sphere x_e, arbitrarily closed sphere of radius ε in the domain S (ε), i.e.: $\| x(t; x_0, t_0) - x_e \| \le \varepsilon$, $t \ge t_0$.

Then balanced state x_e of the system is stable in Lyapunov. If the equilibrium state of the system is not only Lyapunov stability and have:

$$\lim_{t\to\infty} \| x(t; x_0, t_0) - x_e \| = 0$$

This state of equilibrium is called asymptotically stable. When this equilibrium is asymptotically stable large-scale initial conditions extended to the entire state space and a state of equilibrium has the asymptotic stability. In this case, δ→∞, S (δS→∞). When t→∞, the trajectories starting from any point in the state space to converge to x_e.

In this study, using the second method of Lyapunov method for researching the system is discussed. The second method of Lyapunov does not require solving the differential equations of the system to determine the stability of the system brought great convenience. The positive definiteness, the semi-positive definiteness, the negative definiteness and the semi-negative definiteness of the functions are defined.

Positive definiteness: The scalar function V(x) has V(x) >0 and V (0) = 0 for all non-zero status x in the domain S and the scalar function V (x) is called positive definiteness.

Negative definiteness: If V (x) is a positive-definite function, the scalar function V(x) is called negative-definite function.

Semi-positive definiteness: If the scalar function V (x) is positive definiteness in all states of the domain S in addition to the origin and some state zero and V (x) is called semi-positive definite function.

Semi-negative definiteness: If V (x) is semi-positive definite function, the scalar function V (x) is called the semi-negative definite function.

The following stability determinant theorem can be gotten by Lyapunov function:

Theorem 1: For continuous-time nonlinear time-varying autonomous system (1), if there is a continuous first-order partial derivatives of the scalar function V (x), V (0) = 0 and all nonzero-state point x in the entire state space X satisfies the following conditions:

- V (x) is Positive definiteness
- $\bar{V}(x)$ is Negative definiteness
- When ||x||→∞, there is V (x, t) →∞

The origin equilibrium state x = 0 of the system is asymptotically stable.

Theorem 2: For continuous-time nonlinear time-varying autonomous system (1), if there is a consecutive order partial derivative of the scalar function V (x), V (0) = 0 and for all non-zero status in the entire state space X the point x satisfies the following conditions:

- V (x) is Positive definiteness
- $\bar{V}(x)$ is Negative definiteness
- Any nonzero x ∈ X, $\bar{V}$ (x (t; x_0, 0), t) x∈X, is not identically zero
- When ||x|| →∞, there is V (x, t) →∞

The origin equilibrium state x = 0 of the system is asymptotically stable.

THEORETICAL BASIS

We consider the system as follows Yang and Xu (2007a), Liu and Hill (2007) and Dashkovskyiy *et al.* (2012):

$$\begin{cases} \tilde{x}(t) = (A_1 + \Delta A_1)x(t) + (A_2 + \Delta A_2) \\ x(t-\tau) + (B + \Delta B)u(t) + Hw(t), t \neq t_k \\ z(t) = Gx(t), t \neq t_k \\ \Delta x = C_k x(t), t = t_k \\ x_i(t) = \phi_i(t), t \in [t_{k-1}, t_k] \end{cases} \tag{2}$$

where,
$X(t) \in R^n$: The state vector
$u \in R^{l_1}$: The control input
$w(t) \in R^{l_2}$: The disturbance input
$z \in R^{l_3}$: The controlled output variable
τ : The time delay constant
ΔA & ΔB : The uncertain functions with the appropriate dimension, which indicates the uncertainty of the system in the model
$t_k (k = 0, 1, 2\ldots)$: The pulse time meeting $0 = t_0 < t_1 <, \ldots < t_k, < \ldots, \lim_{k\to\infty} t_k = \infty$
$A, B \in R^{n\times n}$ & $E \in R^{l_3 \times n}$: The real constant matrices with appropriate dimension

Assume that the considered parameters are norm-bounded and has the following form:

$$\Delta A = F_A \Lambda_A(t) E_A, \Delta B = F_B \Lambda_B(t) E_B$$

where, F_A, F_B, F_{Ck} E_A, E_B, E_{Ck} are the known constant matrices with appropriate dimension and they reflect the uncertainty of the structural information. $\Lambda_A(t)$, $\Lambda_B(t)$, $\Lambda_{Ck}(t)$ are the time varying unknown matrix and satisfy:

$$\Lambda_A(t)\Lambda_A^T(t) \le I, \Lambda_B(t)\Lambda_B^T(t) \le I$$

The main purpose of this section is to design memory-feedback control law:

$$u(t) = Kx(t)$$

This makes the closed-loop system:

$$\begin{cases} \tilde{x}(t) = \overline{A}_1 x(t) + \overline{A}_2 x(t-\tau) + Hw(t), t \neq t_k \\ z(t) = Ex(t) \\ \Delta x = C_k x(t), t = t_k \end{cases} \tag{3}$$

Equation (3) is robust exponential stability.

where,

$$\overline{A}_1 = A_1 + \Delta A_1 + BK + \Delta BK, \overline{A}_2 = A_2 + \Delta A_2$$

MAIN RESULTS

First, consider the following system uncertainties (Chen and Zheng, 2011; Chen and Xu, 2012; Cheng *et al.*, 2012):

$$\begin{cases} \tilde{x}(t) = A_1 x(t) + A_2 x(t-\tau) + Bu(t) + Hw(t), t \neq t_k \\ \Delta x(t) = I_k(x,t) = C_k x(t), t = t_k \\ z(t) = Ex(t) \\ x(t_0^+) = x_0, t_0 = 0 \end{cases} \tag{4}$$

When state feedback controller is joined:

$$u(t) = Kx(t)$$

System (4) can be changed:

$$\begin{cases} \tilde{x}(t) = A_1 x(t) + A_2 x(t-\tau) + Hw(t), t \neq t_k \\ \Delta x(t) = I_k(x,t) = C_k x(t), t = t_k \\ z(t) = Ex(t) \\ x(t_0^+) = x_0, t_0 = 0 \end{cases} \tag{5}$$

where, $\bar{A}_1 = A_1 + BK$, where, the relevant symbols are the same with the system (2).

Theorem 3: Given $\alpha>0$, if there is the symmetric positive definite matrix $P \in R^{n\times n}$, $Q \in R^{n\times n}$ makes:

$$\begin{vmatrix} 2\alpha P + \overline{A}^T P + P\overline{A}_1 + Q + E^T E + r^2 s^2 I & PA_2 & \frac{1}{r}PH \\ A_2^T P & -e^{-2\alpha\tau}Q & 0 \\ \frac{1}{r}H^T P & 0 & -I \end{vmatrix} < 0 \tag{6}$$

$$\begin{vmatrix} -P & (I + C_k)^T P \\ * & -P \end{vmatrix} < 0 \tag{7}$$

It is establishment. The system (5) is robust exponential stable. Here $\bar{A}_1 = A_1 + BK$.

Proof: Take Lyapunov function:

$$V(t) = e^{2\alpha t} x^T(t) P x(t) + \int_{t-\tau}^{t} e^{2\alpha s} x^T(s) Q x(s) ds$$

where, P>0, Q>0 will be determined, when $t \neq t_k$, the derivative of t is V (t) along the closed-loop system (5), there is:

$$V'(t) = e^{2\alpha t}(2\alpha x(t)^T Px(t) + x'(t)^T Px(t) + x(t)^T$$
$$Px'(t)) + e^{2\alpha t}(x^T(t)Qx(t) - e^{-2\alpha\tau} x^T(t-\tau)Qx(t-\tau)$$

(5) is substituted into the above formula, there is:

$$V^{'}(t) = e^{2\alpha t} 2\alpha x(t)^T Px(t) + e^{2\alpha t}$$
$$[\overline{A}_1 x(t) + A_2 x(t-\tau) + Hw]^T$$
$$Px(t) + e^{2\alpha t} x(t)^T P[\overline{A}_1 x(t)$$
$$+ A_2 x(t-\tau) + Hw(t)] + e^{2\alpha t}(x(t)^T Qx(t) - e$$
$$^{-2\alpha t} x^T(t-\tau)Qx(t-\tau))$$
$$= e^{2\alpha t}[x(t)^T(2\alpha P + \overline{A}_1^T P + P\overline{A}_1 + Q)$$
$$x(t) + 2x(t)^T PA_2^T x(t-\tau)$$
$$+ 2x^T(t)PHw - e^{-2\alpha\tau} x^T(t-\tau)Qx(t-\tau)] \quad (8)$$

Lemma 1: Let Q ∈ $R^{n\times n}$ for a given matrix, S ∈ $R^{m\times m}$ for any positive definite symmetric matrix, then play for any u∈R^n v∈R^m, the following inequality holds:

$$2u^T Qv \le u^T QS^{-1} u + v^T Sv$$

We can get the following by this Lemma 1:

$$2x(t)^T PA_2^T x(t-\tau) \le e^{-2\alpha\tau} x(t)^T PA_2^T Q^{-1}$$
$$A_2 Px(t) + e^{-2\alpha\tau} x^T(t-\tau)Qx(t-\tau) \quad (9)$$

Lemma 2: For the appropriate dimension matrix F meeting $F^T F \le I$, there is:

$$2x^T DFEy \le \varepsilon x^T D^T Dx + \frac{1}{\varepsilon} y^T E^T Ey$$

For any vector x∈R^p, y∈R^q and constant accounting ∈>0, the above formula is effective. Where, D and E are constant matrices with appropriate dimension. We can get the following by this Lemma 2:

$$2x^T(t)PHw \le \frac{1}{r^2} x^T(t)PHH^T Px(t) + r^2 w^T w \quad (10)$$

Equation (9) and (10) are substituted into the formula (7), we can obtain:

$$\tilde{V} \le e^{2\alpha\tau}[x(t)^T(2\alpha P + \overline{A}_1^T P + P\overline{A}_1 + Q + \frac{1}{r} PHH^T$$
$$P + e^{-2\alpha\tau} PA_2^T Q^{-1} A_2 P)x(t) + r^2 w^T w] \quad (11)$$

We can get by the complement theorem of Sehur based on (5):

$$2\alpha P + \overline{A}_1^T P + P\overline{A}_1 + Q + \frac{1}{r} PHH^T$$
$$P + e^{-2\alpha\tau} PA_2^T Q^{-1} A_2 P + E^T E < 0 \quad (12)$$

Equation (12) is substituted into (11), we have:

$$\tilde{V} < e^{2\alpha\tau}[-x^T(t)E^T Ex(t) + r^2 w^T w]$$
$$= e^{2\alpha\tau}[-\| Ex(t) \| + r^2 \| w \|^2]$$
$$< -\| z \|^2 + r^2 \| w \|^2 \quad (13)$$

Thus, when ω = 0, there is $\tilde{V}<0$. When t = t_k, formula (7) is known based on Schur complement:

$$(I + C_k)^T P(I + C_k) \le P \quad (14)$$

So:

$$V(t_k, x(t_k)) = e^{2\alpha t_k} x^T(t_k)Px(t_k) + \int_{t_k-\tau}^{t_k} e^{2\alpha s} x(s)^T Qx(s)ds$$
$$= e^{2\alpha t_k}[(I + \overline{C}_k)x(t_k^-)]^T P[(I + \overline{C}_k)x(t_k^-)]$$
$$+ \int_{t_k-\tau}^{t_k} e^{2\alpha s} x(s)^T Qx(s)dss$$
$$\le e^{2\alpha t_k} x^T(t_k^-)Px(t_k^-) + \int_{t_k-\tau}^{t_k} e^{2\alpha s} x(s)^T Qx(s)ds$$
$$= \lim_{t\to t_k^-} V(x, x(t))$$

So:

$$V(t) < V(t_0) = x^T(t_0)Px(t_0) + \int_{t-\tau}^{t} e^{2\alpha s} x^T(s)Qx(s)ds$$

$$< [\lambda_{\max}(P) + \tau\lambda_{\max}(Q)] \| x(t_0) \|^2$$

Lemma 3: If P is a positive definite matrix with order n and Q is a symmetric matrix with order n, for any x ∈ R^n, there is:

$$\lambda_{\min}(P^{-1}Q)x^T Px \le x^T Qx \le \lambda_{\max}(P^{-1}Q)x^T Px$$

Proof: P is positive definite, so there is full rank matrix P_1 existing, which makes that P = $P'_1 P_1$. Let $P_1 x = y$, then:

$$x^T Qx = y^T (P_1^{-1})^T QP_1^{-1} y$$

For,

$$(P_1^{-1})^T QP_1^{-1} = P_1 P_1^{-1}(P_1^T)^{-1} QP_1^{-1}$$
$$= P_1(P_1^T P_1)^{-1} QP_1^{-1} = P_1 P_1^{-1} QP_1^{-1}$$

So the matrix $(P_1^{-1})^T QPP_1^{-1}$ and the matrix $P^{-1}Q$ are similar, so they have the same Eigen values. Then:

$$x^T Qx = y^T (P_1^{-1})^T QP_1^{-1} y \le \lambda_{\max}$$
$$((P_1^{-1})^T QP_1^{-1})y^T y = \lambda_{\max}(P_1^{-1}Q)x^T Px$$

We similarly can get $\lambda_{\min}(P^{-1}Q)x^T Px \le x^T Qx$, so the Lemma is established. We can get the following by this Lemma 3:

$$V(t) \ge e^{2\alpha\tau} x^T(t)Px(t) \ge \lambda_{\min}(P)e^{2\alpha\tau} \| x(t) \|^2$$

So, we have:

$$\| x(t) \| < \sqrt{\frac{\lambda_{\max}(P) + \tau\lambda_{\max}(Q)}{\lambda_{\min}(P)}} \| x(t_0) \| e^{-\alpha t}, t \ge 0$$

Definition 1 Let r>0 is a constant, x (t) = x (t, t_0, φ is the (t_0, φ) solution of system (1), for any ε>0, $t_0 \ge 0$ exist σ>0, when $\|\varphi\| < \delta$, $|x(t)| < \varepsilon . e^{-r(t-t_0)}$, $t \ge t_0$ and the solution of the system (1) is called exponential stability.

Theorem 4: There is appropriate positive r>0, if there is a positive definite symmetric matrix $X \in R^{n\times n}$, $Y \in R^{n\times n}$, $W \in R^{n\times n}$, which makes that the following linear matrix inequalities (Dashkovskyiy *et al*., 2012):

$$\begin{vmatrix} U_{11} & A_2 & \frac{1}{r}H & XE & rsX \\ A_2^T & -e^{-2\alpha\tau}W & 0 & 0 & 0 \\ \frac{1}{r}H^T & 0 & -I & 0 & 0 \\ E^T X & 0 & 0 & -I & 0 \\ rsX & 0 & 0 & 0 & -I \end{vmatrix} < 0 \quad (15)$$

$$\begin{vmatrix} X & X(I + C_K)^T \\ (I + C_K)X & X \end{vmatrix} < 0 \quad (16)$$

Equation (16) is efficient, then the system (4) is the robust exponential stabilization.
Where,

$$U_{11} = 2\alpha X + XA_1^T + A_1 X + Y^T B^T + BY + W$$

Proof: Formula (6) can be obtained by Schur lemma:

$$\begin{vmatrix} 2\alpha P + (A_1 + BK)^T P + P(A_1 + BK) + E^T E + Q & PA_2 & \frac{1}{r}PH & rsI \\ A_2^T P & -e^{-2\alpha\tau}Q & 0 & 0 \\ \frac{1}{r}H^T P & 0 & 0 & 0 \\ rsI & 0 & 0 & -I \end{vmatrix} < 0 \quad (17)$$

Formula (17) is multiplied by daig [P^{-1}, P^{-1}, I, I] and we can obtain:
where,

$$\overline{U}_{11} = 2\alpha P^{-1} + P^{-1}(A_1 + BK)^T P + (A_1 + BK) + P^{-1}E^T E P^{-1} + P^{-1}Q$$

Let P^{-1} = X, Y = KX, Q = PWP and we can get (15) (6) is multiplied by diag [P^{-1}, I] and we can get (16).

The following theorem gives out system (1) feedback controller design.

Theorem 3 Let given scalar ε>0, r>0, σ>0, the system (2) is robust exponentially stable, if there is a positive definite symmetric matrix X>0, Y>0, W>0, such that the following linear matrix inequality holds:

$$\begin{vmatrix} \overline{U}_{11} & \frac{1}{r}H & \varepsilon F_1 & \sigma F_B & \sigma^{-1}XK^T E_B^T & \varepsilon^{-1}XE_1 & XE \\ \frac{1}{r}H^T & -I & 0 & 0 & 0 & 0 & 0 \\ \varepsilon F_1^T & 0 & -I & 0 & 0 & 0 & 0 \\ \sigma F_B^T & 0 & 0 & -I & 0 & 0 & 0 \\ \sigma^{-1}E_B^T KX & 0 & 0 & 0 & -I & 0 & 0 \\ \varepsilon^{-1}E_1 X & 0 & 0 & 0 & 0 & -I & 0 \\ E^T X & 0 & 0 & 0 & 0 & 0 & -I \end{vmatrix} < 0 \quad (18)$$

$$\begin{vmatrix} -X & X(I + C_K)^T & XE_{C_k}^T \\ (I + C_K)X & -X + \varepsilon_{C_k} F_{C_k} F_{C_k}^T & 0 \\ E_{C_k} X & 0 & -\varepsilon_{C_k} I \end{vmatrix} < 0 \quad (19)$$

where,

$$\overline{U}_{11} = 2\alpha X + XA_1^T + A_1 X + W + e^{-2\alpha\tau}[A_2^T (Q - \mu F_2 F_2^T)^{-1} A_2 + \mu^{-1} E_2^T E_2\} + Y^T B^T + BY$$

$$X = P^{-1}, Y = KX, Q = PWP$$

Proof: Take Lyapunov function:

$$V(t) = e^{2\alpha\tau} x^T(t)Px(t) + \int_{t-\tau}^{t} e^{2\alpha s} x^T(s)Qx(s)ds$$

where, P>0, Q>0 to be determined, when $t \neq t_k$, the solution of V (t) along the closed-loop system (3) for t derivative:

$$\tilde{V}(t) = e^{2\alpha\tau}(2\alpha x(t)^T Px(t) + \tilde{x}(t)^T Px(t) + x(t)^T P\tilde{x}(t)) + e^{2\alpha\tau}(x^T(t)Qx(t) - e^{-2\alpha\tau}x^T(t-\tau) \times Qx(t-\tau))$$

$$= e^{2\alpha\tau}[x^T(t)(2\alpha P + A_1^T P + PA_1 + Q + K^T B^T P + PBK) x(t) + x^T(t)(E_1^T \wedge_1 F_1^T P + PF_1 \wedge_1 E_1$$

$$+ K^T E_B^T \wedge_B F_B^T P + PF_B \wedge_B E_B K)x(t) + 2x^T(t) P(A_2 + F_2 \wedge_2 E_2)x(t) + 2x^T(t)PHw]$$

Lemma 3: Let A, F, ∧, E are the matrices of the appropriate dimension and $\wedge^T \wedge \le I$, for any meet $S - \mu FF^T > 0$ positive symmetric matrix S and the real number μ>0 and the following inequality holds:

$$(A + F\wedge E)^T S^{-1}(A + F\wedge E) \le A^T(S - \mu FF^T)^{-1}A + \mu^{-1}E^T E$$

We can get the following by Lemma 1 and 3:

$$\begin{aligned}&2x^T(t)(PA_2+F_2\wedge_2 E_2)x(t)\\&\le e^{-2\alpha\tau}x^T(t)P(A_2+F_2\wedge_2 E_2)^T\\&Q^{-1}(A_2+F_2\wedge_2 E_2)\\&Px(t)+e^{-2\alpha\tau}x^T(t-\tau)Qx(t-\tau)\\&\le e^{-2\alpha\tau}x^T(t)\quad P\left[A_2^T(Q-\mu F_2F_2^T)^{-1}A_2+\mu^{-1}E_2^TE_2\right]\\&Px(t)+e^{-2\alpha\tau}x^T(t-\tau)Qx(t-\tau)\end{aligned}\tag{20}$$

Lemma 4: Let F, ∧, E are the matrices of the appropriate dimension and $\wedge^T\wedge\le I$, for any real number $\mu<0$, the following inequality holds:

$$F\Lambda E+E^T\Lambda^TF^T\le \mu FF^T+\mu^{-1}E^TE$$

We can get the following by this Lemma 4:

$$\begin{aligned}&x^T(t)(E_1^T\Lambda_1F_1^TP+PF_1\Lambda_1E_1+K^TE_B^T\Lambda_BF_B^TP\\&+PF_B\Lambda_BE_BK)x(t)\\&\le x^T(t)(\varepsilon^2PF_1F_1^TP+\varepsilon^{-2}E_1^TE_1+\\&\sigma^2PF_BF_B^TP+\sigma^{-2}K^TE_B^TE_BK)x(t)\end{aligned}\tag{21}$$

We can get the following by this Lemma 2:

$$2x^T(t)PHw\le\frac{1}{r^2}x^T(t)PHH^TPx(t)+r^2w^Tw\tag{22}$$

Equation (20), (21) and (21) are substituted and finished:

$$\begin{aligned}&\tilde V(t)\le e^{2\alpha\tau}\{x(t)(2\alpha P+A_1^TP+PA_1+Q+K^TB^TP\\&+PBK+e^{-2\alpha\tau}P[A_2^T(Q-\mu F_2F_2^T)^{-1}A_2+\mu^{-1}E_2^TE_2]P\\&+\varepsilon^2PF_1F_1^TP+\varepsilon^{-2}E_1^TE_1+\sigma^2PF_BF_B^TP\\&+\sigma^{-2}K^TE_B^TE_BK+\frac{1}{r^2}PHH^TP)x(t)+r^2w^Tw\}\end{aligned}\tag{23}$$

Equation (18) multiplicates diag [P, P, P, I, P, P, P] can be obtained by the Schur complement theorem:

$$\begin{aligned}&2\alpha P+A_1^TP+PA_1+Q+K^TB^TP+PBK\\&+e^{-2\alpha\tau}P[A_2^T(Q-\mu F_2F_2^T)^{-1}A_2+\mu^{-1}E_2^TE_2]P\\&+\varepsilon^2PF_1F_1^TP+\varepsilon^{-2}E_1^TE_1+\sigma^2PF_BF_B^TP\\&+\sigma^{-2}K^TE_B^TE_BK+\frac{1}{r^2}PHH^TP+E^TE<0\end{aligned}\tag{24}$$

Formula (24) is brought into Eq. (23), we have:

$$\begin{aligned}&\tilde V(t)\le e^{2\alpha\tau}[-x^T(t)G^TGx(t)+r^2w^Tw]\\&=e^{2\alpha\tau}[-\|z\|^2+r^2\|w\|^2]\end{aligned}\tag{25}$$

So when w = 0, $\tilde V(t)<0$.

In addition, the formula (19) is known by Schur complement:

$$\begin{aligned}&(I+C_k)^T(P^{-1}-\varepsilon_{C_k}F_{C_k}F_{C_k}^T)^{-1}(I+C_k)\\&+\varepsilon_{C_k}E_{C_k}^TE_{C_k}\le P,k=1,2,\cdots\end{aligned}\tag{26}$$

When $t = t_k$ (k = 1, 2,…), the certification process is similar to Theorem 2 and the system (2) is a robust exponential stability.

NUMERICAL SIMULATION

Consider the system (2), where, the initial function x (t) = $[0\ 0]^T$, τ = 0.9 and the system related matrix:

$$A_1=\begin{bmatrix}1&1\\0&-1\end{bmatrix},A_2=\begin{bmatrix}1&1\\-1&0\end{bmatrix},B=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$
$$H=\begin{bmatrix}0&1\\1&0.5\end{bmatrix},E=\begin{bmatrix}1&0\\-1&1\end{bmatrix}$$

$$F_1=F_2=F_B=\begin{bmatrix}0.3&0\\0&0.3\end{bmatrix},E_1=E_2$$
$$=E_B=\begin{bmatrix}0&0.5\\0.5&0.3\end{bmatrix},C_k=\begin{bmatrix}-1&0.8\\0.2&-0.5\end{bmatrix}$$

Let $\varepsilon=\sigma=r=\varepsilon_{Ck}=1$

When the control input u (t) = 0, the system state trajectory is shown in Fig. 1:

We can see from Fig. 1 that the system is unstable. The following conclusion by Theorem 3 designs the state feedback controller to make the system robust exponential stability. We can get the following relations by using LMITOOL of MATLAB as shown in Fig. 2:

$$X=\begin{bmatrix}0.4721&0.1887\\0.1887&0.2831\end{bmatrix}>0$$

$$Y=\begin{bmatrix}1.0387&0.0142\\-0.0141&1.0387\end{bmatrix}$$

$$W=\begin{bmatrix}-0.0113&-1.4863\\-1.4863&0.0028\end{bmatrix}>0$$

Corresponding controller gain matrix:

$$K=YX^{-1}=\begin{bmatrix}2.9728&-1.9318\\-2.0421&-5.0288\end{bmatrix}$$

As can be seen from Fig. 2, the system can be quickly stabilized.

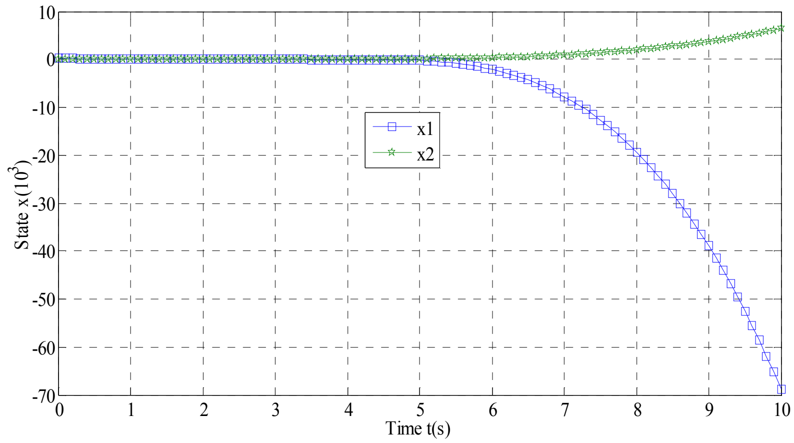

Fig. 1: The system without feedback controller

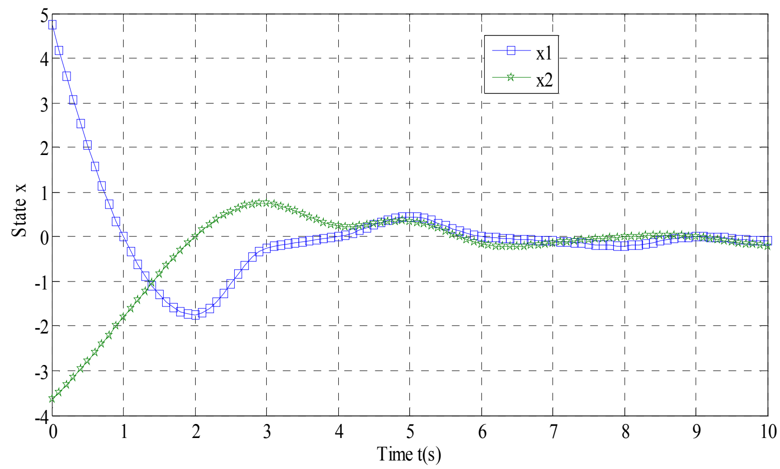

Fig. 2: The system with feedback controller

Note 1: literature (Dashkovskyiy *et al*., 2012) set the pulse effect matrix is a constant matrix assumptions meeting: $-2\leq \|C_k\| \leq 0$ and the conditions do not require the limit of the condition. In fact, when the pulse effect matrix is a diagonal matrix, condition (6) becomes $(1 + c_k)^2 < 1$, i.e., $-2\leq c_k \leq 0$, k = 1, 2, ... Therefore, to some extent, greater improvement and expansion method is proposed.

CONCLUSION

This paper studies the robust control problem of the uncertain linear impulsive delay system. Using Lyapunov stability theory, Lyapunov function method and linear matrix inequalities techniques based on the impulsive delay system stability analysis designs the robust feedback controller to eliminate pulse and the impact of the delay on the stability of the system. The controller is reached conclusion through MATLAB toolbox for solving on the form of linear matrix inequalities, adding robust controller after the system robust exponential stability sufficient condition to verify the validity of the results obtained by numerical examples.

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