Research Journal of Applied Sciences, Engineering and Technology 6(9): 1620-1625, 2013
DOI:10.19026/rjaset.6.3879
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: December 20, 2012 Accepted: February 01, 2013 Published: July 15, 2013

Research Article

Learning Evaluation of Distance Education Based on AHP and Fuzzy Theory

Fan Tongke
Modern Education Technology Center, Xi'an International University, Xi'an 710077, China

Abstract: Based on features of distance education and its disadvantages in terms of learning evaluation, this study uses Analytic Hierarchy Process (AHP) and fuzzy mathematics theory to study distance education. First, an electric file is generated for each learner. Then a practical multi-hierarchy evaluation model is built based on AHP. Finally, evaluation of on-line education is done based on information collected by the model with fuzzy comprehensive evaluation method of fuzzy mathematics. It helps to evaluate students' learning more quickly, accurately and scientifically.

Keywords: Analytic hierarchy process, distance education, fuzzy mathematics, fuzzy comprehensive evaluation, learning evaluation

INTRODUCTION

Modern distance education is a new form of education that integrates education and network. With such advantages as resource share, fast delivery of education message, high integration of educational media, distance education has become a major part of education for all and lifelong education. Since distance education was adopted in colleges and universities in 1999, 68 of them have been approved by Ministry of Education modern to carry out distance education trial. Over 2000 learning centers enrolling more than 2 million students have been established throughout China. On-line education has become a major educational method and platform. However, its quality guarantee system is far from perfect. Quality has always been the top priority for distance education development (Teng, 2011). Therefore, traditional evaluation is no longer suitable for distance education. For on-line education, it is hard for teachers to monitor learners and give advices for improvement. As a result, learners tend to get inattentive and lost after showing a lot of confidence at the beginning. At the 17^{th} and 20^{th} International Distance Education Conference, attention is called for on learning quality. It is proposed to focus on learners and establish lifelong learning and conduct effective evaluation of them (Zhang, 2007a). It's necessary to make full use of computer and network to design an effective monitoring system for learners (Chen and Aijie, 2012).

Evaluation and feedback are necessary to guarantee educational quality. However, modern distance education still uses traditional summarative evaluation method rather than process evaluation. It has no scientific theory or quantitative analysis. Participation of teachers and administrators is insufficient. Therefore, it's unable to monitor students' learning process on evaluation platform (Wang, 2011). In this thesis, a distance education evaluation model is established with AHP (Saaty, 1994). Fuzzy comprehensive evaluation method is used to evaluate distance learning. This model and evaluation method can work as human in decision making while integrate qualitative analysis and quantitative analysis to truly reflect objective information so as to make evaluation simpler and result more scientific.

INTRODUCTION TO AHP

AHP was put forward by U.S operational research expert T.L. Saatty in early 1970s. AHP refers to a decision making method which decomposes elements in relations to decision making into various hierarchies including target, criteria and plan and conduct quantitative and qualitative analysis based on this. It can evaluate weight scientifically and make the evaluation results more accurate and objective. There're three steps in using AHP:

Step 1: Analyze interrelationship of various elements in the system and compare elements on the same hierarchy and compare them to one another two at a time, with respect to their impact on a criteria above them in the hierarchy, building a judgment matrix in pairwise comparison

Step 2: Compute relative weight of the elements with respect to the criteria based on the judgment matrix and test the consistency of the matrix.

Step 3: Compute sequencing weight of each hierarchy to the overall goal.

Table 1: 1-9 scale

Degree of importance	Definition	Description
1	Same in degree of importance	The two factors have the same effect
3	Higher	The effect of one factor is a little bit higher than the other one
5	Much higher	The effect of one factor is much higher than the other one
7	Remarkably higher	The effect of one factor is remarkably higher than the other one
9	Absolutely higher	Higher than the other factor possibly controllable
2/4/6/8	Median of the above degrees of importance	
Reciprocal	When comparing “i" and “j” and give them one of the scale values above, then the reciprocal of the scale should be the weight.	

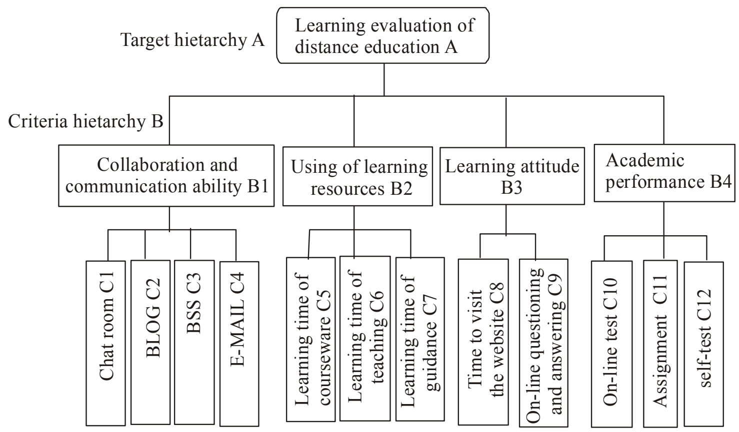

Fig. 1: Hierarchy structure for distance learning evaluation index

Fundamental theory of fuzzy mathematics: Fuzzy mathematics is a new discipline of mathematics. It was first introduced in 1965 by US computer and cybernetics professional L.A.Zadeh in a paper titled Fuzzy Sets which was published on *Information and Control*. It laid a foundation for classical mathematics and made a breakthrough in introducing computer science to natural mechanism. Fuzziness refers to a feature existed in transitional period of differences which is “this and that at the same time”. Fuzzy mathematics is a method used to study and handle this fuzziness. Fuzzy comprehensive evaluation method takes fuzzy mathematics as the basis and uses composition of fuzzy relation theory to quantize indefinite and non-quantitative factors for comprehensive evaluation. It contains a fuzzy set which is composed of multiple factors or indices (known as factor set U) and a fuzzy set of evaluation consisted of evaluation grade from which factor set chooses (known as judgment set V) (Hu, 2011).

LEARNING EVALUATION INDEX SYSTEM IN DISTANCE EDUCATION

Build evaluation index hierarchy model selecting a template: Evaluation to students’ learning is a key part of modern distance education evaluation, the essence of which is to evaluate learning effect of the students. By collecting and processing information concerning the learning process, quantitative analysis is done concerning students' learning attitude, behavior and effect based on instructional objective and then evaluation result is given. All evaluations must be done in the same evaluation index system, meaning that building scientific and feasible evaluations index system is of vital importance to learning evaluation. In this study, AHP is used to build hierarchy as shown in Diagram 1 to achieve distance learning evaluation from such perspectives as students’ collaboration and communication ability, using of learning resource, learning attitude and academic performance (Li *et al.*, 2009; Huang and Taijun, 2010).

This model is composed of two levels of indices, of which collaboration and communication ability, using of learning resource, learning attitude and academic performance belong to primary index and are referred to as criteria level, expressed in B_i (i = 1, 2, 3, 4); each primary index includes a number of j secondary indices (j = 1, 2, 3… 12), expressed in c_j (Fig. 1).

Building of judgment matrix: Building of judgment matrix is a key step for AHP. The process of building is actually a pairwise comparison of elements on the same hierarchy with respect to their priority in sequence. First, compare elements on the criteria hierarchy to one another two at a time and build relative importance judgment matrix; second, compare index factors under each criteria hierarchy to one another two at a time and build relative importance judgment matrix. In order to compare the elements to one another two at a time to get a judgment matrix, Satty's 1-9 scale method is going to be used for grading (Satty and Alexander, 2007). The content of scale method is shown in Table 1.

Table 2: Primary index judgment matrix A

A	B1	B2	B3	B4
B1	1	1/2	1/3	1/5
B2	2	1	1/2	1/4
B3	3	2	1	1/3
B4	5	4	3	1

Table 3: Secondary index judgment matrix

B1	C1	C2	C3	C4
C1	1	1/2	2	1/2
C2	2	1	3	1
C3	1/2	1/3	1	1/3
C4	2	1	3	1
B2	C5	C6	C7	
C5	1	1/2	2	
C6	2	1	3	
C7	1/2	1/3	1	
B3	C8	C9		
C8	1	1/2		
C9	2	1		
B4	C10	C11	C12	
C10	1	4/2	4	
C11	2/4	1	2/1	
C12	1/4	1/2	1	

Table 4: RI index

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Table 5: Matlab program

1	Format short
2	A = [1 1/2 1/3 1/5; 2 1 1/2 1/4;3 2 1 1/3; 5 4 3 1]
3	[v,d] = eigs(A)
4	Tzmax = max(d(:))
5	[m,n] = size(v)
6	Sum = 0;
7	For I = 1:m Sum = sum + v(i,1); end
8	Stand = v(:,1);
9	For I = 1:m stand(i,1) = v(i,1)/sum; end
10	disp ('input matrix is : ')
11	A
12	Disp ('eigenvectors and eigenvalues: ')
13	v
14	d
15	Disp ('largest eigenvalue is : ')
16	tzmax
17	Disp (' the largest eigenvalue (after standardizain) is: ')
18	Stand

Based on the scale in Table 1, expert meeting law is used to compare the indices to one another two at a time and grade them. As a result, primary and secondary judgment matrixes are built, as shown in Table 2 and 3.

Solve judgment matrix by using matlab software: The largest eigenvalue λ_{max} and eigenvector W of the judgment matrix, after being normalized, become the sequencing weight of elements of the same hierarchy with respect to an element of the above hierarchy. The basic problem of AHP is to solve the eigenvector (weight vector) of judgment matrix. The eigenvector is effective only when the judgment matrix meets consistency requirement, otherwise, the judgment matrix needs to be adjusted. The process of calculation and normalization of the largest eigenvalue and eigenvector is quite complicated and errors often rise in the process. In this study, Matlab program is used to accurately complete these calculations in a short period of time. Consistency index *CI*, random consistency index *RI* and consistency ratio *CR* are introduced.
The calculation formula is as follows:

Consistency index:

$$CI = \lambda_{max-}n/n-1 \text{ ("n" refers to order of matrix)} \quad (1)$$

Consistency ratio:

$$CR = CI/RI \quad (2)$$

The judgment matrix is fully consistent when $CR = 0$; satisfactory when $CR<0.1$; the consistency is extremely satisfactory when CR> = 0.1. The values of *RI* are given in Table 4 (Chen and Shiping, 2012).

Take judgment matrix A as an example, the Matlab program for solving the largest eigenvalue and eigenvector is as Table 5.

The largest eigenvalues of the judgment matrix are: $\lambda_{max} = 4.0511$, $W = (0.0838, 0.1377, 0.2323, 0.5462)^T$, CI = 0.017, CR = 0.018<0.1, all of which meet consistency requirement. The weight of the four factors on the criteria hierarchy is $W = (0.0838, 0.1377, 0.2323, 0.5462)^T$. The result corresponding to the matrix on the criteria hierarchy can be calculated as follows:

B1 matrix: $\lambda_{max} = 4.0104$, $W = (0.1891, 0.3509, 0.1091, 0.3509)^T$, CI = 0.0034, CR = 0.0038<0.1.

B2 matrix: $\lambda_{max} = 3.0092$, $W = (0.2970, 0.5396, 0.1634)^T$, CI = 0.0046, CR = 0.0079<0.1

B3 matrix: $\lambda_{max} = 2$, $W = (0.3333, 0.6667)^T$, second-order matrix is full consistent.

B4 matrix: $\lambda_{max} = 3.0092$, $W = (0.5396, 0.1634, 0.2970)^T$, CI = 0.0046, CR = 0.0052<0.1.

Calculation of synthetic weight: With the above calculations, we can obtain the weight of criteria hierarchy to target hierarchy and weight of index hierarchy to criteria hierarchy. The formula for weight of various index hierarchies to target hierarchy is:

$$a_k = \beta_i \times w_{ki} \quad (3)$$

where, β_i stands for weight of various factors on the criteria hierarchy to target hierarchy; w_{ki} stands for weight of various factors on the index hierarchy to criteria hierarchy. The specific weight for each index is shown in Table 6.

FUZZY COMPREHENSIVE EVALUATION METHOD

Establishment of evaluation index factor set and evaluation set (Zhang, 2007b):

Table 6: Synthetic weight for various indices

Index hierarchy	B1	B2	B3	B4	Synthetic weight
	0.0838	0.1377	0.2323	0.5462	
C1	0.1891				0.015847
C2	0.3509				0.029405
C3	0.1091				0.009143
C4	0.3509				0.029405
C5		0.2970			0.040897
C6		0.5396			0.074303
C7		0.1634			0.0225
C8			0.3333		0.077426
C9			0.6667		0.154874
C10				0.5396	0.29473
C11				0.1634	0.089249
C12				0.2970	0.162221

Table 7: Evaluation grades and corresponding scores

Range of score	Grade	Represented score
90≦X＜100	Excellent	95
80≦X＜90	Good	85
70≦X＜80	Medium	75
60≦X＜70	Pass	65
X＜60	Fail	50

- Define primary index set as B = (B1, B2, B3, B4) = (collaboration and communication ability, using of learning resources, learning attitude, performance), and corresponding weight set as Bw = (B1, B2, B3, B4) = (0.0838, 0.1377, 0.2323, 0.5462).
- Define secondary index set as C = (C1, C2……C11, C12) = (chatting room, blog……, homework, self-test), and corresponding weight set as Cw = (C1, C2, …, C11, C12) = (0.1891, 0.3509……0.1634, 0.2970).
- Define fuzzy evaluation set as A = (a1, a2, a3, a4, a5). Based on characteristics and requirements of distance education system, fuzzy numbers must be used to replace grades used by teachers. By making use of the currently used five-grade evaluation mode which consists of excellent, good, medium, pass, fail, a grade score matrix G = $(95, 85, 75, 65, 50)^T$ is established, as shown in Table 7.

Membership function of fuzzy evaluation matrix: Rank all students based on their performance in a particular course. Classify the ranking sequence into five grades, namely (0～10%], (10～30%], (30～60%], (60～90%], (90～100%]. Students' ranking and their real level should meet normal distribution in a test. This is equivalent to defining a membership function by using fuzzy statistics method of degree of membership. We can use [0,1] interval to measure indefiniteness. Based on degree of membership, if a student's rank rages within (10～30%], then the possibility for his or her real level to fall within (10～30%] is 0.7 and 0.15 for (0～10%] and (30%～60%]. Therefore, the student's real level in the class can be described with vector (0.15, 0.7, 0.15, 0, 0) (Zhang *et al*., 2011; Wu, 2011).

EVALUATION CASE

Here's a description of fuzzy comprehensive evaluation with an example of a student's performance in a particular course, as shown in Table 8. The Table 8 shows that: Evaluation matrixes for primary indices are R_1, R_2, R_3 and R_4 as shown below:

$$R_1 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0.85 & 0.15 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.85 & 0.15 & 0 & 0 & 0 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \end{bmatrix}$$

Weights for evaluation indices are:

$$C_{1-4} = (0.1891, 0.3509, 0.1091, 0.3509)$$
$$C_{5\text{-}7} = (0.2970, 0.5396, 0.1634)$$
$$C_{8\text{-}9} = (0.3333, 0.6667)$$
$$C_{10\text{-}12} = (05396, 0.1634, 0.2970)$$

Calculate T_1 :

$$T_1 = C_{1-4} \times R_1 = (0.1891, 0.3509, 0.1091, 0.3509) \times \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix} = (0.0974, 0.5070, 0.3430, 0.0526, 0)$$

After normalization of T_1:

$$T_1' = (0.0974, 0.5070, 0.3430, 0.0526, 0)$$

This indicates that possibility for this student's collaborating and communication ability is 10% for being excellent, 51% for good, 34% for medium, 5% for pass and 0% for fail. Based on rank and score matrix, this student's collaboration and communication ability B_1 in this course is:

$$B_1 = T_1' \times G = (0.0974, 0.5070, 0.3430, 0.0526, 0) \times \begin{bmatrix} 95 \\ 85 \\ 75 \\ 65 \\ 50 \end{bmatrix} = 81.4910$$

It falls into the grade of good.

Table 8: A student's performance in distance learning

Primary index weight	Secondary index weight	Integral	Ranking	Percentage distribution (%)	Degree of membership
Collaboration and communication ability (B1) 0.0838	Chat room (C1) 0.1891	78	16	31	0.15, 0.7, 0.15, 0, 0
	BLOG (C2) 0.3509	20	20	40	0, 0.15, 0.7, 0.15, 0
	BBS (C3) 0.1091	30	9	23	0.15, 0.7, 0.15, 0, 0
	E-Mail (C4) 0.3509	30	12	32	0.15, 0.7, 0.15 ,0, 0
Using of learning resource (B2) 0.1377	Learning time of courseware (C5) 0.2970	20	10	40	0, 0.15, 0.7, 0.15, 0
	Learning time of teaching material (C6) 0.5396	20	17	45	0, 0.15, 0.7, 0.15 ,0
	Learning time of guidance material (C7) 0.1634	12	5	10	0.85, 0.15, 0, 0, 0
Learning attitude (B3) 0.2323	Time to visit the website(C8) 0.3333	40	10	18	0.15, 0.7, 0.15, 0, 0
	On-line question and answer (C9) 0.6667	20	9	12	0.85, 0.15, 0, 0, 0
Academic performance (B4) 0.5462	On-line test (C10) 0.5396	80	13	32	0.15, 0.7, 0.15, 0, 0
	Assignment (C11)0.1634	30	20	47	0, 0.15, 0.7, 0.15, 0
	Self-test (C12) 0.2970	85	15	40	0, 0.15, 0.7, 0.15, 0

Table 9: Student's performance in a particular course in primary index system

Primary index and weight	Score	Ranking	Percentage distribution (%)	Degree of membership
Collaboration and communication ability	81.4910	5	13	0.85, 0.15, 0, 0, 0
Using of learning resource	78.0229	20	45	0, 0.15, 0.7, 0.15, 0
Learning attitude	90.6669	6	18	0.15, 0.7, 0.15, 0, 0
Academic performance	80.396	9	20	0.15, 0.7, 0.15, 0, 0

With the same method, we can get the scores for using of learning resources B2, learning attitude B3 and performance B4 as 78.0229, 90.6669, 80.396, respectively. Rank the score in primary index system again to gain Table 9.

We can get evaluation matrix A for overall performance of the student in this particular course from the Table 9:

$$R = \begin{bmatrix} 0.85 & 0.15 & 0 & 0 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

Primary index weight vector of overall performance is:

B = (B1, B2, B3, B4) = (0.0838, 0.1377, 0.2323, 0.5462)

Result vector T for overall performance A is:

$$T = B \times R = (0.0838, 0.1377, 0.2323, 0.5462) \times \begin{bmatrix} 0.85 & 0.15 & 0 & 0 & 0 \\ 0 & 0.15 & 0.7 & 0.15 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix} = (0.1880, 0.5782, 0.2132, 0.0207, 0)$$

Therefore, the student's overall performance A in this course is:

$$A = T \times G = (0.1880, 0.5782, 0.2132, 0.0207, 0) \times \begin{bmatrix} 95 \\ 85 \\ 75 \\ 65 \\ 50 \end{bmatrix} = 84.3353$$

The student's overall performance is good, which is end of evaluation.

CONCLUSION

This study uses Analytic Hierarchy Process (AHP) to build a practical multi-hierarchy evaluation model and Matlab software to solve matrix with better efficiency. It helps to evaluate students' learning in distance education more quickly, accurately and scientifically.

ACKNOWLEDGMENT

This study was supported by Research Subject 2012, Twelfth Five-year Plan for Shaanxi Education Science (NO.SGH12534) and the Natural Science Foundation Research Program 2012 of Shaanxi (NO.2012JM8045).

REFERENCES

Chen, Y. and L. Aijie, 2012. Application of modern information technology in distance education. Sci. Technol. Infor., 13: 27-42.

Chen, W. and C. Shiping, 2012. Research on quality evaluation of academic thesis based on AHP. J. Zhejiang Univ. Technol., 4: 222-225.

Hu, B., 2010. Fuzzy Theory. Wuhan University Press, Wuhan.

Huang, C. and L. Taijun, 2010. Research on AHP model application in on-line course evaluation system. J. Hainan Normal Univ. Natural Sci., 9: 351-354.

Li, T., Q. Wang and J. Ding, 2009. Evaluation model of on-line learning based on fuzzy mathematics. Shanxi Electro. Technol., 6: 48-50.

Saaty, T.L., 1994. How to make a decision: The analytic hierarchy process. Interfaces, 24(6): 19-43.

Satty, T.L. and I.M. Alexander, 2007. Thinking with Models: Mathematical Models in the Physical, Biological and Social Sciences. Pergamon Press, London.

Teng, X., 2011. On current situation and future trend of distance education in China. China Educ. Technol. Equipment, 1(3): 20-22.

Wang, B., 2011. On significance of teachers in modern distance education. China Adult Educ., 1: 50-53.

Wu, B., 2011.Comprehensive evaluation of audience satisfaction survey based on fuzzy mathematics model. Natural Sci. J. Hainan Univ., 2(29): 127-131.

Zhang, H., 2007a. Monitor Strategy for On-Line Self Learners. Nanjing Normal University Press, China.

Zhang, L., 2007b. The Method of Educational Statistics Analysis. Science Press, Beijing, pp: 255-256.

Zhang, T., X. Yunyun, L. Zancheng and L. Zhenrong, 2011. A method of digital imaging and image processing for detection data based on fuzzy mathematics. J. Projectiles Rockets Missile Guidance, 4: 27-29.