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Research Article
Learning Evaluation of Distance Education Based on AHP and Fuzzy Theory
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Abstract: Based on features of distance education and its disadvantages in terms of learning evaluation, this study uses Analytic Hierarchy Process (AHP) and fuzzy mathematics theory to study distance education. First, an electric file is generated for each learner. Then a practical multi-hierarchy evaluation model is built based on AHP. Finally, evaluation of online education is done based on information collected by the model with fuzzy comprehensive evaluation method of fuzzy mathematics. It helps to evaluate students’ learning more quickly, accurately and scientifically.

Keywords: Analytic hierarchy process, distance education, fuzzy mathematics, fuzzy comprehensive evaluation, learning evaluation

INTRODUCTION

Modern distance education is a new form of education that integrates education and network. With such advantages as resource share, fast delivery of education message, high integration of educational media, distance education has become a major part of education for all and lifelong education. Since distance education was adopted in colleges and universities in 1999, 68 of them have been approved by Ministry of Education modern to carry out distance education trial. Over 2000 learning centers enrolling more than 2 million students have been established throughout China. On-line education has become a major educational method and platform. However, its quality guarantee system is far from perfect. Quality has always been the top priority for distance education development (Teng, 2011). Therefore, traditional evaluation is no longer suitable for distance education. For on-line education, it is hard for teachers to monitor learners and give advices for improvement. As a result, learners tend to get inattentive and lost after showing a lot of confidence at the beginning. At the 17th and 20th International Distance Education Conference, attention is called for on learning quality. It is proposed to focus on learners and establish lifelong learning and conduct effective evaluation of them (Zhang, 2007a). It’s necessary to make full use of computer and network to design an effective monitoring system for learners (Chen and Aijie, 2012).

Evaluation and feedback are necessary to guarantee educational quality. However, modern distance education still uses traditional summative evaluation method rather than process evaluation. It has no scientific theory or quantitative analysis. Participation of teachers and administrators is insufficient. Therefore, it’s unable to monitor students’ learning process on evaluation platform (Wang, 2011). In this thesis, a distance education evaluation model is established with AHP (Saaty, 1994). Fuzzy comprehensive evaluation method is used to evaluate distance learning. This model and evaluation method can work as human in decision making while integrate qualitative analysis and quantitative analysis to truly reflect objective information so as to make evaluation simpler and result more scientific.

INTRODUCTION TO AHP

AHP was put forward by U.S operational research expert T.L. Saatty in early 1970s. AHP refers to a decision making method which decomposes elements in relations to decision making into various hierarchies including target, criteria and plan and conduct quantitative and qualitative analysis based on this. It can evaluate weight scientifically and make the evaluation results more accurate and objective. There’re three steps in using AHP:

Step 1: Analyze interrelationship of various elements in the system and compare elements on the same hierarchy and compare them to one another two at a time, with respect to their impact on a criteria above them in the hierarchy, building a judgment matrix in pairwise comparison

Step 2: Compute relative weight of the elements with respect to the criteria based on the judgment matrix and test the consistency of the matrix.

Step 3: Compute sequencing weight of each hierarchy to the overall goal.
Table 1: 1-9 scale

<table>
<thead>
<tr>
<th>Degree of importance</th>
<th>Definition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Same in degree of importance</td>
<td>The two factors have the same effect</td>
</tr>
<tr>
<td>3</td>
<td>Higher</td>
<td>The effect of one factor is a little bit higher than the other one</td>
</tr>
<tr>
<td>5</td>
<td>Much higher</td>
<td>The effect of one factor is much higher than the other one</td>
</tr>
<tr>
<td>7</td>
<td>Remarkably higher</td>
<td>The effect of one factor is remarkably higher than the other one</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely higher</td>
<td>Higher than the other factor possibly controllable</td>
</tr>
<tr>
<td>2/4/6/8</td>
<td>Median of the above degrees of importance</td>
<td>When comparing &quot;i&quot; and &quot;j&quot; and give them one of the scale values above, then the reciprocal of the scale should be the weight.</td>
</tr>
</tbody>
</table>

Fundamental theory of fuzzy mathematics: Fuzzy mathematics is a new discipline of mathematics. It was first introduced in 1965 by US computer and cybernetics professional L.A.Zadeh in a paper titled Fuzzy Sets which was published on Information and Control. It laid a foundation for classical mathematics and made a breakthrough in introducing computer science to natural mechanism. Fuzziness refers to a feature existed in transitional period of differences which is “this and that at the same time”. Fuzzy mathematics is a method used to study and handle this fuzziness. Fuzzy comprehensive evaluation method takes fuzzy mathematics as the basis and uses composition of fuzzy relation theory to quantize indefinite and non-quantitative factors for comprehensive evaluation. It contains a fuzzy set which is composed of multiple factors or indices (known as factor set U) and a fuzzy set of evaluation consisted of evaluation grade from which factor set chooses (known as judgment set V) (Hu, 2011).

Learning Evaluation Index System in Distance Education

Build evaluation index hierarchy model selecting a template: Evaluation to students’ learning is a key part of modern distance education evaluation, the essence of which is to evaluate learning effect of the students. By collecting and processing information concerning the learning process, quantitative analysis is done concerning students' learning attitude, behavior and effect based on instructional objective and then evaluation result is given. All evaluations must be done in the same evaluation index system, meaning that building scientific and feasible evaluations index system is of vital importance to learning evaluation. In this study, AHP is used to build hierarchy as shown in Diagram 1 to achieve distance learning evaluation from such perspectives as students’ collaboration and communication ability, using of learning resource, learning attitude and academic performance (Li et al., 2009; Huang and Taijun, 2010).

This model is composed of two levels of indices, of which collaboration and communication ability, using of learning resource, learning attitude and academic performance belong to primary index and are referred to as criteria level, expressed in $B_i$ ($i = 1, 2, 3, 4$); each primary index includes a number of $j$ secondary indices ($j = 1, 2, 3… 12$), expressed in $c_j$ (Fig. 1).

Building of judgment matrix: Building of judgment matrix is a key step for AHP. The process of building is actually a pairwise comparison of elements on the same hierarchy with respect to their priority in sequence. First, compare elements on the criteria hierarchy to one another two at a time and build relative importance judgment matrix; second, compare index factors under each criteria hierarchy to one another two at a time and build relative importance judgment matrix. In order to compare the elements to one another two at a time to get a judgment matrix, Satty's 1-9 scale method is going to be used for grading (Satty and Alexander, 2007). The content of scale method is shown in Table 1.
The basic problem of AHP is to solve the eigenvector judgment matrix, after being normalized, become the accurately complete these calculations in a short period of time. Consistency index $CI$, random consistency index $RI$ and consistency ratio $CR$ are introduced. The calculation formula is as follows:

**Consistency index:**

$$CI = \lambda_{\text{max}} n/n-1 \quad ("n" \text{ refers to order of matrix}) \quad (1)$$

**Consistency ratio:**

$$CR = CI/RI \quad (2)$$

The judgment matrix is fully consistent when $CR = 0$; satisfactory when $CR<0.1$; the consistency is extremely satisfactory when $CR> = 0.1$. The values of $RI$ are given in Table 4 (Chen and Shiping, 2012).

Take judgment matrix A as an example, the Matlab program for solving the largest eigenvalue and eigenvector is as Table 5.

The largest eigenvalues of the judgment matrix are: $\lambda_{\text{max}} = 4.0511, W = (0.0838, 0.1377, 0.2323, 0.5462)^T$, $CI = 0.017, CR = 0.018<0.1$, all of which meet consistency requirement. The weight of the four factors on the criteria hierarchy is $W = (0.0838, 0.1377, 0.2323, 0.5462)^T$. The result corresponding to the matrix on the criteria hierarchy can be calculated as follows:

- **B1 matrix:** $\lambda_{\text{max}} = 4.0104, W = (0.1891, 0.3509, 0.1091, 0.3509)^T$, $CI = 0.0034, CR = 0.0038<0.1$.
- **B2 matrix:** $\lambda_{\text{max}} = 3.0092, W = (0.2970, 0.5396, 0.1634)^T$, $CI = 0.0046, CR = 0.0079<0.1$.
- **B3 matrix:** $\lambda_{\text{max}} = 2, W = (0.3333, 0.6667)^T$, second-order matrix is full consistent.
- **B4 matrix:** $\lambda_{\text{max}} = 3.0092, W = (0.5396, 0.1634, 0.2970)^T$, $CI = 0.0046, CR = 0.0052<0.1$.

**Calculation of synthetic weight:** With the above calculations, we can obtain the weight of criteria hierarchy to target hierarchy and weight of index hierarchy to criteria hierarchy. The formula for weight of various index hierarchies to target hierarchy is:

$$a_i = \beta \times w_i \quad (3)$$

where, $\beta_i$ stands for weight of various factors on the criteria hierarchy to target hierarchy; $w_i$ stands for weight of various factors on the index hierarchy to criteria hierarchy. The specific weight for each index is shown in Table 6.

**FUZZY COMPREHENSIVE EVALUATION METHOD**

Establishment of evaluation index factor set and evaluation set (Zhang, 2007b):
particular course. Classify the ranking sequence into five

Table 6: Synthetic weight for various indices

<table>
<thead>
<tr>
<th>Index hierarchy</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>Synthetic weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.0838</td>
<td>0.1377</td>
<td>0.2323</td>
<td>0.5462</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>0.1891</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>0.3509</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>0.3509</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>0.2970</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>0.5396</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>0.1634</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C8</td>
<td>0.3333</td>
<td>0.077426</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C9</td>
<td>0.6667</td>
<td>0.154874</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C10</td>
<td>0.5396</td>
<td>0.29473</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td>0.1634</td>
<td>0.089249</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C12</td>
<td>0.2970</td>
<td>0.162221</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Evaluation grades and corresponding scores

<table>
<thead>
<tr>
<th>Range of score</th>
<th>Grade</th>
<th>Represented score</th>
</tr>
</thead>
<tbody>
<tr>
<td>90≤X&lt;100</td>
<td>Excellent</td>
<td>95</td>
</tr>
<tr>
<td>80≤X&lt;90</td>
<td>Good</td>
<td>85</td>
</tr>
<tr>
<td>70≤X&lt;80</td>
<td>Medium</td>
<td>75</td>
</tr>
<tr>
<td>60≤X&lt;70</td>
<td>Pass</td>
<td>65</td>
</tr>
<tr>
<td>X&lt;60</td>
<td>Fail</td>
<td>50</td>
</tr>
</tbody>
</table>

- Define primary index set as $B = (B_1, B_2, B_3, B_4)$ = (collaboration and communication ability, using of learning resources, learning attitude, performance), and corresponding weight set as $B_w = (B_1, B_2, B_3, B_4) = (0.0838, 0.1377, 0.2323, 0.5462)$.
- Define secondary index set as $C = (C_1, C_2,..., C_{11}, C_{12}) = (chatting room, blog……, homework, self-test), and corresponding weight set as $C_w = (C_1, C_2, ..., C_{11}, C_{12}) = (0.1891, 0.3509……0.1634, 0.2970)$.
- Define fuzzy evaluation set as $A = (a_1, a_2, a_3, a_4, a_5)$. Based on characteristics and requirements of distance education system, fuzzy numbers must be used to replace grades used by teachers. By making use of the currently used five-grade evaluation mode which consists of excellent, good, medium, pass, fail, a grade score matrix $G = (95, 85, 75, 65, 50)^t$ is established, as shown in Table 7.

Membership function of fuzzy evaluation matrix:
Rank all students based on their performance in a particular course. Classify the ranking sequence into five grades, namely (0~10%), (10~30%), (30~60%), (60~90%), (90~100%). Students’ ranking and their real level should meet normal distribution in a test. This is equivalent to defining a membership function by using fuzzy statistics method of degree of membership. We can use [0,1] interval to measure indefiniteness. Based on degree of membership, if a student’s rank ranges within (10~30%), then the possibility for his or her real level to fall within (10~30%) is 0.7 and 0.15 for (0~10%) and (30%~60%). Therefore, the student’s real level in the class can be described with vector (0.15, 0.7, 0.15, 0, 0) (Zhang et al., 2011; Wu, 2011).

**EVALUATION CASE**

Here's a description of fuzzy comprehensive evaluation with an example of a student's performance in a particular course, as shown in Table 8. The Table 8 shows that: Evaluation matrixes for primary indices are $R_1, R_2, R_3$ and $R_4$ as shown below:

$$R_1 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

Weights for evaluation indices are:

$$C_{1.4} = (0.1891, 0.3509, 0.1091, 0.3509)$$

$$C_{5.7} = (0.2970, 0.5396, 0.1634)$$

$$C_{8.9} = (0.3333, 0.6667)$$

$$C_{10.12} = (0.05396, 0.1634, 0.2970)$$

Calculate $T_1$ :

$$T_1 = C_{1.4} \times R_1 = (0.1891 \times 0.3509 \times 0.1091 \times 0.3509)$$

$$= \begin{bmatrix} 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \\ 0.15 & 0.7 & 0.15 & 0 & 0 \end{bmatrix}$$

After normalization of $T_1$:

$$T_1' = (0.09740.50700.34300.05260)$$

This indicates that possibility for this student’s collaborating and communication ability is 10% for being excellent, 51% for good, 34% for medium, 5% for pass and 0% for fail. Based on rank and score matrix, this student’s collaboration and communication ability $B_1$ in this course is:

$$B_1 = T_1' \times G = (0.09740.50700.34300.05260) \times \begin{bmatrix} 95 \\ 85 \\ 65 \\ 50 \end{bmatrix}$$

$$= 81.4910$$

It falls into the grade of good.
Table 8: A student’s performance in distance learning

<table>
<thead>
<tr>
<th>Primary index weight</th>
<th>Secondary index weight</th>
<th>Integral</th>
<th>Ranking</th>
<th>Percentage distribution (%)</th>
<th>Degree of membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collaboration and communication ability (B1)</td>
<td>Chat room (C1) 0.1891</td>
<td>78</td>
<td>16</td>
<td>31</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td></td>
<td>BLOG (C2) 0.3509</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td>0, 0.15, 0.7, 0.15, 0</td>
</tr>
<tr>
<td></td>
<td>BBS (C3) 0.1091</td>
<td>30</td>
<td>9</td>
<td>23</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td></td>
<td>E-Mail (C4) 0.3509</td>
<td>30</td>
<td>12</td>
<td>32</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td>Using of learning resource (B2) 0.1377</td>
<td>Learning time of courseware (C5) 0.2970</td>
<td>20</td>
<td>10</td>
<td>40</td>
<td>0, 0.15, 0.7, 0.15, 0</td>
</tr>
<tr>
<td></td>
<td>Learning time of teaching material (C6) 0.5396</td>
<td>12</td>
<td>5</td>
<td>10</td>
<td>0.85, 0.15, 0, 0, 0</td>
</tr>
<tr>
<td></td>
<td>Learning time of guidance material (C7) 0.1634</td>
<td>40</td>
<td>10</td>
<td>18</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td>Learning attitude (B3) 0.2323</td>
<td>Time to visit the website (C8) 0.3333</td>
<td>40</td>
<td>10</td>
<td>18</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td></td>
<td>On-line question and answer (C9) 0.6667</td>
<td>20</td>
<td>9</td>
<td>12</td>
<td>0.85, 0.15, 0, 0, 0</td>
</tr>
<tr>
<td>Academic performance (B4) 0.5462</td>
<td>On-line test (C10) 0.5396</td>
<td>80</td>
<td>13</td>
<td>32</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td></td>
<td>Assignment (C11) 0.1634</td>
<td>30</td>
<td>20</td>
<td>47</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
<tr>
<td></td>
<td>Self-test (C12) 0.2970</td>
<td>85</td>
<td>15</td>
<td>40</td>
<td>0.15, 0.7, 0.15, 0, 0</td>
</tr>
</tbody>
</table>

With the same method, we can get the scores for using of learning resources B2, learning attitude B3 and performance B4 as 78.0229, 90.6669, 80.396, respectively. Rank the score in primary index system again to gain Table 9.

We can get evaluation matrix A for overall performance of the student in this particular course from the Table 9:

\[
A = T \times G = (0.1880, 0.5782, 0.2132, 0.0207, 0)
\]

\[
\begin{bmatrix}
95 \\
85 \\
65 \\
50
\end{bmatrix}
\times \begin{bmatrix}
0.1880 \\
0.5782 \\
0.2132 \\
0.0207
\end{bmatrix} = \begin{bmatrix}
84.3353
\end{bmatrix}
\]

The student’s overall performance is good, which is end of evaluation.

**CONCLUSION**

This study uses Analytic Hierarchy Process (AHP) to build a practical multi-hierarchy evaluation model and Matlab software to solve matrix with better efficiency. It helps to evaluate students’ learning in distance education more quickly, accurately and scientifically.

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