

Research Article

Analysis of Bifurcation and Nonlinear Control for Chaos in Gear Transmission System

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Abstract: In order to study the bifurcation characteristics and control chaotic vibration of the gear transmission system. The complex dynamics characters of gear transmission system are studied. The dynamical equation and the state equation of gear transmission system are established according to Newton's rule. The route to chaos of the system is studied by the bifurcation diagram, phase portrait, time course diagram and Poincaré map. A method of controlling chaos by nonlinear feedback controller is developed to guide chaotic motions towards regular motions. Numerical simulation shows that with the increase of meshing stiffness, gear transmission system will be from the periodic motion to chaotic motion by doubling bifurcation, the effectiveness and feasibility of the strategy to get rid of chaos by stabilizing the related unstable periodic orbit.

Keywords: Bifurcation, chaos, chaos control, gear, poincaré map

INTRODUCTION

Gear transmission system is widely used in machinery, automobile, aerospace, robotics and other areas. The vibration characteristic of gear transmission system directly influences the system performance and reliability. Accordingly, people have been studied on a large number of theoretical analysis and test research of vibration characteristic in the gear transmission system for a long time and have achieved many important results. In recent years, with the development of nonlinear vibration theory, domestic and foreign scholars have done a lot of research on the nonlinear vibration of gear transmission system (Chang-Jian, 2010; Luo and O'Connor, 2009; Byrtus and Zeman, 2011; De Souza *et al.*, 2004; Wen and Feng, 2004).

The nonlinear factors, such as faults in meshing stiffness (Litak and Friswell, 2005), backlash (Theodossiades and Natsiavas, 2001), time-varying mesh parameters (Wang *et al.*, 2007), turbulent flow (Chang-Jian and Hsu, 2012), can cause the system to produce very strong nonlinear vibration, in certain parameter regions, the gear transmission system is in the form of chaotic motion, which has a great influence on working performance and reliability of the transmission system. Therefore, people need to take more effective and feasible control method to the gear transmission system. Scholars have done a lot of research on parametric vibration and backlash nonlinear vibration of the cylindrical gear transmission system, the bevel gear transmission

system and planetary gear transmission system, which mainly concentrated in the modeling method, solving method and system characteristic. However, research on how to effectively control chaos in the gear transmission system is relatively less.

This study discusses the forming process of bifurcation and chaos in the single degree of freedom spur gear transmission system and the use of nonlinear feedback control method for chaotic control. The method can be used to control the chaotic motion to a stable orbit of period 1, which verifies the feasibility and effectiveness of the method.

THE MECHANICAL MODEL AND DIFFERENTIAL EQUATION OF MOTION

The single degree of freedom spur gear transmission system is shown in Fig. 1 (Kahraman and Blankenship, 1997). The model only considers the torsional vibration of gear, ignoring the transverse and axial elastic deformation of transmission shaft, as well as the supporting system of elastic deformation and other factors. The lumped mass method was adopted to establish the torsional vibration model of gear pair.

In the graph, T_1 and T_2 are the torque for the role in active and passive gear, θ_1 and θ_2 are the torsional vibration displacement, I_1 and I_2 are the active and passive gear rotation inertia, R_{b1} and R_{b2} are the base circle radius of active and passive gear, $e(t)$ is the gear integrated error, $k(t)$ is the gear pair time-varying meshing stiffness, C_g is the gear pair meshing damping.

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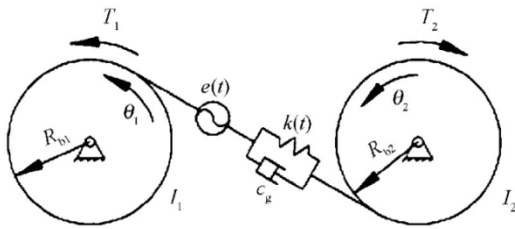


Fig. 1: Dynamic model of the system

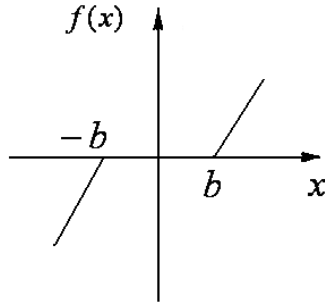


Fig. 2: Gear side clearance and force diagram

According to the Newton's rule, the dynamic equations of the system are given by:

$$\begin{cases} T_1 = I_1 \ddot{\theta}_1 + c_g R_{b1} (R_{b1} \dot{\theta}_1 - R_{b2} \dot{\theta}_2 - \dot{e}(t)) + k(t) R_{b1} f(R_{b1} \theta_1 - R_{b2} \theta_2 - e(t)) \\ T_2 = -I_2 \ddot{\theta}_2 + c_g R_{b2} (R_{b1} \dot{\theta}_1 - R_{b2} \dot{\theta}_2 - \dot{e}(t)) - k(t) R_{b2} f(R_{b1} \theta_1 - R_{b2} \theta_2 - e(t)) \end{cases} \quad (1)$$

The gear meshing force is mainly composed of the elastic meshing force caused by time-varying meshing stiffness, whereby the viscous meshing force caused by the gear dynamic load and meshing damping. With elastic meshing force P, viscous meshing force Q, the expression is:

$$\begin{cases} P = k(t) f(R_{b1} \theta_1 - R_{b2} \theta_2 - e(t)) \\ Q = c_g R_{b2} (R_{b1} \dot{\theta}_1 - R_{b2} \dot{\theta}_2 - \dot{e}(t)) \end{cases} \quad (2)$$

In order to eliminate the rigid body displacement, the system is treated by non-dimensional. Introduce the generalized coordinates relative of gear meshing line to the system. Suppose that:

$$x = (R_{b1} \theta_1 - R_{b2} \theta_2 - e(t)) / b_e \quad (3)$$

$$\omega_n = \sqrt{\frac{k_{av}}{m_e}}, m_e = \frac{m_1 m_2}{m_1 + m_2} \quad (4)$$

where,

- b_e = The given nominal size
- ω_n = The natural frequency of the system
- k_{av} = The average mesh stiffness of gear pair

- m_e = Equivalent mass
- m_1 and m_2 = The active and passive gear quality, respectively

Let $\tau = \omega_n t$, $\bar{b} = b/b_e$, $\omega_h = \omega_e/\omega_n$. where, $2b$ is the gear clearance, as shown in Fig. 2, \bar{b} is the dimensional gap, ω_h is dimensional frequency, ω_e is the meshing frequency of gear pair.

Clearance nonlinear function is:

$$f(x) = \begin{cases} x - \bar{b} & x > \bar{b} \\ 0 & -\bar{b} \leq x \leq \bar{b} \\ x + \bar{b} & x < -\bar{b} \end{cases} \quad (5)$$

Then Eq.1 is rewritten in non-dimensional form:

$$\ddot{x} + 2\zeta \dot{x} + \bar{k}(\tau) f(x) = \bar{F}_m + \bar{F}_h(\tau) \quad (6)$$

where the non-dimensional quantities $\zeta = \frac{c_g}{2m_e \omega_n}$, $\bar{k}(\tau) = \frac{k(\tau)}{m_e \omega_n^2}$, $\bar{F}_m = \frac{F_m}{m_e b_e \omega_n^2}$, $\bar{F}_h(\tau) = \frac{F_h(\tau)}{b_e}$ have been introduced. F_m is gear equivalent external incentive, $F_h(\tau)$ is equivalent internal error excitation caused by gear integrated error. In Eq. (1), (2) and (6), a dot (·) denotes differentiation with the non-dimensional time τ .

If the stiffness and gear meshing error take the first-order harmonic component, there are:

$$\bar{k}(\tau) = 1 + \varepsilon \cos(\omega_h \tau + \phi_h) \quad (7)$$

$$\bar{F}_h(\tau) = \bar{F}_e \omega_h^2 \cos(\omega_h \tau + \phi_h) \quad (8)$$

where the non-dimensional quantities $\varepsilon = k_a/k_{av}$, $\bar{F}_e = e_a/b_e$ have been introduced. k_a is the time-varying meshing stiffness of amplitude, e_a is error amplitude. Considering the relationship between meshing stiffness and meshing error, let meshing phase angle $\phi_h = \gamma_h + \pi$, γ_h is the variable stiffness initial phase, with $\gamma_h = 0$. Then Eq.2 is transformed into state equation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = F_m + F_e \cos(\omega_h t) - 2\zeta x_2 - [1 - \varepsilon \cos(\omega_h t)] f(x_1) \end{cases} \quad (9)$$

NONLINEAR DYNAMIC ANALYSIS

The gear transmission system, with parameters $F_m = 0.063$, $F_e = 0.1$, $\zeta = 0.34$, $\omega_h = 2.4$, $\bar{b} = 1$, has been chosen to be analyzed. The system parameter ε is taken as the bifurcation parameter. Supposing the initial point of the system is $x_1(0) = 0$, $x_2(0) = 0$. We applied fourth-order Runge-Kutta method to the numerical simulation of the gear transmission system, the local bifurcation diagram is illustrated in Fig. 3.

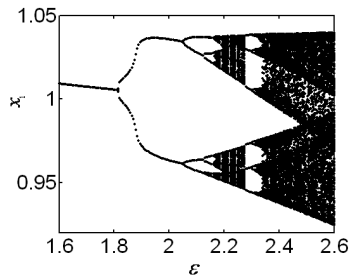


Fig. 3: Bifurcation diagram

From the above diagram, the initial movement of the system is the stable periodic 1 motion, as shown in Fig. 4. As the bifurcation parameter ε increasing, the system is periodic 2 motion via the period-doubling bifurcation. When $\varepsilon = 2.0$, the phase portrait, time course diagram and Poincaré map of the system are shown in Fig. 5. Then the system experiences a transition phenomenon. Figure 6 shows the system is a periodic 4 motion. The system is periodic 8 motion (Fig. 7) via the period-doubling bifurcation. Then the

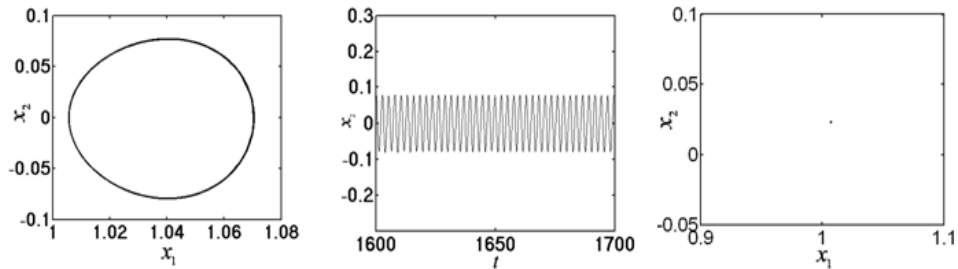


Fig. 4: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 1.7$)

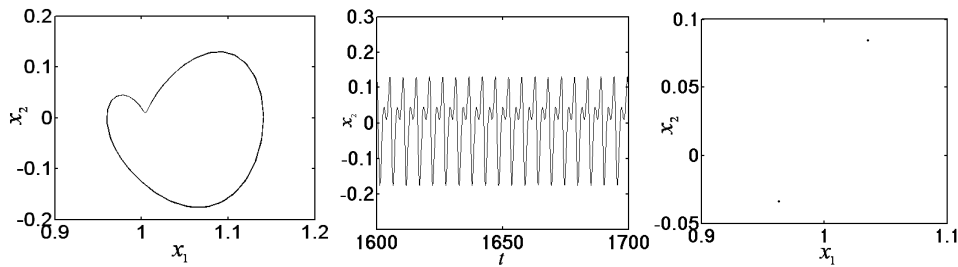


Fig. 5: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.0$)

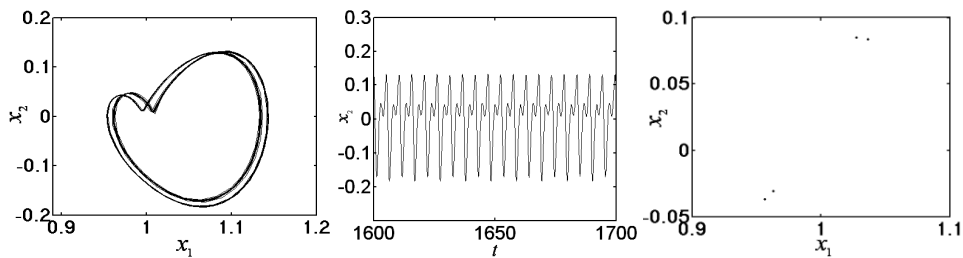


Fig. 6: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.1$)

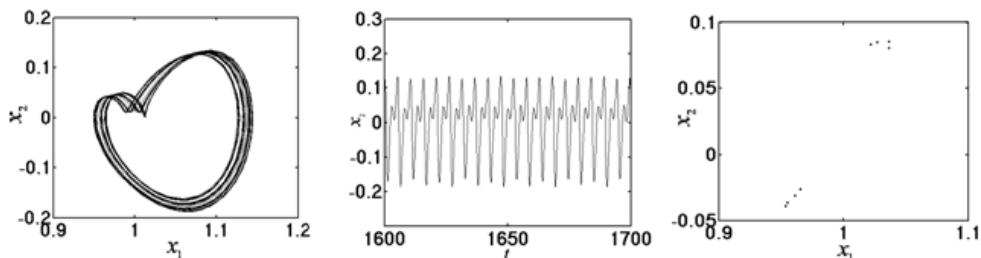


Fig. 7: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.15$)

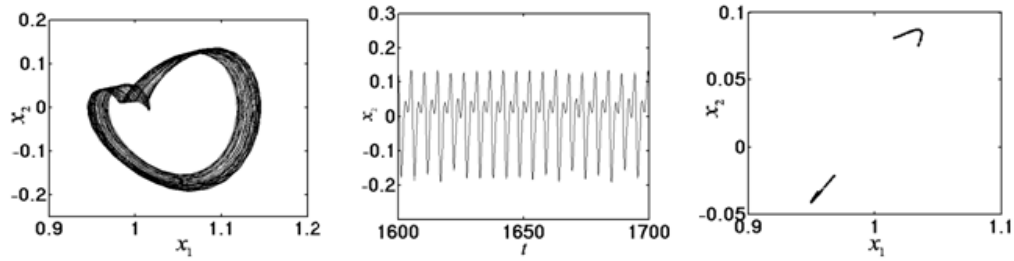


Fig. 8: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.21$)

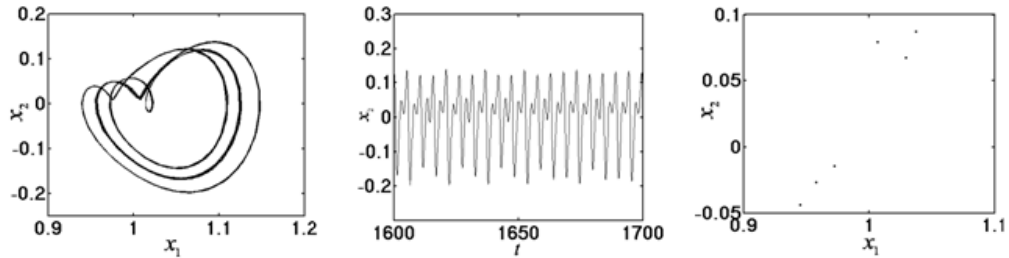


Fig. 9: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.28$)

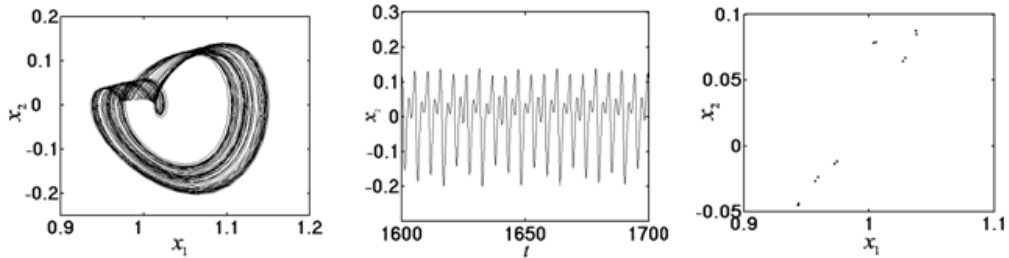


Fig. 10: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.31$)

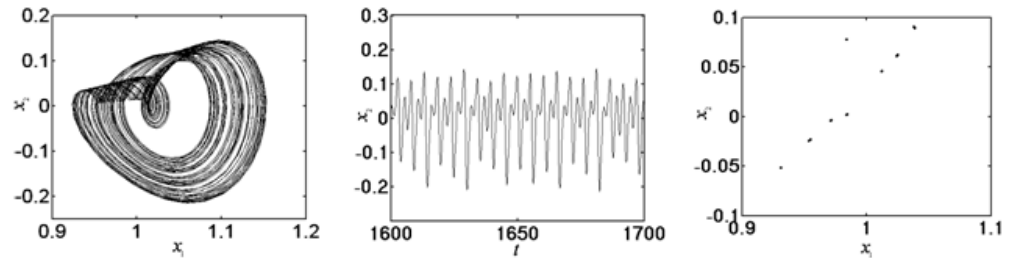


Fig. 11: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.492$)

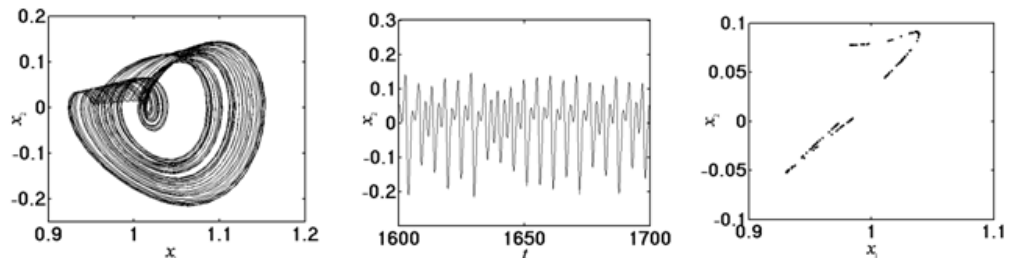


Fig. 12: Phase portrait, time course diagram and Poincaré map of the system ($\varepsilon = 2.15$)

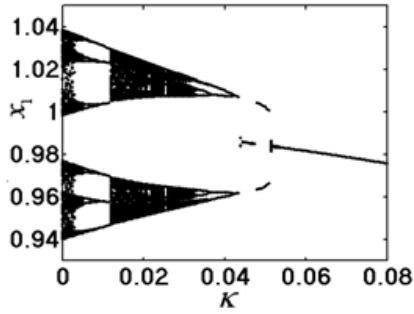


Fig. 13: Bifurcation diagram of the controlled system

system falls into chaotic motion (Fig. 8). With the increase of the bifurcation parameter \mathcal{E} , the system is from chaotic motion to the periodic 6 motion (Fig. 9) and then the periodic 12 motion via the period-doubling bifurcation (Fig. 10). When $\varepsilon = 2.492$, the system is phase locking as shown in Fig. 11. As \mathcal{E} is increased progressively, the system falls into chaotic motion again via period-doubling bifurcation (Fig. 12). Figure 8 and 12 show the chaos attractor morphology in different parameters. Although the system is in the state of chaos, but in different parameters, the chaos attractor morphology is not the same.

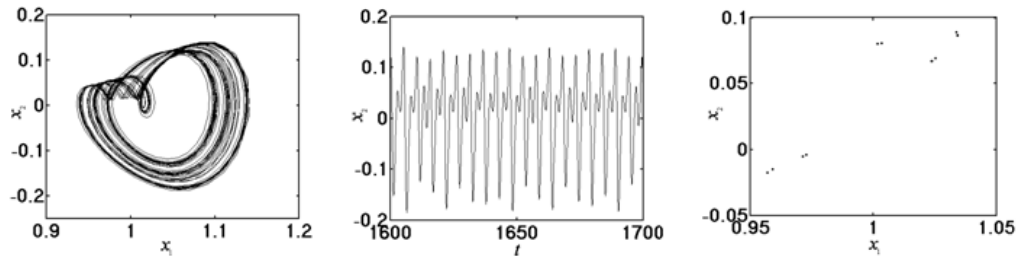


Fig. 14: Phase portrait, time course diagram and Poincaré map of the controlled system ($k = 0.006$)

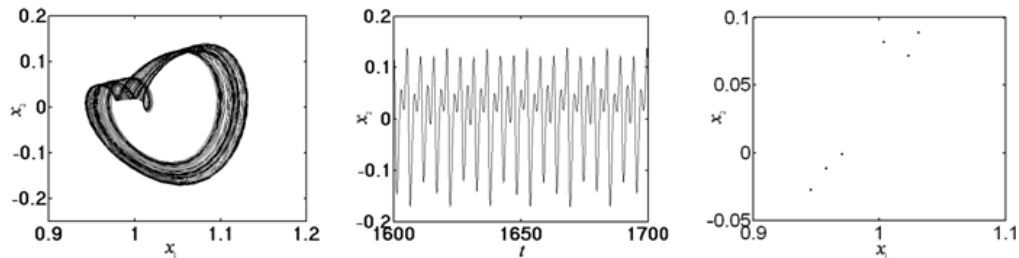


Fig. 15: Phase portrait, time course diagram and Poincaré map of the controlled system ($k = 0.01$)

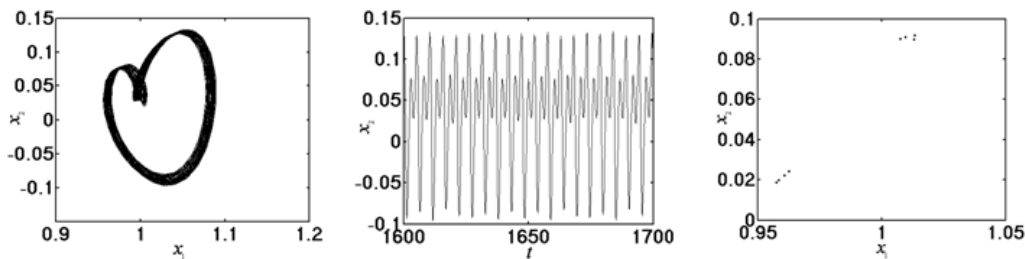


Fig. 16: Phase portrait, time course diagram and Poincaré map of the controlled system ($k = 0.035$)

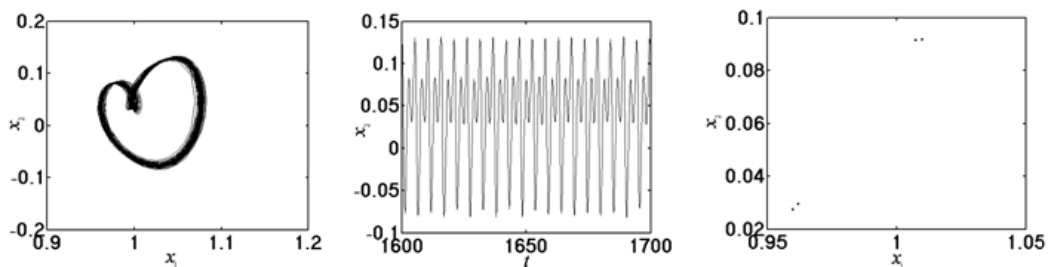


Fig. 17: Phase portrait, time course diagram and Poincaré map of the controlled system ($k = 0.04$)

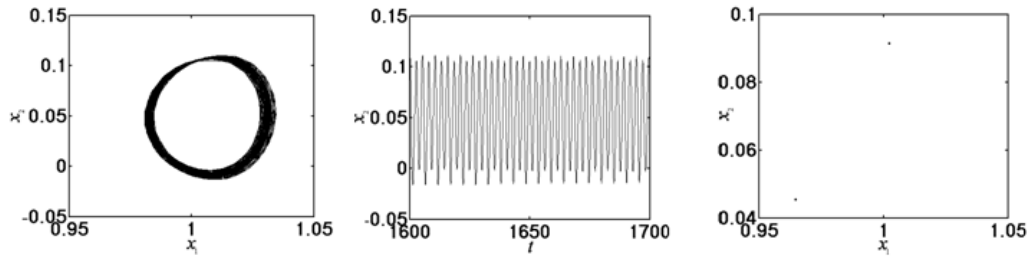


Fig. 18: Phase portrait, time course diagram and Poincaré map of the controlled system (k= 0.05)

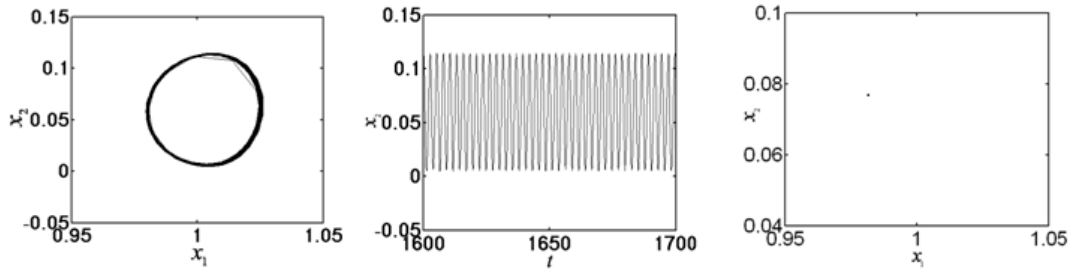


Fig. 19: Phase portrait, time course diagram and Poincaré map of the controlled system (k = 0.06)

CHAOS CONTROL

The definition of an n-dimensional non-linear chotic system as follow:

$$\begin{cases} \dot{x} = F(X(t),t) \\ \dot{y} = DX \end{cases} \quad (10)$$

where, F is a nonlinear smooth vector function, X is system state variable, $X = [x_1, x_2, \dots, x_n]^T$. y is the output of system. The constant matrix D is $1 \times n$. The nonlinear feedback controller of the system is $U = Ky|y|$. Where, K is a feedback gain matrix. The nonlinear feedback controller is added to the system. Then the controlled system is:

$$\dot{x} = F(X(x),t) - U \quad (11)$$

If $K = [k, 0]^T$, X is the system output, the equation of the controlled system is:

$$\begin{cases} \dot{x}_1 = x_2 - kx_1|x_1| \\ \dot{x}_2 = F_m + F_d \cos(\omega_n t) - 2\zeta x_2 - [1 - \varepsilon \cos(\omega_n t)]f(x_1) \end{cases} \quad (12)$$

When k = 0.25, the system is chaotic. After the introduction of the nonlinear feedback controller, using the fourth-order Runge-Kutta method of numerical simulation, the local bifurcation diagram of the controlled system is shown in Fig. 13. By selecting different control parameters, the chaotic motion of the system is effectively control to the stable periodic orbits. The phase portrait, time course diagram and Poincaré map of the controlled system are obtained, as shown in Fig. 14 to 19. For k = 0.006, the system is

controlled to periodic 12 motion. When k = 0.01, the system is controlled to periodic 6 motion. As k = 0.035, the system is controlled to periodic 8 motion. For k = 0.04, the system is controlled to periodic 4 motion. If k = 0.05, the system is controlled to periodic 2 motion. When k = 0.06, the system is controlled to periodic 1 motion.

CONCLUSION

In the study, considering the gear side clearance, time-varying mesh stiffness and gear error and other factors, a single-DOF spur gear transmission system mechanics model and dynamical equations are established according to the Newton's rule. Using fourth-order Runge-Kutta algorithm, the differential equations of motion of the gear transmission system are solved numerically. Chaos formation process is analyzed by bifurcation diagram, phase plane, time course diagram and Poincaré map of the system in time-varying meshing stiffness case. In order to improve the dynamic performance of the gear transmission system, to avoid the chaotic motion, this study uses nonlinear feedback control method to control the chaos in the gear transmission system. The chaotic motion of the system is effectively control to the stable periodic orbits. The research results in the reasonable choice of gear parameters and chaos control in practical engineering has certain guiding significance.

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