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Research Article

Covariance Intersection Fusion Kalman Estimators for Multi-Sensor System with Colored Measurement Noises

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Abstract: For multi-sensor system with colored measurement noises, using the observation transformation, the system can be converted into an equivalent system with correlated measurement noises. Based on this method, using the classical Kalman filtering, this study proposed a Covariance Intersection (CI) fusion Kalman estimator, which can handle the fused filtering, prediction and smoothing problems. The advantage of the proposed method is that it can avoid the computation of the cross-covariances among the local filtering errors and can reduce the computational burden significantly, as well as the CI fusion algorithm can be used in the uncertain system with unknown cross-covariances. Based on classical Kalman filtering theory, the centralized fusion and three weighted fusion (weighted by matrices, scalars and diagonal) estimators are also presented respectively. Their accuracy comparisons are given. The geometric interpretations based on covariance ellipses are also given. The experiment results show that the accuracy of the CI fuser is higher than that of the each local smoothers and is lower that that of the centralized fusion Kalman smoother or the optimal fuser weighted by matrix. The MSE curves show that the accuracy of the CI fuser is close to the optimal fuser weighted by matrix in most instances, which means that our proposed method has higher accuracy and good performance.

Keywords: Covariance intersection fusion, colored measurement noises, the centralized fusion, weighted fusion

INTRODUCTION

Multisensor information fusion filtering has been widely applied to many fields, including guidance, navigation, GPS positioning and so on. Now the commonly used method of information fusion is centralized and distributed fusion Kalman methods. The centralized fusion Kalman filters can give the globally optimal state estimation by directly combing all the local measurement equations, but the computation burden is larger. Distributed fusion Kalman filters are given by three weighted fusion algorithms with the matrix weights, scalar weights or diagonal matrix weights (Deng *et al*., 2005; Sun *et al*., 2010; Deng *et al*., 2012). Compared with the centralized fuser, the weighted fuser can reduce the calculation burden, but they are globally suboptimal.

The above weighting fusion filters require to calculate the cross-covariances of local filtering errors. However, in many theoretical and application problems, the cross-covariance is unknown, or the computation of the cross-covariances is very complicated (Sun *et al*., 2010).

In order to overcome the above drawback and limitation, the covariance intersection fusion Kalman method is presented in Julier and Uhlman (1997, 2009), which can avoid computing local cross-covariance and can solve the fused filtering problems for multi-sensor systems with unknown cross-covariance. The accuracy comparison in Deng *et al*. (2012) is given only for systems with uncorrelated white measurement noises. In this study, a Covariance Intersection (CI) fusion Kalman estimator is presented for multi-sensor system with colored measurement noises, whose accuracy is higher than that of each local Kalman estimator and is lower than that of the centralized fusion estimator or the optimal Kalman fuser weighted by matrices and the accuracy comparison is given.

PROBLEM FORMULATION

Consider a multi-sensor tracking system with colored measurement noises:

$$x(t+1)=\Phi x(t)+\Gamma w(t) \tag{1}$$

$$z_i(t)=\bar{H}_i x(t)+\eta_i(t),\ i=1,2\cdots,L \tag{2}$$

$$\eta_i(t+1)=P_i\eta_i(t)+\xi_i(t),\ i=1,2\cdots,L \tag{3}$$

where,

t = The discrete time
$x(t)\in R^n$ = The state
$z_i(t)\in R^{m_i}$ = The measurement of the i th sensor

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$w(t) \in R^{e}$, $\xi_i(t) \in R^{m_i}$ = White noises with zero mean and variance Q and $Q_{\xi i}$, respectively

$\eta_i(t) \in R^{m_i}$ = The colored measurement noise of the i th sensor

$\Phi, \Gamma, \bar{H}_i, P_i$ = The known appropriate dimension constant matrices

From Eq. (2) introducing the observation transformation:

$$y_i(t) = z_i(t+1) - P_i z_i(t) \tag{4}$$

we have the new measurement equation:

$$y_i(t) = H_i x(t) + v_i(t) \tag{5}$$

$$H_i = \bar{H}_i \Phi - P_i \bar{H}_i \tag{6}$$

$$v_i(t) = \bar{H}_i \Gamma w(t) + \xi_i(t) \tag{7}$$

From (7) we easily obtain w(t) and $v_i(t)$ are correlated noise with zeros mean and variance:

$$R_i = \bar{H}_i \Gamma Q \Gamma^{\mathrm{T}} \bar{H}_i^{\mathrm{T}} + Q_{\xi i} \tag{8}$$

$$R_{ij} = \mathrm{E}\left[v_i(t) v_j^{\mathrm{T}}(t)\right] = \bar{H}_i \Gamma Q \Gamma^{\mathrm{T}} \bar{H}_j^{\mathrm{T}} \tag{9}$$

$$S_i = \mathrm{E}\left[w(t) v_i^{\mathrm{T}}(t)\right] = Q \Gamma^{\mathrm{T}} \bar{H}_i^{\mathrm{T}} \tag{10}$$

where, E is the mathematical expectation operator and the superscript T denotes the transpose.

Based on the observation transformation, the system with colored noises (1)-(3) is transformed into the system with correlated measurement noises (1) and (5). The aim is to find the local, centralized fusion, three weighted fusion and CI fusion smoothers $\hat{x}_i(t|t+N)$ i = 1, 2, …, L, c, m, s, d, CI, N>0. And compare their accuracies.

The local steady-state kalman filters: Assume that the system (1) and (5) is completely observable and completely stable, then the local steady-state Kalman filter of the i th sensor is given as:

$$\hat{x}_i(t+1|t+1) = \Psi_{fi} \hat{x}_i(t|t) + \left[I_n - K_{fi} H_i\right] J_i y_i(t) + K_{fi} y_i(t+1) \tag{11}$$

$$\Psi_{fi} = \left[I_n - K_{fi} H_i\right] \bar{\Phi}_i, \quad \bar{\Phi}_i = \Phi - J_i H_i \tag{12}$$

$$J_i = \Gamma S_i R_i^{-1}, K_{fi} = \Sigma_i H_i^{\mathrm{T}} \left(H_i \Sigma_i H_i^{\mathrm{T}} + R_i\right)^{-1} \tag{13}$$

where, Σ_i is the one-step predicting error variance and satisfies the steady-state Riccati equation:

$$\Sigma_i = \bar{\Phi}_i \left[\Sigma_i - \Sigma_i H_i^{\mathrm{T}} \left(H_i \Sigma_i H_i^{\mathrm{T}} + R_i\right)^{-1} H_i \Sigma_i\right] \bar{\Phi}_i^{\mathrm{T}} + \Gamma\left(Q - S_i R_i^{-1} S_i^{\mathrm{T}}\right) \Gamma^{\mathrm{T}} \tag{14}$$

The local filtering error variances are given as:

$$P_i(0) = \left[I_n - K_{fi} H_i\right] \Sigma_i, i = 1, 2 \cdots, L \tag{15}$$

and the local filtering error cross-covariance satisfies the Lyapunov equation:

$$P_{ij}(0) = \Psi_{fi} P_{ij}(0) \Psi_{fj}^{\mathrm{T}} + \Delta_{fij}, i, j = 1, \cdots, L, \ i \neq j \tag{16}$$

$$\begin{aligned}\Delta_{fij} = {} & \Psi_{fi} K_{fi} \left[R_{ij} J_j^{\mathrm{T}} - S_i^{\mathrm{T}} \Gamma^{\mathrm{T}}\right]\left[I_n - K_{fj} H_j\right]^{\mathrm{T}} \\ & + \left[I_n - K_{fi} H_i\right]\left[J_i R_{ij} - \Gamma S_j\right] K_{fj}^{\mathrm{T}} \Psi_{fj}^{\mathrm{T}} \\ & + \left[I_n - K_{fi} H_i\right]\left[\Gamma Q \Gamma^{\mathrm{T}} - J_i S_i^{\mathrm{T}} \Gamma^{\mathrm{T}} - \Gamma S_j J_j^{\mathrm{T}} \right. \\ & \left. + J_i R_{ij} J_j^{\mathrm{T}}\right] \times \left[I_n - K_j H_j\right]^{\mathrm{T}} + K_{fi} R_{ij} K_{fj}^{\mathrm{T}}\end{aligned} \tag{17}$$

The local steady-state kalman predictors: For multi-sensor system (1) and (5), the local steady-state Kalman one-step predictor of the i th sensor is given as:

$$\hat{x}_i(t+1|t) = \Psi_{pi} \hat{x}(t|t-1) + K_{pi} y_i(t) \tag{18}$$

$$\Psi_{pi} = \Phi - K_{pi} H_i, \ K_{pi} = \left(\Phi \Sigma_i H_i^{\mathrm{T}} + \Gamma S_i\right) Q_{\xi i}^{-1} \tag{19}$$

$$Q_{\varepsilon i} = H_i \Sigma_i H_i^{\mathrm{T}} + R_i \tag{20}$$

The local one-step predictor error variances are computed by Eq. (14)

The local one-step predictor error cross-covariance satisfies the Lyapunov equation:

$$\Sigma_{ij} = \Psi_{pi} \Sigma_{ij} \Psi_{pj}^{\mathrm{T}} + \Delta_{pij}, \ \ i, j = 1, \cdots, L, \ i \neq j \tag{21}$$

$$\Delta_{pij} = \begin{bmatrix}\Gamma & -K_{pi}\end{bmatrix} \begin{bmatrix} Q & S_j \\ S_i^{\mathrm{T}} & R_{ij}\end{bmatrix} \begin{bmatrix}\Gamma^{\mathrm{T}} \\ -K_{pj}^{\mathrm{T}}\end{bmatrix} \tag{22}$$

and the N-step predictor of the i th sensor is:

$$\hat{x}_i(t|t+N) = \Phi^{-N-1} \hat{x}_i(t+N+1|t+N), N \leq -2 \tag{23}$$

The N-step predictor error variances and cross-covariance are given as the following formula:

$$P_i(N) = \Phi^{-N-1} \Sigma_i \left(\Phi^{-N-1}\right)^{\mathrm{T}} + \sum_{k=0}^{-N-2} \Phi^k \Gamma Q \Gamma^{\mathrm{T}} \left(\Phi^k\right)^{\mathrm{T}}, \quad N \leq -2 \tag{24}$$

$$P_{ij}(N) = \Phi^{-N-1} \Sigma_{ij} \left(\Phi^{-N-1}\right)^{\mathrm{T}} + \sum_{k=0}^{-N-2} \Phi^k \Gamma Q \Gamma^{\mathrm{T}} \left(\Phi^k\right)^{\mathrm{T}}, \ \mathrm{N} \leq -2 \tag{25}$$

where the Σ_i and Σ_{ij} are the one-step error variances and cross-covariance which are obtained by the Eq. (14) and (21).

The local steady-state kalman smoothers: For multi-sensor system (1) and (5), the local steady-state Kalman N-step smoother of the i th sensor is given as:

$$\hat{x}_i(t|t+N)=\hat{x}_i(t|t-1)+\sum_{k=0}^{N}K_i(k)\varepsilon_i(t+k), N>0, i=1,2\cdots,L \tag{26}$$

$$K_i(k)=\Sigma_i\Psi_{pi}^{\mathrm{T}k}H_i^{\mathrm{T}}Q_{\varepsilon i}^{-1}, k=0,\cdots,N \tag{27}$$

where, we define that $\Psi_{pi}^{Tk}=(\Psi_{pi}^{T})k$, $\sum_i$ is the one-step predicting error variance and $\hat{x}_i(t|t-1)$ is the local steady-state one-step predictor.

N-step error variance matrix and error cross-covariance of smother are obtained by:

$$P_i(N)=\Sigma_i-\sum_{k=0}^{N}K_i(k)Q_{\varepsilon i}K_i^{\mathrm{T}}(k), N>0 \tag{28}$$

$$\begin{aligned}P_{ij}(N)=\Sigma_{ij}&-\sum_{r=0}^{N}K_i(r)H_i\Psi_{pi}^{r}\Sigma_{ij}-\sum_{s=0}^{N}\Sigma_{ij}\Psi_{pj}^{\mathrm{T}s}H_j^{\mathrm{T}}K_j^{\mathrm{T}}(s)\\&+\sum_{r=0}^{N}\sum_{s=0}^{N}K_i(r)E_{ij}(r,s)K_j^{\mathrm{T}}(s), N>0\end{aligned} \tag{29}$$

where, $\sum_{ij}$ is the one-step predicting error cross-covariance and $E_{ij}(r,s)=\mathrm{E}[\varepsilon_1(t+r)\varepsilon_2^T(t+s)]$. when min (r, s)>0, we have:

$$\begin{aligned}E_{ij}(r,s)=H_i\Psi_{pi}^{r}\Sigma_{ij}\Psi_{pj}^{\mathrm{T}s}H_j^{\mathrm{T}}+\sum_{k=1}^{\min(r,s)}H_i\Psi_{pi}^{r-k}\begin{bmatrix}\Gamma & -K_{pi}\end{bmatrix}\\\times\begin{bmatrix}Q & S_j\\S_i^{\mathrm{T}} & R_{ij}\end{bmatrix}\begin{bmatrix}\Gamma^{\mathrm{T}}\\-K_{pj}^{\mathrm{T}}\end{bmatrix}\Psi_{pj}^{\mathrm{T}(s-k)}H_j^{\mathrm{T}}+R_{ij}\delta_{rs}\end{aligned} \tag{30}$$

when min (r, s) = 0:

$$E_{ij}(0,0)=H_i\Sigma_{ij}H_j^{\mathrm{T}}+R_{ij} \tag{31}$$

$$E_{ij}(r,0)=H_i\Psi_{pi}^{r}\Sigma_{ij}H_j^{\mathrm{T}}+H_i\Psi_{pi}^{r-1}\left[\Gamma S_j-K_{pi}R_{ij}\right] \tag{32}$$

$$E_{ij}(0,s)=H_i\Sigma_{ij}\Psi_{pj}^{\mathrm{T}s}H_j^{\mathrm{T}}+\left[S_i^{\mathrm{T}}\Gamma^{\mathrm{T}}-R_{ij}K_{pj}^{\mathrm{T}}\right]\Psi_{pj}^{\mathrm{T}(s-1)}H_j^{\mathrm{T}} \tag{33}$$

The centralized fusion steady-state kalman estimators: Introducing the augmented measurement equation:

$$y_c(t)=H_cx(t)+v_c(t) \tag{34}$$

with the definitions:

$$y_c(t)=\left[y_1^{\mathrm{T}}(t),\cdots,y_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{35}$$

$$H_c=\left[H_1^{\mathrm{T}},\cdots,H_L^{\mathrm{T}}\right]^{\mathrm{T}} \tag{36}$$

$$v_c(t)=\left[v_1^{\mathrm{T}}(t),\cdots,v_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{37}$$

For the system (1) and (34), the centralized fusion steady-state Kalman filter is given as:

$$\begin{aligned}\hat{x}_c(t+1|t+1)=\Psi_{fc}\hat{x}_c(t|t)+\left[I_n-K_{fc}H_c\right]J_cy_c(t)\\+K_{fc}y_c(t+1)\end{aligned} \tag{38}$$

$$\Psi_{fc}=\left[I_n-K_{fc}H_c\right]\bar{\Phi}_c, \bar{\Phi}_c=\Phi-J_cH_c \tag{39}$$

$$J_c=\Gamma S_cR_c^{-1}, K_{fc}=\Sigma_cH_c^{\mathrm{T}}\left[H_c\Sigma_cH_c^{\mathrm{T}}+R_c\right]^{-1} \tag{40}$$

$$R_c=\begin{bmatrix}R_1 & \cdots & R_{1L}\\\vdots & \ddots & \vdots\\R_{L1} & \cdots & R_L\end{bmatrix}, S_c=\left[S_1,\cdots,S_L\right] \tag{41}$$

where, $\sum_c$ satisfies the steady-state Riccati equation:

$$\begin{aligned}\Sigma_c=\bar{\Phi}_c\left[\Sigma_c-\Sigma_cH_c^{\mathrm{T}}\left(H_c\Sigma_cH_c^{\mathrm{T}}+R_c\right)^{-1}H_c\Sigma_c\right]\bar{\Phi}_c^{\mathrm{T}}\\+\Gamma\left(Q-S_cR_c^{-1}S_c^{\mathrm{T}}\right)\Gamma^{\mathrm{T}}\end{aligned} \tag{42}$$

The centralized fusion error variance is given by:

$$P_c(0)=\left[I_n-K_cH_c\right]\Sigma_c \tag{43}$$

For the system (1) and (34), the centralized fusion steady-state Kalman N-step predictor is given as:

$$\hat{x}_c(t|t+N)=\Phi^{-N-1}\hat{x}_c(t+N+1|t+N), N\le-2 \tag{44}$$

$$\Psi_{pc}=\bar{\Phi}_c-\bar{K}_{pc}H_c, \bar{K}_{pc}=\bar{\Phi}_cK_{fc} \tag{45}$$

$$P_c(N)=\Phi^{-N-1}\Sigma_c\left(\Phi^{-N-1}\right)^{\mathrm{T}}+\sum_{k=0}^{-N-2}\Phi^k\Gamma Q\Gamma^{\mathrm{T}}\left(\Phi^k\right)^{\mathrm{T}}, N\le-2 \tag{46}$$

For the system (1) and (34), the centralized fusion steady-state Kalman smoother is given as:

$$\hat{x}_c(t|t+N)=\hat{x}_c(t|t-1)+\sum_{k=0}^{N}K_c(k)\varepsilon_c(t+k), N>0 \tag{47}$$

$$K_c(k)=\Sigma_c\Psi_{pc}^{\mathrm{T}k}H_c^{\mathrm{T}}Q_{\varepsilon c}^{-1}, k=0,\cdots,N \tag{48}$$

$$\Psi_{pc}=\Phi-K_{pc}H_c \tag{49}$$

$$K_{pc}=\left(\Phi\Sigma_cH_c^{\mathrm{T}}+\Gamma S_c\right)Q_{\xi c}^{-1} \tag{50}$$

$$Q_{\varepsilon c}=H_c\Sigma_cH_c^{\mathrm{T}}+R_c \tag{51}$$

$$R_c=\begin{bmatrix}R_1 & \cdots & R_{1L}\\\vdots & \ddots & \vdots\\R_{L1} & \cdots & R_L\end{bmatrix}, S_c=\left[S_1,\cdots,S_L\right] \tag{52}$$

where, $\hat{x}_c(t|t-1)$ is the certralized fusion steady-state one step predictor:

$$\hat{x}_c(t|t-1) = \Psi_{pc}\hat{x}_c(t-1|t-2) + K_{pc}y_c(t-1) \tag{53}$$

The fused error variance is given by:

$$P_c(N) = \Sigma_c - \sum_{k=0}^{N} K_c(k) Q_{\varepsilon c} K_c^{\mathrm{T}}(k) \tag{54}$$

The three weighted fusion steady-state kalman estimators: When the local smoothing error variance Pi(N) (i = 1, …, L) and cross-covariances $P_{ij}(N)$ (i≠j) are exactly known, the three weighted fusion steady-state Kalman estimators are given as follows:

The optimal fusion estimator weighted by matrices is:

$$\hat{x}_m(t|t+N) = \sum_{i=1}^{L} \Omega_i \hat{x}_i(t|t+N), \quad N=0, N<0, N>0 \tag{55}$$

with the constraints $\sum_{i=1}^{L} \Omega_i = I_n$.
The optimal matrix weights are given by:

$$[\Omega_1, \cdots, \Omega_L] = (e^{\mathrm{T}} P^{-1} e)^{-1} e^{\mathrm{T}} P^{-1} \tag{56}$$

where, $e^{\mathrm{T}} = [I_n, \ldots, I_n]$, $P = (P_{ij}(N))_{nL\times Ln}$ the optimal fused error variance matrix is:

$$P_m(N) = (e^{\mathrm{T}} P^{-1} e)^{-1} \tag{57}$$

The optimal fusion smoother weighted by scalars is:

$$\hat{x}_s(t|t+N) = \sum_{i=1}^{L} \omega_i \hat{x}_i(t|t+N) \tag{58}$$

with the constraints $\sum_{i=1}^{L} \omega_i = 1$.
The optimal scalar weights are given by:

$$[\omega_1, \cdots \omega_L] = (e^{\mathrm{T}} P_{\mathrm{tr}}^{-1} e)^{-1} e^{\mathrm{T}} P_{\mathrm{tr}}^{-1} \tag{59}$$

where, the symbol tr denotes the trace of matrix and $e^{\mathrm{T}} = [1, \ldots, 1]$, $P_{\mathrm{tr}} = (\mathrm{tr}P_{ij}(N))_{L\times L}$.

The optimal fused error variance is given as:

$$P_s(N) = \sum_{i=1}^{L}\sum_{j=1}^{L} \omega_i \omega_j P_{ij}(N) \tag{60}$$

The optimal fusion smoother weighted by diagonal matrix is:

$$\hat{x}_d(t|t+N) = \sum_{i=1}^{L} A_i \hat{x}_i(t|t+N) \tag{61}$$

with the constraints:

$$A_i = diag(a_{i1}, \cdots, a_{in}), \quad \sum_{i=1}^{L} a_{ij} = 1, j = 1, \cdots, n \tag{62}$$

The optimal diagonal matrix weights are given by:

$$[a_{1j}, \cdots, a_{Lj}] = \left[e^{\mathrm{T}} (P^{ii})^{-1} e\right]^{-1} e^{\mathrm{T}} (P^{ii})^{-1} \tag{63}$$

where $e^{\mathrm{T}} = [1, \ldots, 1]$, $P^{ii} = P^{ii} = (P_{sk}^{ii})_{L\times L}$, $s, k = 1, \cdots, L$, P_{sk}^{ii} is the (i,i) diagonal element of P_{sk} and the fused error variance is given by:

$$P_d(N) = \sum_{i=1}^{L}\sum_{j=1}^{L} A_i P_{ij}(N) A_j^{\mathrm{T}} \tag{64}$$

The Covariance Intersection (CI) fusion steady-state kalman smoother: When the local smoothing error variance $P_i(i = 1, \cdots, L)$ are exactly known, but the cross-covariances $P_{ij}, (i \neq j)$ are unknown, using the CI fusion algorithm ,the CI fusion Kalman smoother without cross-covariances is presented as follows:

$$\hat{x}_{CI}(t|t+N) = P_{CI}(N) \sum_{i=1}^{L} \omega_i P_i^{-1}(N) \hat{x}_i(t|t+N) \tag{65}$$

$$P_{CI}(N) = \left[\sum_{i=1}^{L} \omega_i P_i^{-1}(N)\right]^{-1} \tag{66}$$

with the constraints $\sum_{i=1}^{L} \omega_i = 1$, $\omega_i \geq 0$, where, the optimal weighted coefficients $\omega_i(1, \ldots, L)$ are determined by minimizing the performance index such that:

$$\min_{\omega_i} \mathrm{tr}\, P_{CI}(N) = \min_{\substack{\omega_i \in [0,1] \\ \omega_1 + \cdots + \omega_L = 1}} \mathrm{tr}\left\{\left[\sum_{i=1}^{L} \omega_i P_i^{-1}(N)\right]^{-1}\right\} \tag{67}$$

This is a nonlinear optimization problem with constraints in Euclidean space R^L, it can be solved by “fmincon” function in MATLAB toolbox.

The cross-covariances can be obtained by the local steady-state Kalman smoothing formula (29), so the actual fused error variance $\bar{P}_{CI}(N)$ is given by:

$$\begin{aligned} \bar{P}_{CI}(N) &= \mathrm{E}\left[\tilde{x}_{CI}(t|t+N)\tilde{x}_{CI}^{\mathrm{T}}(t|t+N)\right] \\ &= P_{CI}(N)\left[\sum_{i=1}^{L}\sum_{j=1}^{L} \omega_i P_i^{-1}(N) P_{ij}(N) P_j^{-1}(N) \omega_j\right] P_{CI}(N) \end{aligned} \tag{68}$$

Form Eq. (66) the $P_{CI}(N)$ only depend on the local error variance P_i ($i = 1, \ldots, L$), but is independent on cross-covariances P_{ij}, (i ≠ j). Form (68) the actual error variance $\bar{P}_{CI}(N)$ not only depend on the local error variance $P_i(i = 1, \ldots, L)$ but also dependent on cross-covariances P_{ij}, $(i \neq j)$. So $P_{CI}(N)$ is the common upper bound of the actual fused error variance.

The accuracy comparison of local and fused kalman smoothers: For the multi-sensor system (1)and (5)with exactly known local error variances $P_i(i = 1, \ldots, L)$ and cross-covariance P_{ij}, $(i \neq j)$, the accuracy relations based on error covariance matrix and their trace are given as:

$$\operatorname{tr} P_c(N) \le \operatorname{tr} P_m(N) \le \operatorname{tr} P_d(N) \le \operatorname{tr} P_s(N) \le \operatorname{tr} P_i(N) \quad (69)$$

$$\operatorname{tr} P_c(N) \le \operatorname{tr} P_m(N) \le \operatorname{tr} \bar{P}_{CI}(N) \le \operatorname{tr} P_{CI}(N) \le \operatorname{tr} P_i(N) \quad (70)$$

$$P_c(N) \le P_m(N) \le \bar{P}_{CI}(N) \le P_{CI}(N) \quad (71)$$

$$P_m(N) \le P_d(N), P_m(N) \le P_s(N), P_m(N) \le P_i(N) \quad (72)$$

where, the matrix inequality $A \le B$ means that $B-A \ge 0$ is positive semi-definite.

Remark1: the accuracy of the local and fused smoothers is defined as the trace of their error variances, the smaller trace means higher accuracy and the larger trace means lower accuracy.

SIMULATION RESULTS

Consider the 3-sensor tracking system with colored measurement noises:

$$x(t+1) = \Phi x(t) + \Gamma w(t) \quad (73)$$

$$z_i(t) = \bar{H}_i x(t) + \eta_i(t), \; i = 1,2,3 \quad (74)$$

$$\eta_i(t+1) = P_i \eta_i(t) + \xi_i(t), i = 1,2,3 \quad (75)$$

In the simulation, we take:

T0 = 0.2, $\Phi = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}$, $\bar{H}_1 = [1\ 0]$, $\bar{H}_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\bar{H}_3 = [1\ 0]$, N = 0, −2, 2, $P_1 = 0.3$, P_2 = diag (0.16, 0.3), P3 = 0.4, t = 1, …, 300, Q = 1, $Q_{\zeta 1} = 1$, $Q_{\zeta 2}$ = diag (2, 0.15), $Q_{\zeta 1} = 0.49$

The accuracy comparison of the local, the centralized fuser, three weighted fusers and the CI fuser are compared in Table 1.

Form Table 1, we see that the accuracy of the centralized fuser $trP_c(N)$ is the highest, the actual accuracy $tr\bar{P}_{CI}(N)$ is higher than the local smoothers, but lower than the optimal fusion smoother weighted by matrix $trP_m(N)$. The accuracy weighted by scalars $trP_s(N)$ is close to that weighted by diagonal matrix $trP_d(N)$ and both of them are lower than the accuracy weighted by matrix. Table 1 verifies the accuracy relations (69) and (70).

In order to give a geometric interpretation of the accuracy relations, the variance ellipse is defined as the locus of points $\{x: x^TP^{-1}x = c\}$, where P is the variance matrix and c is a constant. Generally, we select c = 1. It has been proved in Deng *et al*. (2012) that $P_1 \le P_2$ is equivalent to that the variance ellipse of P_1 is enclosed in that of P_2 .the accuracy comparison of the variance ellipses is shown in Fig. 1-3.

Form Fig. 1-3, we see that the covariance ellipse of $P_m(N)$ is enclosed in the ellipse of $\bar{P}_{CI}(N)$ and the ellipse of $\bar{P}_{CI}(N)$ is enclosed in that of $P_{CI}(N)$. The ellipse of $P_s(N)$ and $P_d(N)$ encloses the ellipse of $P_m(N)$. the ellipse of $P_c(N)$ is enclosed in all ellipses for $P_i(N)$, i

Table 1: The accuracy comparison of the local and fused Kalman smoothers

$trP_1(-2)$	$trP_2(-2)$	$trP_3(-2)$	$trP_c(-2)$	$trP_m(-2)$
0.83743	0.75414	0.64807	0.2902	0.31242
$trP_d(-2)$	$trP_s(-2)$	$tr\bar{P}_{CI}(-2)$	$trP_{CI}(-2)$	
0.35446	0.39103	0.41673	0.60245	
$trP_1(0)$	$trP_2(0)$	$trP_3(0)$	$trP_c(0)$	$trP_m(-2)$
0.57428	0.61503	0.43132	0.18285	0.20153
$trP_d(0)$	$trP_s(0)$	$tr\bar{P}_{CI}(0)$	$trP_{CI}(-2)$	
0.21842	0.25805	0.2703	0.4048	
$trP_1(2)$	$trP_2(2)$	$trP_3(2)$	$trP_c(-2)$	$trP_m(2)$
0.40457	0.54129	0.29834	0.13706	0.152
$trP_d(0)$	$trP_s(0)$	$tr\bar{P}_{CI}(0)$	$trP_{CI}(0)$	
0.1564	0.18525	0.20226	0.27976	

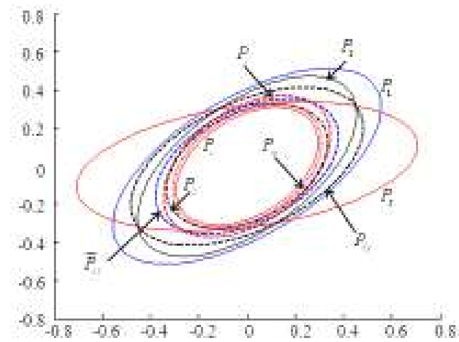

Fig. 1: The variance ellipses of filters

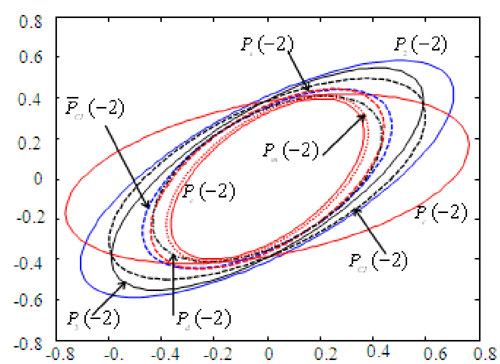

Fig. 2: The variance ellipses of predictors

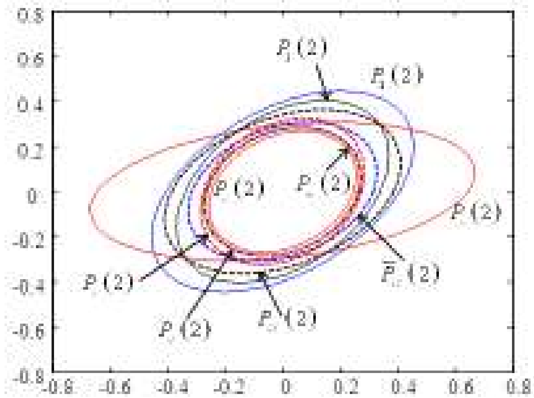

Fig. 3: The variance ellipses of smoothers

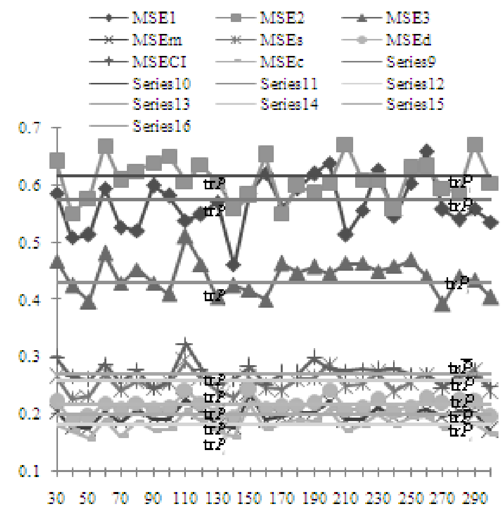

Fig. 4: The MSE curves of the local and fused filters

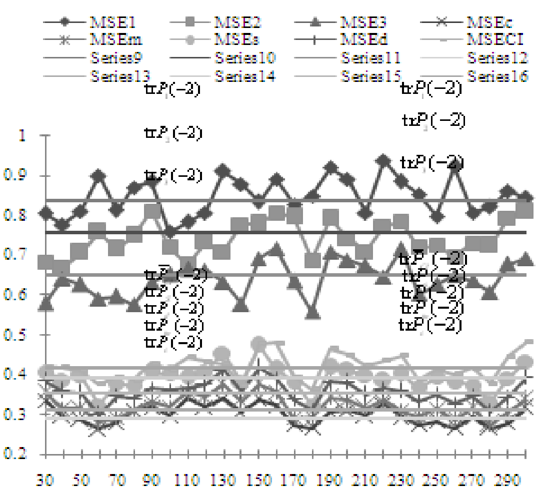

Fig. 5: The MSE curves of the local and fused predictors

= 1, 2, 3, P_θ(N), m, s, d, $\bar{P}_{CI}(N)$ and P_{CI}(N), which verifies the correctness of matrix relations (71) and (72).

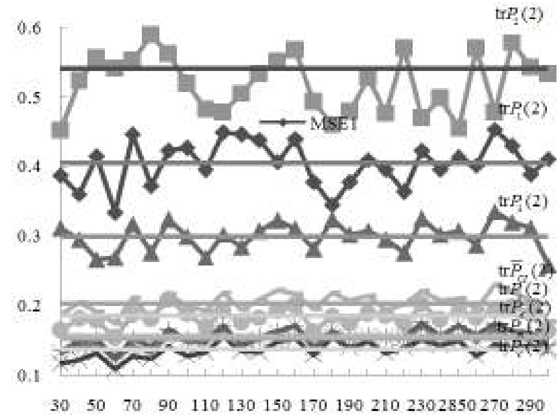

Fig. 6: The MSE curves of local and fused smoothers

In order to verify the above theoretical accuracy relations, the Mean Square Error (MSE) value at time t, ρ = 200 for local and fused Kalman smoothers is defined as:

$$\mathrm{MSE}_i(t) = \frac{1}{\rho}\sum_{j=1}^{\rho}\left(x^{(j)}(t) - \hat{x}_i^{(j)}(t|t+N)\right)^{\mathrm{T}} \times \left(x^{(j)}(t) - \hat{x}_i^{(j)}(t|t+N)\right), N = 0, -2, 2 \quad (76)$$

$\hat{x}_i^{(j)}(t|t+N)$ or $x^{(j)}(t)$ denotes the jth realization of $\hat{x}_i^{(j)}(t|t+N)$or x(t).

The MSE curves of the local and fused estimators are shown in Fig. 4-6.

Form Fig. 4-6, we see that the $\mathrm{MSE}_i(t)$ values of the local and fused Kalman estimators are close to the corresponding theoretical trace values, when t is large enough, according to the ergodicity of the sample function, we have:

$$\mathrm{MSE}_i(t) \to \mathrm{tr}P_i(N), \rho \to \infty, t \to \infty \quad i = 1,2,3,m,s,d \quad (77)$$

$$\mathrm{MSE}_{CI}(t) \to \mathrm{tr}\bar{P}_{CI}(N), \rho \to \infty, t \to \infty \quad (78)$$

Form Fig4-6, we see that the accuracy relations (69) and (70) hold.

CONCLUSION

For the multi-sensor system with colored measurement noises, it converted into an equivalent system with correlated noises by the observation transformation. Based on the classical Kalman filtering, the CI fuser without cross-covariance has been presented. The centralized fusion Kalman filter and three weighted fusers have been also presented and the accuracy comparisons of these fusers were given by a Mote-Carlo simulation example. In this study, the CI fusion results of the Deng *et al*. (2012) have been extended to the case with colored measurement noises.

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