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Research Article

Optimal Deployment Problems of Radar Network

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Abstract: The deployment problems of the radar sets are important in the air defense of the military. The target detection joint probability of several radar sets is studied. The optimum deployment models of circle, line and sector have been built up. The area of space, which is determined by the detection joint probability is more than 0.9, is got by the use of Monte Carlo method. The optimum deployments problems of the circle deployment, line deployment and sector deployment can be solved by entire enumerate method, golden section method and coordinate alternation method. On condition that the number of radar is less in the twice-ines deployment model it can be solved by combine entire enumerate method with coordinate alternation method. Contrarily, it can be solved by combining genetic algorithm with coordinate alternation method.

Keywords: Coordinate alternation method, genetic algorithm, golden section method, Monte Carlo method, target detection probability

INTRODUCTION

The modern air attack environment is getting more and more complex. Attackers may fly at very low to very high altitude and they may be bombers, interceptors, or air to ground missiles etc. It is impossible that a single radar set has a satisfactory capability against all possible threats. So several radar sets are deployed to detect the attackers (Zou and Chakrabarty, 2004). The optimal deploy problem of two sets of radar is solved by the use of Monte Carlo method (Gao, 1999). Various radar and jammer parameters for effective luring away of the missile are studied (Vijaya *et al*., 2011). In this study, the target detection joint probability of several radar sets is studied and the optimum deployment models of circle, line and sector have been built up.

DETECTION PROBABILITY OF SINGLE RADAR SET

Detection probability of single radar set is AFSC TR-65-2 (1965) and Nelson (1954).

$$p = 1 - e^{-\frac{ap_T}{2z^2r^4}} \tag{1}$$

where,

- p = Detection probability
- a = Available reflection area of target
- z = Standard deviation of noise amplitude
- p_T = Power of transmitter
- r = Target distance

The detection probability is computed using (1), as the parameter of radar and target is given. As the parameter of radar and target is known, $ap_T/2z^2$ is a fixed number. If we assume k = $ap_T/2z^2$, the detection probability of target relates only to target distance. It can be calculated simply as:

$$p = 1 - e^{-\frac{k}{r^4}} \tag{2}$$

In general, the value of k can be got by measurement. Suppose that detection probability of target at range of 32 km on the surface is 0.684 by testing. Applying (2) result in: $k = -\ln 0.316 \times 32^4$

DETECTION JOINT PROBABILITY OF SEVERAL RADAR SETS

If all radar sets are independent to each other, the detection joint probability can be obtained as follows (Ravindran and Phillips, 1987; Zou and Chakrabarty, 2003):

$$p_L = 1 - \prod_{i=1}^{n} (1 - p_i) = 1 - e^{-k\sum_{i=1}^{n}\frac{1}{r_i^4}} \tag{3}$$

where,

- p_i = Detection probability of radar i
- r_i = Range of target i
- n = Number of radar sets

Therefore,

$$p_L = 1 - e^{-k\sum_{i=1}^{n}\frac{1}{\left[(x-x_i)^2+(y-y_i)^2+z^2\right]^2}} \tag{4}$$

where, (x_i, y_i) is coordinate of radar i and (x, y, z) is coordinate of target.

The detection joint probability at a definite height is discussed mainly. The detection joint probability at altitude $z = h_0$ can be expressed as:

$$p_L = 1 - e^{-k\sum_{i=1}^{n}\frac{1}{\left[(x-x_i)^2+(y-y_i)^2+h_0^2\right]^2}} \tag{5}$$

MONTE CARLO METHOD

The detection joint probability must be high (for example $p_L \geq 0.09$) enough to detect target. The space determined by $p_L \geq p_0$ can be expressed as:

$$p_L = 1 - e^{-k\sum_{i=1}^{n}\frac{1}{\left[(x-x_i)^2+(y-y_i)^2+h_0^2\right]^2}} \geq p_0$$

we get

$$\sum_{i=1}^{n}\frac{1}{[(x-x_i)^2+(y-y_i)^2+h_0^2]^2} \geq -\frac{\ln(1-p_0)}{k} \tag{6}$$

Now key problem is how to deploy radar sets as to get area of space determined by $p_L \geq p_0$ maximum.

It is difficult to compute area by multiple integral method. We use Monte Carlo method to resolve this problem. The general algorithm follows:

Step 1: Set N = 10000, m = 0 and j = 0.

Step 2: If j > N , go to step 4. Otherwise generate $X_{mj} \sim U[0, X_{max}]$, $Y_{mj} \sim U[0, Y_{max}]$. (Where $U[0, X_{max}]$ = uniform distribution on the interval $[0, X_{max}]$, $[0, Y_{max}]$ = uniform distribution on the interval $[0, Y_{max}]$) and replace j by j + 1.

Step 3: If

$$\sum_{i=1}^{n}\frac{1}{[(x_{mj}-x_i)^2+(y_{mj}-y_i)^2+h_0^2]^2} \geq -\frac{\ln(1-p_0)}{k}$$

replace m s by m + 1. Otherwise return to step 2.

Step 4: Let $S \approx 4m/N\ X_{max}\ Y_{max}$ (Where S = area of space determined by $p_L \geq p_0$ maximum) and stop.

THE OPTIMUM DEPLOYMENT MODEL OF THE CIRCLE

The optimum deployment models are many, such as the circle, line and sector deployment. Suppose that several radar sets are deployed on the circle (Fig. 1).The radius is r. The coordinate of radar i is:

Table 1: Optimum circle deployment

n	r(km)	S_{max} (km^2)
2	29	4288.5
3	34	6931.8
4	40	9382.5
5	46	11782.8
6	52	14066.1
7	65	16292.7
8	72	18609.3
9	81	20890.8

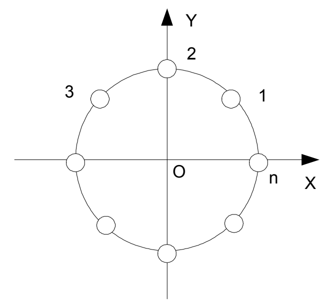

Fig. 1: Circle deployment

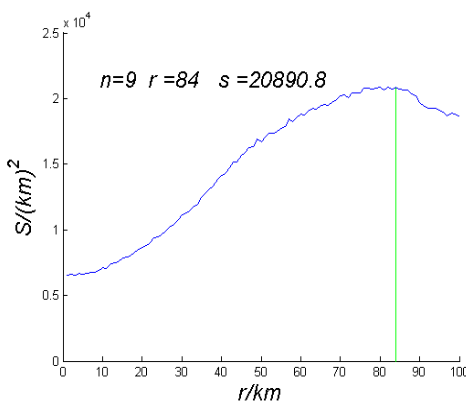

Fig. 2: Relation between area of detection with r when n = 9

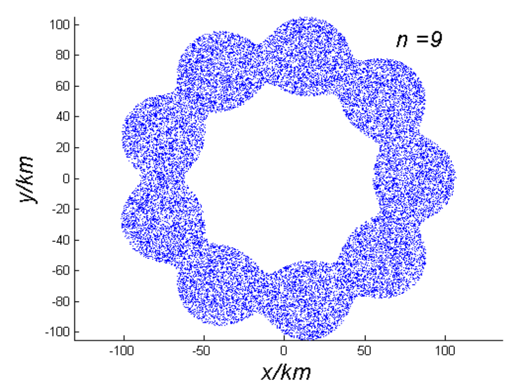

Fig. 3: Maximal area of detection when n = 9

$$\begin{cases} x_i = r\cos(\frac{2\pi}{n}\times i) & i = 1,2,\cdots,n \\ y_i = r\sin(\frac{2\pi}{n}\times i) \end{cases} \tag{7}$$

The detection joint probability of typical target at altitude z = 10 km can be obtained using (5). The space determined by $p_L \geq p_0$ maximum can be also obtained using (6). The area can be estimated by Monte Carlo method, as r_0 from 0 to 120 km and step is l km. We can obtain the maximum area. Table 1 show the solution. The relation between area of detection with r when n = 9 be shown in Fig. 2. The shape of the maximum area can be shown in Fig. 3. The number of S_{max}/ n will increase according as n increase. In other words, the average effectiveness of radar will increase according as the number of radar set increase. But as radar sets are deployed on the circle, the detection joint probability is lower than 0.9 at its centre (Fig. 2).

THE OPTIMUM DEPLOYMENT MODEL OF THE LINE

Suppose that several radar sets are deployed on the line (Fig. 4). The interval is d, This line is axis X and the center of radars is zero. The coordinate of radar i is

$$\begin{cases} x_i = (-\dfrac{n+1}{2} + i) \times d \quad i = 1,2,\cdots,n \\ y_i = 0 \end{cases} \tag{8}$$

The area can be estimated also by Monte Carlo method, as d from 0 to 120 km and step is l km. But the runtime is long. We can use golden section method to solve this problem. The golden section method is described as follows:

Step 1: Set α = 0, b = 100 and ε > 0 is given.

Step 2: Let r_1 = α + 0.382(b - α) and r_2 = α + 0.318(b - α). The area $S_1(r_1)$ and $S_2(r_2)$ can be estimated also by Monte Carlo method.

Step 3: If $/r_2 - r_1/ < \varepsilon$, then $r^* = r_1 + r_2/\ 2$, Stop. Otherwise go to Step (4).

Step 4: If S1 > S_2, then b = r_2, r_2 = r_1, r_1 = α + 0.382 (b - α) and S_2 = S_1. The area S_1 (r_1) can be estimated also by Monte Carlo method and then go to step (3). Otherwise α = r_1, r_1 = r_2, r_2 = α + 0.618(b - α) and S_1 = S_2. The area $S_2(r_2)$ can be estimated also by Monte Carlo method and then go to step (3).

Table 2 shows the solution. The maximal area of detection when n = 3, z = 10 km and d = 56.6 km is shown in Fig. 5.

The Optimum Deployment Model of the Sector: Suppose that several radar sets are deployed on the sector (Fig. 6). The angle is α. The coordinate of radar i is:

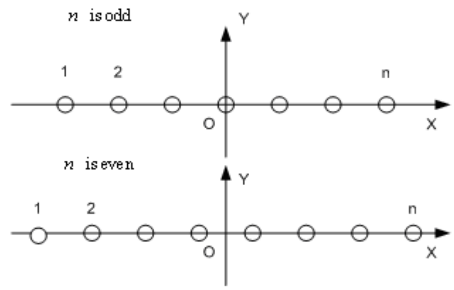

Fig. 4: Line deployment

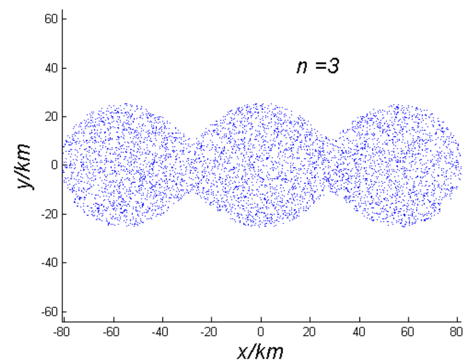

Fig. 5: Maximal area of detection when n = 3, z = 10 km, and d = 52.7 km

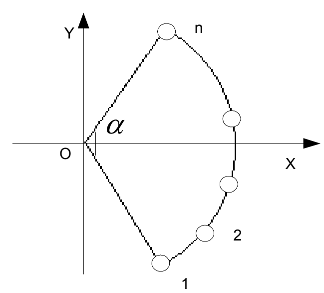

Fig. 6: Sector deployment

Table 2: Optimum line deployment

n	r (km)	S_{max} (km^2)
2	56.5	4435
3	52.7	6760.8
4	54.3	9090
5	52.3	11405
6	54.6	13752
7	55.2	16168
8	56.2	18662
9	52.3	20736

$$\begin{cases} x_i = r\cos[-\dfrac{\alpha}{2} + \dfrac{\alpha}{n-1} \times (i-1)] \\ y_i = r\sin[-\dfrac{\alpha}{2} + \dfrac{\alpha}{n-1} \times (i-1)] \end{cases} \tag{9}$$

Table 3 shows the solution by golden section method. The maximal area of detection when n = 3, z = 10 km and r = 58.3 km is shown in Fig. 7.

Table 3: Optimum sector deployment

n	r (km)	S_{max} (km^2)
2	34.2	4347.2
3	58.3	6678.4
4	84.4	9028.8
5	105.5	11459

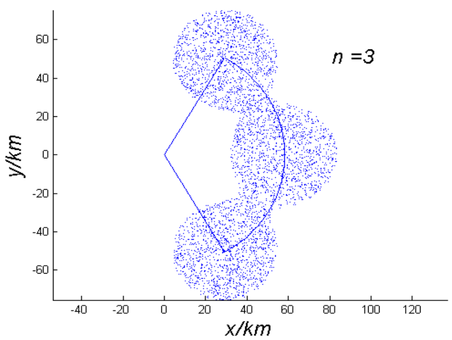

Fig. 7: Maximal area of detection when n = 3, z = 10 km and r = 58.3 km

THE TWICE-LINES DEPLOYMENT MODEL

In order to defend important spot, we can deploy the radar sets in twice-lines manner. Suppose that several radar sets are deployed on the sector in multi-lines (Fig. 8).

Suppose the inner radius is r_1 and the outer radius is $r_2(r_2 > r_1)$. The inner numbers of radar sets is n_1 and the outer numbers of radar sets is n_2 $(n_2 \geq n_1)$ n_2.
The coordinate of radar i is:

$$\begin{cases} x_i = r_1 \cos[-\frac{\alpha}{2} + \frac{\alpha}{n_1 - 1} \times (i-1)] \\ y_i = r_1 \sin[-\frac{\alpha}{2} + \frac{\alpha}{n_1 - 1} \times (i-1)] \end{cases}$$

$i = 1, 2, \ldots, n_1$ (inner) (10a)

$$\begin{cases} x_i = r_2 \cos[-\frac{\alpha}{2} + \frac{\alpha}{n_2 - 1} \times (i - n_1 - 1)] \\ y_i = r_2 \sin[-\frac{\alpha}{2} + \frac{\alpha}{n_2 - 1} \times (i - n_1 - 1)] \end{cases}$$

$i = n_1 + 1, n_1 + 2, \cdots, n$ (outer) (10b)

The space determined by $p_L \geq p_0$ maximum can be also obtained using (6). Dimension of determined area is $S(n_1, n_2, r_1, r_2)$. The optimal model is

$$\begin{aligned} &\max S(n_1, n_2, r_1, r_2) \\ &s.t. \quad n_1 + n_2 = n \\ &\qquad r_2 > r_1 \\ &\qquad n_2 \geq n_1 \geq 1 \end{aligned} \tag{11}$$

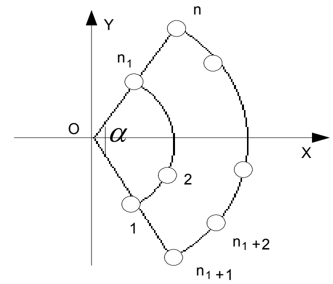

Fig. 8: Twice-lines deployment

- If r_1, r_2, are known and the n is not big, we can use entire enumerate method. Given a group (n_1, n_2), we can got the dimension by Monte Carlo method. For example, as $n = 8$,there are four situation (n_1, n_2) = {(1,7), (2,6), (3,5), (4,4) }. We simulate respectively and find out the optimal deployment. If the n is big, the genetic algorithm is used to solve it.
- If n1, n_2, are known, we can use the coordinate alternation method [5]. Supposed, $\alpha_2 \leq r_2 \leq b_2$, $\alpha_2 \leq r_2 \leq b_2$ and $\varepsilon > 0$ is given. The coordinate alternation method is described as follows:

Step 1: Let $r^{(0)}_1 = \alpha_1 + b1/2$.
Step 2: r_2 is got by golden section method to get area of space S maximum. That is $S(r^{(0)}_1 - r^{(1)}_2) = \max S(r^{(0)}_1 - r_2)$.
Step 3: Set $r_2 = r^{(1)}_1$. r_1 is got by golden section method to get area of space S maximum. That is $S(r^{(1)}_1 - r^{(1)}_2) = \max (r_1 - r^{(1)}_2)$
Step 4: If $/r^{(1)}_1 - r^{(0)}_1/ < \varepsilon$, Stop. The solution is ($r^{(1)}_1$, $r^{(1)}_2$). Otherwise let $r^{(0)}_1 = r^{(1)}_1$ go to step 2.

- If r_1, r_2, n_1, and n_2 is unknown, the model (11) is a hybrid nonlinear programming model. If the n is big, we can use the entire enumerate method with coordinate alternation method. When n are known, we can use the combining genetic algorithm with coordinate alternation method.

CONCLUSION

The optimum deployment models of circle, line and sector have been built up. The other deployment can be deal with similarly. The model of detection probability of radar is simple in this study and the precise model will be study in the future. The multi-lines deployment model can be solved similarly.

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