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Research Article

A Closed-form Expression for BER of FSO Links over Gamma-Gamma Atmospheric Turbulence Channels with Pointing Errors

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Abstract: FSO communication systems links is vulnerable due to degrading effects of atmospheric turbulence and point errors. We investigate the error rate performance of Free-Space Optical (FSO) links over gamma-gamma turbulence fading channels in the presence of pointing error. Assuming Intensity-Modulation/Direct Detection (IM/DD) with On-Off Keying (OOK), a novel closed-form expression for BER of FSO is derived. Another work is a study of how the BER are affected by the atmospheric turbulence and other parameters such as the normalized beamwidth, the average transmitted optical power, the normalized jitter. Numerical examples are further provided to verify the derived analytical expressions. The results show that optimizing the beamwidth can achieve the minimum BER for a given average transmitted optical power.

Keywords: BER, free space optical communication, gamma-gamma distribution, OOK, pointing error

INTRODUCTION

Free Space Optical communication (FSO) has received considerable attention recently as attractive solution for high–rate last–mile terrestrial communications due to its large bandwidth, unregulated spectrum, low cost, ease of redeployment. However, the performance of FSO communication systems is susceptible to atmospheric turbulence and point errors. Atmospheric turbulence causes irradiance fluctuations in the received signals as a result of variations in the refractive index. Building sway causes vibrations of the transmitter beam (Sandalidis, 2008) and, therefore, misalignment (point errors) between the transmitter and receiver. The combined effects of atmospheric turbulence and point errors degrade severely the link performance of FSO.

Many statistical models for the intensity fluctuation through FSO channels have been proposed such as the log-normal distribution, K distribution, exponential distribution, IK distribution, gamma-gamma distribution mode. The gamma-gamma distribution has been found to be a suitable turbulence channels model. Tsiftsis (2008) have evaluate the performance of FSO systems in terms of the outage probability and Bit-Error Rate (BER) using this turbulence model. Nistazakis *et al.* (2009) have studied the FSO performance in terms of average capacity and outage probability over gamma-gamma atmospheric turbulence channels. A closed-form expression (Han *et al.*, 2010) for outage probability of FSO over gamma-gamma turbulence channels has been simplified. Uysal *et al.* (2006) has investigated error rate performance analysis of coded free-space optical links over gamma-gamma turbulence channels.

The joint effects of point errors and atmospheric turbulence on link performance (Shlomi, 2003) have been investigated for first time. Farid and Hranilovic (2007) consider a FSO channel model affected by misalignment fading (pointing error) effects. They have considered log-normal distributed and a closed-form expression for the outage probability is presented. But they have not given a closed-form expression for the outage probability over gamma-gamma distributed turbulence channel. Ergodic capacity (Borah and Voelz, 2009) is numerically evaluated for turbulence channels with pointing errors using OOK formats. In Sandalidis (2008), BER performances of FSO links over K distribution atmospheric turbulence channels with pointing errors have been investigated. Chao *et al.* (2010) has analysised average capacity of FSO Links over gamma-gamma distribution atmospheric turbulence channels with pointing errors. Sandalidis *et al.* (2009) has presented a closed-form expression of the BER over gamma-gamma turbulence channels for an Intensity-Modulation/Direct Detection (IM/DD) FSO system with Differential Phase-Shift Keying (DPSK).

Unlike previous study, we have investigated BER performance of FSO links over gamma-gamma atmospheric turbulence channels in the presence of pointing errors using OOK formats, which is widely used in commercial systems.

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SYSTEM AND CHANNEL FADING MODEL

We consider a FSO system using IM/DD with OOK. The laser beams propagate along a horizontal path through a gamma-gamma turbulence channel with Additive White Gaussian Noise (AWGN) in the presence of pointing errors. The received electrical signal of the FSO system is given by:

$$y = hx + n \tag{1}$$

where,

x = The binary transmitted signal

h = The normalized channel fading coefficient considered to be constant over a large number of transmitted bits

n = AWGN with variance σ^2n

The transmitted signal is taken as symbols drawn equiprobably from an OOK constellation such that $x \in (0,2P_t)$ and Pt is the average transmitted optical power. The channel state can be formulated as:

$$h = h_a h_p \tag{2}$$

where,

h_a = The random attenuation due to atmospheric turbulence

h_p = The random attenuation due to pointing errors

In various statistical models to describe FSO atmospheric turbulence channels, the gamma–gamma distribution provides a good agreement between theoretical and experimental data for moderate-to-strong turbulence strength. The gamma-gamma turbulence model has the following pdf:

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{[(\alpha+\beta)/2]-1} \mathrm{K}_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right) \tag{3}$$

where,

h_a = The normalized irradiance

$\Gamma(\cdot)$ = The well-known Gamma function

$K_{\alpha-\beta}$ = Modified Bessel function of the second kind and assuming plane wave propagation

The parameters α, β can be directly related to atmospheric conditions through the expressions below:

$$\alpha = \frac{1}{\sigma_x^2} = \left[\exp\left(\frac{0.49\sigma_0^2}{\left(1+1.11\sigma_0^{12/5}\right)^{7/6}}\right) - 1\right]^{-1} \tag{4}$$

$$\beta = \frac{1}{\sigma_y^2} = \left[\exp\left(\frac{0.51\sigma_0^2}{\left(1+0.69\sigma_0^{12/5}\right)^{5/6}}\right) - 1\right]^{-1} \tag{5}$$

where the parameter σ_0^2 is called Rytov variance and is used as a metric of the strength of turbulence fluctuations. It is given by:

$$\sigma_0^2 = 1.23 C_n^2 \kappa^{7/6} L^{11/6} \tag{6}$$

where,

C_n^2 = The index of refraction structure parameter

$k = 2\pi/\lambda$ = The optical wave number

λ = The wavelength

L = The propagation distance

If we assume a circular detection aperture of radius *r* and a Gaussian beam, the pdf of h_p is given by:

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_0^{\gamma^2}} h_p^{\gamma^2-1} \tag{7}$$

where $\gamma = w_{zep}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver, $w_{zep}^2 = w_z^2 \sqrt{\pi}\, erf(v)/2v \exp(-v^2)$, $v = \sqrt{\pi} r/\sqrt{2} w_z$ $A_0 = [erf(v)]^2$ with being the error function, w_z is beam waist (radius calculated at e^{-2}) on the receiver plane at distance z from the transmitter and a circular aperture of radius r.

Using the previous pdfs for atmospheric turbulence and pointing error, a close-form expression for the pdf of $h = h_a h_p$ is given as:

$$f_h(h) = \frac{\alpha\beta\gamma^2}{A_0\Gamma(\alpha)\Gamma(\beta)} \times G_{1,3}^{30}\left[\frac{\alpha\beta}{A_0} h \,\middle|\, \begin{matrix} \gamma^2 \\ \gamma^2-1, \alpha-1, \beta-1 \end{matrix}\right] \tag{8}$$

AVERAGE BER

The average BER of IM/DD with OOK is given by:

$$P_b(e) = \int_0^\infty f_h(h) Q\left(\frac{\sqrt{2}P_t h}{\sigma_n}\right) dh \tag{9}$$

where $Q(\cdot)$ is the Gaussian Q function which is related to the complementary error function erfc(·) by $erfc(x) = 2Q(\sqrt{2}x)$. By substituting Eq. (8) in (9), we obtain Eq. (10) below:

$$\begin{aligned} P_b(e) = \int_0^\infty & \frac{\alpha\beta\gamma^2}{A_0\Gamma(\alpha)\Gamma(\beta)} \times G_{1,3}^{30}\left[\frac{\alpha\beta}{A_0} h \,\middle|\, \begin{matrix} \gamma^2 \\ \gamma^2-1, \alpha-1, \beta-1 \end{matrix}\right] \\ & \times \frac{1}{2\sqrt{\pi}} G_{1,2}^{2,0}\left[\frac{P_t^2}{\sigma_n^2} h^2 \,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right] dh \end{aligned} \tag{10}$$

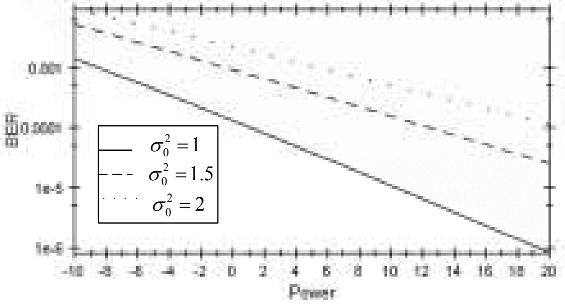

Fig. 1: BER versus the transmitted optical power for various values of the turbulence strength ($\frac{w_z}{r} = 6$, $\frac{\sigma_s}{r} = 2$)

Thus, expressing the erfc(·) as Meijer G-function

$$erfc\left(\sqrt{x}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left[x\Big|_{0,1/2}^{1}\right]$$

and using [ref.12, Eq. 07.34.21.0011.01], a closed-form expression for BER is derived as:

$$P_b(e) == \frac{2^{\alpha+\beta-3}\gamma^2}{\pi^{\frac{3}{2}}\Gamma(\alpha)\Gamma(\beta)} G_{7,4}^{2,6}\left[\frac{16P_t^2A_0^2}{\sigma_n^2\alpha^2\beta^2}\middle|\begin{matrix}1, \frac{1-\gamma^2}{2}, \frac{2-\gamma^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}\\ 0, \frac{1}{2}, -\frac{\gamma^2}{2}, \frac{1-\gamma^2}{2}\end{matrix}\right] \tag{11}$$

Equation (11) can be further simplified using [ref.12, Eq. 07.34.03.0001.01] as:

$$P_b(e) = \frac{2^{\alpha+\beta-3}\gamma^2}{\pi^{\frac{3}{2}}\Gamma(\alpha)\Gamma(\beta)} G_{6,3}^{2,5}\left[\frac{16P_t^2A_0^2}{\sigma_n^2\alpha^2\beta^2}\middle|\begin{matrix}1, \frac{2-\gamma^2}{2}, \frac{1-\alpha}{2}, \frac{2-\alpha}{2}, \frac{1-\beta}{2}, \frac{2-\beta}{2}\\ 0, \frac{1}{2}, -\frac{\gamma^2}{2}\end{matrix}\right] \tag{12}$$

NUMERICAL RESULTS

Using the above-derived closed form for BER, we investigate the performance of a FSO link assuming noise standard deviation $\sigma_n = 10^{-7} A/Hz$. In Fig. 1 the average BER is depicted in terms of the transmitted optical power in dBm for various values of the turbulence strength. It is observed that for a given values of the turbulence strength the better BER performance is achieved because the transmitted signal power is increased. Moreover, the BER increases as turbulence strength gets stronger with the different tubulence conditions: $\sigma_0^2 = 1$, $\sigma_0^2 = 1.5$, $\sigma_0^2 = 2$.

In Fig. 2 the average BER versus the normalized jitter is presented assuming a given valaue of the transmitted signal power $P_t = 20dBm$ and the normalized beamwidth $\sigma_s/r = 6$. When the jitter increase, it is shown that the jitter degrade the BER performance of FSO links.

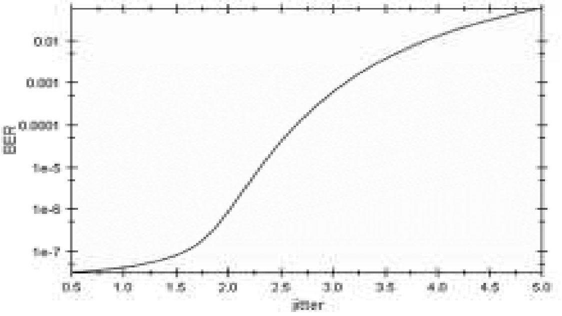

Fig. 2: BER versus the normalized jitter ($\sigma_0^2 = 1$, $\frac{w_z}{r} = 6$, $P_t = 20dBm$)

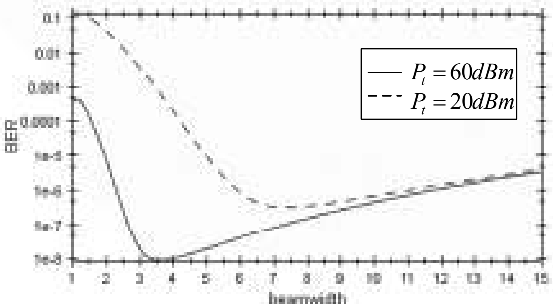

Fig. 3: BER versus the normalized beamwidth for the transmitted signal power ($\sigma_0^2 = 1$, $\frac{\sigma_s}{r} = 1$)

In Fig. 3 the average BER versus the normalized beamwidth and is shown where the transmitted signal power $P_t = 20dBm$ or $P_t = 60dBm$ and the normalized jitter $\sigma_s/r = 1$. From the figure, we find the minimum BER can be achieved by selecting an optimum beamwidth for a given the transmitted signal power value. Using optimization method such as the Nelder-Mead one, we can calculate the optimum value. For example, a minimum BER occurs for the optimum beamwidth 3.56 with a given transmitted signal power value. Another example a minimum BER occurs for the optimum beamwidth 7.49 where a given transmitted signal power value.

CONCLUSION

In this study, we have studied the BER performance of a FSO system over gamma-gamma turbulence fading in the presence of misalignment fading. A closed-form BER expression including both fading conditions was derived. It is demonstrated that optimization of beamwidth values can achieve the minimum BER for a given transmitted signal power value. This study is also a guide for FSO designers who should take into consideration the combined effects of pointing errors and atmospheric turbulence.

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