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Research Article

Numerical Simulation of a Whole Structure with Joints

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Abstract: The aim of this study was to investigate a numerical simulation of a whole structure with joints by the Boundary Element Method (BEM). On the basis of element model, joints characteristics were introduced to boundary dynamic equations by flexible constraint conditions on boundary. Finally, the dynamic model of a whole structure with joints was established by BEM. Finally, the dynamic characteristics of a whole structure with joints were obtained. Results shown that the suggested method can be effectively executed to solve the joints characteristics of a whole structure.

Keywords: BEM, joints characteristics, simulation

INTRODUCTION

At present, most numerical simulation methods of a whole structure include Finite Element Method (FEM) and Boundary Element Method (BEM) and the FEM and BEM have been applied to solve dynamic simulation for many years (Barik and Mukhopadhyay, 1998; Zietsman, 2003; Shen *et al*., 2003; Providakis, 1997). For a whole structure with joints, it is very difficult for FEM to analyze the dynamic behaviors. But the BEM has some main advantages of inputting less data and reducing computational dimensions and it can efficient process the joints characteristics.

Different from a single structure or a component, a whole structure is composed of many components connected with many joints and the dynamic characteristics of joints affect the dynamic behaviors of a whole structure considerably (Damjan and Miha, 2008). The dynamic characteristics of joints are affected by many factors. Here, the dynamic characteristics of joints refer to the dynamic stiffness and damping of joints. Experiments show that dynamic characteristics of joints are intense nonlinearity (Li *et al*., 2010). For the intense non-linearity of dynamic characteristics of joints, the values of the dynamic stiffness and damping of joints, which was based upon the dynamic characteristic parameters of joint surfaces at unit area obtained by experiments, can be determined during the analysis of the dynamic behaviors of a whole structure. Hence, there would be many iterations process during this analysis. How to establish a reasonable dynamic model of a whole structure is a key to analyze the dynamic characteristics of a whole structure. This study solved this problem by establishing boundary dynamic equations of a whole structure with joints, which are introduced in this study. Especially, it is impossible to treat all of these factors by an analyzing method in determining the dynamic characteristics of joints. Joints cannot exist without the mechanical system. The values of the dynamic stiffness and damping of joints obtained directly by experiments are used in similar conditions in the experiment. Since there are so many values obtained by experiments and they are not general for variable conditions, we cannot use them easily to analyze dynamic characteristics of a new whole structure. This study solved this problem by use of the dynamic characteristics parameters of joint surfaces at unit area, which are introduced.

Because of the complexity of a whole structure with joints, a reasonable dynamic model should not only calculate efficiently but also treat the non-linearity behaviors of joints easily for a whole structure. The dynamic model of a whole structure proposed in this study can meet these two requirements. According to the former researches, a whole structure is equivalent as the structural element, joint element, spring element and lumped mass when establishing the dynamic model by the BEM in this study. For the structural element including plate and beam components, they had been earnest analyzed and the detailed papers can be found (Zhang *et al*., 2002; Fang and Wu, 2007; Fang *et al*., 2003). After appropriate elements are chosen depending upon different shapes of components in a whole structure, a reasonable dynamic model is established. Hence, the aim of this study is to investigate a numerical simulation of a whole structure with joints by the Boundary Element Method (BEM).

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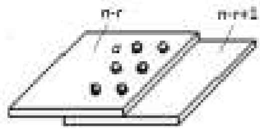

Fig. 1: The location *a*

MODELING OF A WHOLE STRUCTURE WITH JOINTS

Joints characteristics: There will discuss the introduced method of joints characteristics. For a whole structure, let the sum of domain points on joints surface is r, the total area of domain points is A and the formulas of stiffness and damping of the i^{th} domain point can be written as follows: (i = 1-n):

$$K_{ni} = \frac{K_{nd}A}{r}, K_{\tau i} = \frac{K_{\tau d}A}{r} \quad (1)$$

$$C_{ni} = \frac{C_{nd}A}{r}, C_{\tau i} = \frac{C_{\tau d}A}{r} \quad (2)$$

where,

K_{ni}, $K_{\tau i}$: Normal stiffness and tangential stiffness of point i
C_{ni}, $C_{\tau i}$: Normal damping and tangential damping of point i
k_{nd}, $k_{\tau d}$: The normal stiffness and tangential stiffness at unit joint area
c_{nd}, $c_{\tau d}$: The normal damping and tangential damping at unit joint area

Joints information of stiffness and damp is included in Eq. (1) and (2). After confirming the size, pressure, excited frequency and relative displacement, the stiffness and damping of joints can be gotten. At last, the value of complex stiffness matrices can be obtained and these problems for dynamic characteristics analysis of joints will be discussed. It can be seen from the above discussion that main factors of affecting joints dynamic behaviors are included in Eq. (1) and (2). Thereby, the above formulas have specific physical meaning and excellent universal property.

Relational expressions of joints conditions must be established, which is the modeling prerequisite of a whole structure. The dynamic equations of the whole structure can be reduced effectively after all parts are assembled by joints conditions. A complex stiffness matrix can be used to denote joints conditions commonly. In order to illustrate easily, joints between parts No. *n-r*, No. *n-r*+1 are illustrated to establish the relational expressions of joints conditions.

It is easy to establish the equations for strength restraint and the following relational equations are satisfied:

$$(T_{n-r}^{\ i+1})^T = -S(T_{n-r+1}^{\ j})^T \quad (3)$$

where,

$S = (S_a^{i+1})^{-1} (S_a^{j})^{-1}$: Space coordination transforming matrices of location *a*
T_{n-r}^{i+1} : Boundary strength matrix of part *n-r* at point *i*+1
T_{n-r+1}^{j} : Boundary strength matrix of part *n-r*+1 at point *j*, the location *a* is shown in Fig. 1

Based on the literature (Zhang *et al*., 2002; Fang and Wu, 2007), the restraint equations of the strength and displacement can be obtained after dealing with the matrices:

$$S_a^{\ j} KC_a (S_a^{\ i+1} U_{n-r}^{\ i+1} - S_a^{\ j} U_{n-r+1}^{\ j})^{\mathrm{T}} = -(T_{n-r+1}^{\ j})^{\mathrm{T}} \quad (4)$$

where,

U_{n-r}^{i+1} : The boundary displacement matrix of part No. *n-r* at point *i*+1
U_{n-r+1}^{j} : The boundary displacement matrix of part No. *n-r*+1 at point *j*

Introducing of joints: For a whole structure is composed of many components connected with joints, the boundary dynamic equations of part No. *n-r*, No. *n-r*+1 are gotten after being dealt with:

$$\begin{bmatrix} EE_{n-r}^{\ 11,i} - EE_{n-r}^{\ 12,i+1} SRE_{11} & -EE_{n-r}^{\ 12,i+1} SRE_{22} \\ EE_{n-r}^{\ 21,i} - EE_{n-r}^{\ 22,i+1} SRE_{11} & -EE_{n-r}^{\ 22,i+1} SRE_{22} \\ EE_{n-r+1}^{\ 11,j} RE_{11} & EE_{n-r+1}^{\ 12,j+1} + EE_{n-r+1}^{\ 12,j+1} RE_{22} \\ EE_{n-r+1}^{\ 21,j} RE_{11} & EE_{n-r+1}^{\ 22,j+1} + EE_{n-r+1}^{\ 22,j+1} RE_{22} \end{bmatrix} \quad (5)$$

$$\begin{Bmatrix} T_{n-r}^{\ i} \\ T_{n-r+1}^{\ j+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r}^{\ i} \\ U_{n-r}^{\ i+1} \\ U_{n-r+1}^{\ j} \\ U_{n-r+1}^{\ j+1} \end{Bmatrix}$$

Equation (5) can be combined unceasingly according to the joints conditions. Repeating above process until finishing assembled process, the final dynamic equations of a whole structure can be established and the dynamic characteristics of the whole structure with joints can be gotten by solving Eq. (5). Owing to joints characteristics had been introduced to boundary dynamic equations by flexible constraint conditions on boundary, the corresponding BEM dynamic model of the whole structure with joints was educed finally. Considering the flexible joints have nonlinear characteristics as well as the stiffness and damping are the functions of displacement, the values of stiffness and damping are unknown and they have only interdependent relations in the equation. Therefore, a large amount of computing exists in Eq. (5). In order to improve computational precision, joint's displacements must be repeated iteration to obtain the values. Namely, the known values of stiffness and damping are used to act as the iterative initial values firstly. As a result, the iterative calculating of the equations can be executed. The iterative convergence is judged by using the variation of front-to-back twice iterative values of stiffness and damping.

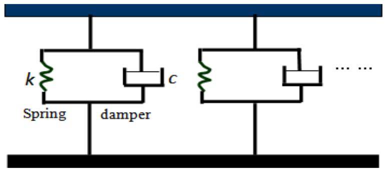

Fig. 2: The joint model

ANALYSIS OF JOINTS CHARACTERISTICS

Although the dynamic equations of a whole structure have been reduced by the above discussion, the values of stiffness and damping of joints in Eq. (5) have still not been determined. As an example, the dynamic characteristics of a bolt joint will be analyzed. Owing to joints surface between components produce small displacement and rotation, it can not only storage energy but also consume energy. So, analysis of joints characteristics is the foundation for discussing dynamic modeling of a whole structure with joints. Dynamic characteristics of joints in a whole structure are determined by many factors and the related theory of joints can be found in literatures (Chlebus and Dybala, 1999). These factors are categorized into two aspects. The first aspect is taken into account as the dynamic fundamental behavior parameters of joint surfaces at unit area and the other aspect is taken into account through analysis of dynamic behaviors of joints.

Joints model are equivalent as springs and dampers commonly, lots of scholars have agreed with the viewpoint for the joint model and the corresponding joint model is shown in Fig. 2.

According to some investigations (Tony *et al*., 2007; Dhupia *et al*., 2008), many kinds of dynamic fundamental behavior parameters of joint surfaces at unit joint area have been gotten by experimental method. At last, the dynamic fundamental parameters of joint surfaces at unit joint area, namely the universal behaviors formulas of stiffness and damp at unit joint area can be given as follows by experiments (Dhupia *et al*., 2008; Guo *et al*., 2011):

$$k_{nd} = \alpha_n p_n^{\beta_n} \omega^{\gamma_n} x_n^{\eta_n}, k_{\tau d} = \alpha_\tau p_n^{\beta_\tau} \omega^{\gamma_\tau} x_\tau^{\eta_\tau} \quad (6)$$

$$c_{nd} = \alpha_{nc} p_n^{\beta_{nc}} \omega^{\gamma_{nc}} x_n^{\eta_{nc}}, c_{\tau d} = \alpha_{\tau c} p_n^{\beta_{\tau c}} \omega^{\gamma_{\tau c}} x_\tau^{\eta_{\tau c}} \quad (7)$$

where,

$\alpha, \beta, \gamma, \eta$: The corresponding behaviors coefficient
p_n : The normal pressure
x_n, x_τ : The normal and tangential amplitude

A bolt joint is very common in a whole structure and the sectional drawing is presented in Fig. 3a. As an example, the dynamic modeling of a bolt joint will be discussed. According to the former analysis, the equivalent model of a bolt joint can be established by equivalent as the number of *n* springs and dampers, as shown in Fig. 3b and 3c shows a coordinate system of dynamic analysis of a bolt joint.

In Fig. 3, D is the relative displacement matrix of joint in the whole coordinate system, $D = \{D_1, D_2, D_3, D_4, D_5, D_6\}$ denotes the amplitudes vector of translational displacement and rotational displacement, D_i is the relative displacement matrix of the i^{th} joint surface, respectively. According to the selected coordinate system, the following equation can be given:

$$\{D_{ji}^R\} = [T_i]^{-1} \cdot \{D_i\} = \{D_{1i}^R, D_{2i}^R, D_{3i}^R, D_{4i}^R, D_{5i}^R, D_{6i}^R\} \quad (8)$$

where,

T_i : The transform matrix
D_{ji}^R : The relative displacement matrix of the i^{th} joint surface in the local coordinate system

For a bolt joint, the relation of deformation and load is nonlinear. Assuming the influence of joint relative vibration displacement for stiffness is ignored, the displacement amplitudes at point A (x_1, x_2, x_3) on the i^{th} joint surface are given as follows:

$$\begin{cases} \lambda_{1i} = \lambda_{P_{1i}} - D_{1i}^R + x_2 D_{6i}^R - x_3 D_{5i}^R \\ \lambda_{2i} = \lambda_{P_{2i}} - D_{2i}^R + x_1 D_{6i}^R - x_3 D_{4i}^R \\ \lambda_{3i} = \lambda_{P_{3i}} - D_{3i}^R + x_1 D_{5i}^R - x_2 D_{4i}^R \end{cases} \quad (9)$$

where,

$\lambda_{1i}, \lambda_{2i}$ & λ_{3i} : The displacement amplitudes at point A (x_1, x_2, x_3) on the i^{th} joint surface
$\lambda_{P1i}, \lambda_{P2i}$ & λ_{P3i} : The initial displacement amplitudes on the i^{th} joint surface

From Eq. (6) to (7), the dynamic stiffness and damping per unit joint area at point A (x_1, x_2, x_3) on the i^{th} joint surface can be gotten:

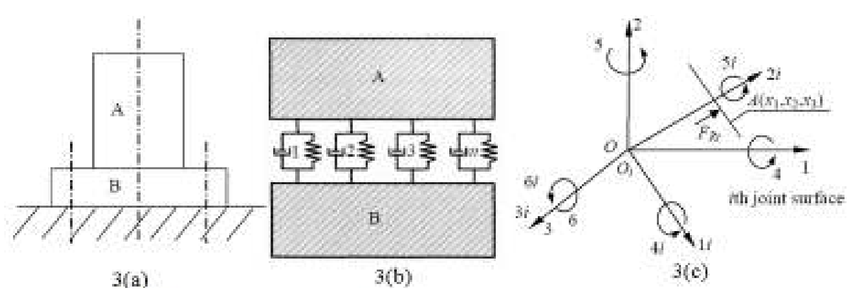

Fig. 3: The equivalent model of a bolt joint

$$\text{Dynamic stiffness}\begin{cases} k_n = \alpha_n p_n^{\beta_n} \omega^{\gamma_n} \left|x_2\right|^{\eta_n} \\ k_\tau = \alpha_\tau p_n^{\beta_\tau} \omega^{\gamma_\tau} \left|x_1\right|^{\eta_\tau} \\ k_3 = \alpha_\tau p_n^{\beta_\tau} \omega^{\gamma_\tau} \left|x_3\right|^{\eta_\tau} \end{cases} \tag{10}$$

$$\text{Damping}\begin{cases} c_n = \alpha_{nc} p_n^{\beta_{nc}} \omega^{\gamma_{nc}} \left|x_2\right|^{\eta_{nc}} \\ c_\tau = \alpha_{\tau c} p_n^{\beta_{\tau c}} \omega^{\gamma_{\tau c}} \left|x_1\right|^{\eta_{\tau c}} \\ c_3 = \alpha_\tau p_n^{\beta_{\tau c}} \omega^{\gamma_{\tau c}} \left|x_3\right|^{\eta_{\tau c}} \end{cases} \tag{11}$$

Based on the above equations, the unit force equations at point A (x_1, x_2, x_3) on the joint surface can be gotten:

$$\begin{cases} P_{1i} = \left(k_n + i\omega c_n\right)\lambda_{2i} \\ P_{2i} = \left(k_\tau + i\omega c_\tau\right)\lambda_{1i} \\ P_{3i} = \left(k_3 + i\omega c_3\right)\lambda_{3i} \end{cases} \tag{12}$$

Based on the Eq. (10) and (12), the forces on the i^{th} joint surface are gotten by integration of the unit forces over the area of the joint surfaces as follows:

$$\begin{cases} F_{1i} = -\iint_{S_i} P_{1i} ds_i \\ F_{2i} = -\iint_{S_i} P_{2i} ds_i + \sum_{j=1}^{N_i}\left(P_{ji} + \Delta P_{ji}\right) \\ F_{3i} = -\iint_{S_i} P_{3i} ds_i \\ F_{4i} = -\iint_{S_i} P_{3i} x_2 ds_i + \iint_{S_i} P_{2i} x_3 ds_i - \sum_{j=1}^{N_i} D_{3i}{}^R\left(P_{ji} + \Delta P_{ji}\right) \\ F_{5i} = -\iint_{S_i} P_{1i} x_3 ds_i + \iint_{S_i} P_{3i} x_1 ds_i \\ F_{6i} = -\iint_{S_i} P_{2i} x_1 ds_i + \iint_{S_i} P_{1i} x_2 ds_i + \sum_{j=1}^{N_i} D_{1i}{}^R\left(P_{ji} + \Delta P_{ji}\right) \end{cases} \tag{13}$$

where,

ΔP_{ij} = α EA/l ($D_2 + D_{1i}^R D_6 - D_{3i}^R D_4$)
j = 1, 2,…N_i
N_i = The total number of contact surfaces on the i^{th} bolt joint surface
P_{ji} = The pretightening force of the i^{th} bolt joint
ΔP_{ji} = The force change of connective bolt
l, E, A, a = The length, basal area, longitudinal elastic ratio, influence coefficient of a bolt joint, respectively

According to the definition of dynamic stiffness, the complex stiffness equations on the i^{th} joint surface can be deduced. Owing to D_{ji}^R are included in λ_{1i}, λ_{2i}, λ_{3i}, the following equations is gotten:

$$K_{J_{ij}} = \frac{\partial F_{ji}}{\partial D_{ji}{}^R} = \frac{\partial F_{ji}}{\partial \lambda_{1i}} \cdot \frac{\partial \lambda_{1i}}{\partial D_{ji}{}^R} + \frac{\partial F_{ji}}{\partial \lambda_{2i}} \cdot \frac{\partial \lambda_{2i}}{\partial D_{ji}{}^R} + \frac{\partial F_{ji}}{\partial \lambda_{3i}} \cdot \frac{\partial \lambda_{3i}}{\partial D_{ji}{}^R} \tag{14}$$

Substitute Eq. (13) into (14), all elements expressions of complex stiffness matrix K_{Jij} can be reduced. At last, the complex stiffness equation for the i^{th} joint surface is expressed as:

$$\left\{F_{ji}\right\} = \left[K_{J_{ij}}\right]\left\{D_{ji}{}^R\right\} \tag{15}$$

The total force on all of bolt joint surfaces can be gotten as follows:

$$\left\{F_j\right\} = \left(\sum_{i=1}^{N}[T_i]\cdot[K_{J_{ij}}]\cdot[T_i]^{-1}\right)\cdot\left\{D_j{}^R\right\} \tag{16}$$

According to Eq. (16), the complex stiffness matrix of the bolt joint is deduced:

$$\left[KC_J\right] = \sum_{i=1}^{N}[T_i]\cdot[K_{J_{ij}}]\cdot[T_i]^{-1} \tag{17}$$

In Eq. (17), $KC_J = K_J + i\omega C_J$. From the above discussion, it can be seen that the complex stiffness

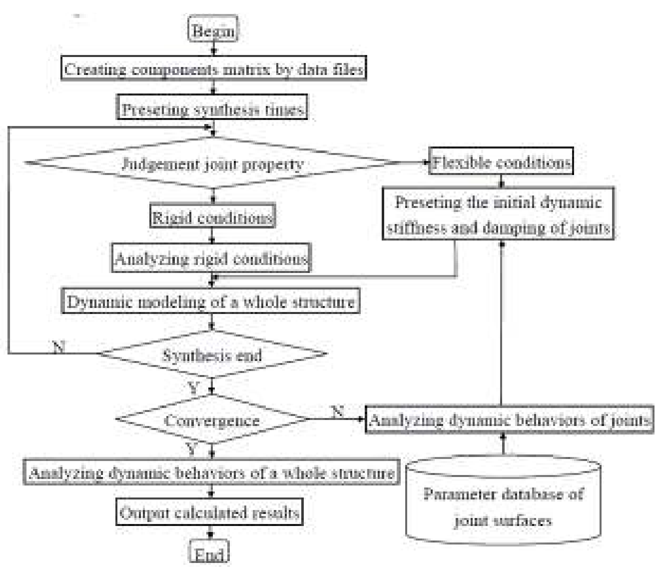

Fig. 4: The flowchart of calculating dynamic characteristics

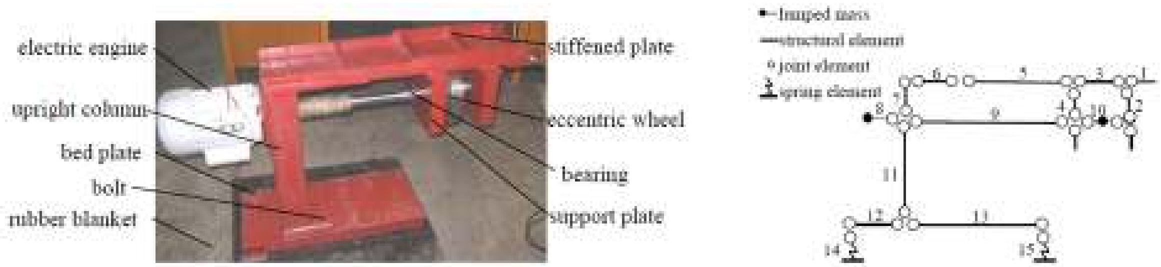

Fig. 5: The entity structure and BEM model

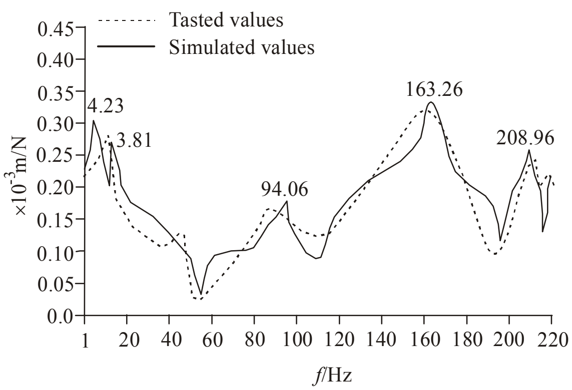

Fig. 6: The curve of frequency characteristics

matrix of the bolt joint includes all the factors that affect the dynamic characteristics of the joint and it has very important physical meaning. Analysis of the dynamic characteristic of joints proposed in this study is also fit for other types of joints, such as cylinder joints, guide way joints, etc. So, the introduced approach can be used to analyze the dynamic characteristics of many joints by the similar modeling method.

NUMERICAL SIMULATION

According to the above modeling process of a whole structure with joints, the corresponding procedure has been developed to analyze dynamic characteristics of the whole structure with joints and the corresponding flowchart is shown in Fig. 4. A whole structure including bolt and cylinder joints will be analyzed by BEM and the entity structure is shown in Fig. 5. In Fig. 5, the electric engine and eccentric wheel are equivalent as the lumped mass, the upright column, bed plate and support plate are equivalent as plate parts, the bridging beam are equivalent as beam parts, the ground base is equivalent as springs. The bolt joints mainly exist in bolt pontes and the cylinder joints mainly exist in the bearing and support plate. At last, the calculating model of the whole structure with joints is established by the BEM, as shown in Fig. 5.

The frequency curve is presented by simulation, as shown in Fig. 6. At the same time, the corresponding vibrational experiments are executed and the experimental results are also presented in Fig. 6. It can be seen from Fig. 6 that the maximum difference between the calculated values and experiments' is 10.1%. Hence, the BEM model calculated method is correct by BEM.

CONCLUSION

The aim of this study is to study the simulation method of a whole structure with joints, the results shows that the proposed method is easy to introduce joints characteristics in the course of dynamic modeling and the method can analyze the dynamic modeling of a whole structure with joints based on the BEM. So, this research can lay good foundation for analyzing dynamic behaviors of other similar structure. But, the proposed model is far from a mature approach. To make it more substantial, it is suggested that the future work focuses on the following areas:

- Determine all kinds of dynamic basic parameters of joints and then modify the proposed model with joints to improve the calculated precision.
- Extend the present model to explore the influence rules of structural modification for dynamic characteristics of a whole structure with joints based on the developed procedure and establish an analytic platform for structural modification and optimum design of a whole structure in future.

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