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Research Article

An Approach for Self-Calibration by a Quartered Circle

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Abstract: The camera calibration is a key step for converting a projective reconstruction into a metric one, which is equivalent to recovering the unknown intrinsic parameters with each image. A circle is a common geometric primitive for the camera self-calibration. To avoid the limit of circle center to the camera self-calibration in a planar template, a method how to solve out the vanishing line is proposed. Then using the property of vanishing line, the camera intrinsic parameters are figured out and the camera self-calibration is achieved. The template contains a quartered circle in the study. Firstly, the camera is used to take photographs of the template from three or more direction. Secondly, using the ellipse and two mutually perpendicular diameters which are extracted from the image, according to the polarity principle and making use of the invariant property of cross-ratio of four lines which are intersected in one same point, the vanishing line can be solved out. In the end, the circular points can be figured out by the intersection between vanishing line and the circle. And using the property of the circular points, the self-calibration is realized.

Keywords: Circular points, cross-ratio, polarity principle, vanishing line, quartered circle

INTRODUCTION

The camera calibration is a key step to realize Three-Dimensional (3D) information from Two-Dimensional (2D) images of objects. Because of the superiority of circular calibration object, it is widely used. The tradition method based on circular calibration object has a high precision, but the setting of circular object as a frame of reference is not convenient (Kim *et al*., 2005; Zhao and Liu, 2010). In contrast, the self-calibration is an easy and flexible method (Pei-Cheng *et al*., 2007). In recent years, the self-calibration method based on circular calibration object has been studied widely (Chen and Zhao, 2011; Lv *et al*., 2011; Zhao *et al*., 2012).

Lv *et al*. (2011) making use of a circle, which is divided into eight equal parts by two diameters and according to the property of the harmonic conjugate points, the vanishing points in the four diameters are figured out to finish the camera self-calibration. Zhao *et al*. (2012) uses a plane template containing three circles of which the centers of circles are different. According to the relationship between pole and polar line, the vanishing line can be solved and then the intrinsic parameters are figured out. But literatures (Lv *et al*., 2011; Zhao *et al*., 2012) are both need the circle center, which requires an accurate method to obtain the centers of circles.

In this study, according to the polarity principle, its corollary and the property of the cross-ratio of four lines intersecting at a common point, the vanishing line in the plane can be gotten by using a quartered circle without the need to know the center of a circle, so that the self-calibration can be finished with the circular points. This method not only has a simple principle, but the template is also easy to make or find out in the real world. And the simulation and the real experiments show that the method is feasible.

CALIBRATION PRINCIPLES

Camera models: The camera is of pinhole model in the study, which is the simplification of optical imaging process. Supposing point $p = [u, v]^T$ is a 2D point on the image of which the homogeneous coordinate is $p = [u, v, 1]^T$ and point $P = [x, y, z]^T$ is a 3D point in space of which the homogeneous coordinate is $P = [x, y, z, 1]^T$, the relationship between p and p is shown as follows:

$$\lambda p^{'} = K[R \quad t]P^{'} \tag{1}$$

where, λ is a non-zero arbitrary proportionality coefficient, $K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$ is the camera intrinsic matrix, S is the parameter describing the skew of two images axes, f_u and n f_v are the scale factors in image u and v axes, (u, v_0) is the coordinate of principal point and [R T] is called the extrinsic matrix, which shows the location of Camera Coordinate System (CCS)

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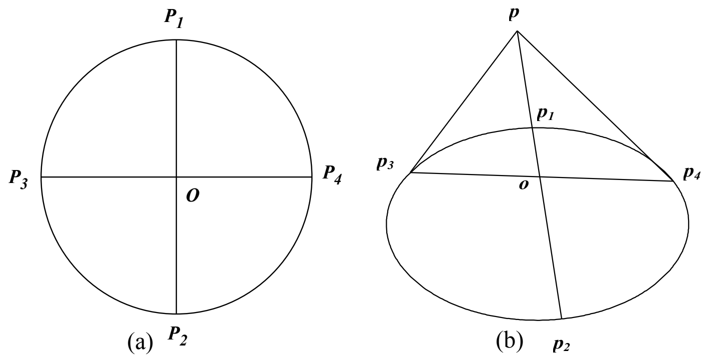

Fig. 1: (a) The template of a quartered circle and (b) the image of the template, p_1p_2, p_3p_4 are two mutually perpendicular diameters, O is the center of a circle; p_1p_2, p_3p_4 are images of p_1p_2, p_3p_4, point p is the image of $P_{1\infty}$ which is the points at infinity of P_1P_2, o is the image of O

relative to the World Coordinate System (WCS), where R is a 3×3 matrix, T is a translation vector (Zhao and Lv, 2012; Li and Zhao, 2012)

Calculating the vanishing line:

Definition 1: (Kneebone and Semple, 1998)**:** supposing P is a point in the plane, Γis a non-degeneracy conic, if P is not in the Γ,then the locus line p of the conjugate point of P about Γ is called the polar line of point P about Γ,otherwise, the P is called the pole of line P about Γ.

Theorem 1: (Kneebone and Semple, 1998) (Polarity Principle): The polar line P of point P about a non-degenerate conic Γ passes through the point $Q\Leftrightarrow$ the polar line q of point Q about Γ passes through the point P. According to Theorem 1, it's easy to get Corollary 1 as follows:

Corollary 1: According to the polar transformation about the non-degenerate conic, the cross-ratio of four collinear points is equal to the cross-ratio of their corresponding four (coplanar and) concurrent lines at one point.

Theorem 2: (Kneebone and Semple, 1998)**:** Let $p_i \in S(p)$ (i = 1, 2, 3, 4) be four lines through one point. If (p_1 p_2, p_3p_4) = k (k ≠ 0, 1, ∞) () and the coordinates of three of the four lines are known, the coordinate of the fourth line can be uniquely determined.

Theorem 3: (Wu, 2008)**:** If (a, b, a+λ_1b, a+λ_2b) are homogeneous coordinates of l_1, l_2, l_3, l_4, which are four different lines through one point, then we have:

$$(l_1l_2,l_3l_4)=\frac{\lambda_1}{\lambda_2},\lambda_1\lambda_2(\lambda_1-\lambda_2)\neq 0 \tag{2}$$

Proposition: If a quartered circle can be detected from an image plane, the vanishing line can be obtained by two mutually perpendicular diameters.

Proof: Suppose the temple is shown as Fig. 1a, where P_1P_2, P_3P_4 are two mutually perpendicular diameters, O is the center of a circle. Let the points at infinity of P_1P_2, P_3P_4 be $P_{1\infty}, P_{2\infty}$, respectively. Figure 1b is the image of the template. Let p_1p_2, p_3p_4 be images of p_1p_2, p_3p_4, respectively, o be the image of O and point p be the image of $P_{1\infty}$, which is the vanishing point in the direction of p_1,p_2. Let the tangents of p_3, p_4 about the image of a circle be l_{P3}, l_{p4} the lines in the direction of $p_1p_2p_3p_4$ be l_{p1p2}, l_{p3p4} and the coordinates of p_1, p_2, p_3, p_4 be (u_{p1}, v_{p1}), (u_{p2}, v_{p2}), (u_{p3}, v_{p3}), (u_{p4}, v_{p4}), respectively. As shown in Fig. 1a, from Definition 1, the polar lines of points P_3, P_4 about circle are the tangents at P_3, P_4 in the plane circle and let them be L_{p3}, L_{p4}, respectively. It's easy to know that the polar line of point O is the line at infinity, the polar line of $P_{1\infty}$ about the circle is the line in the direction of P_3P_4 and let them be L_∞ and L_{p3p4}, respectively. In the circle on the plane, according to the theory of harmonic conjugate in projective geometry, we have:

$$(P_3P_4,OP_\infty)=-1 \tag{3}$$

Then according to the Corollary 1, we get:

$$(L_{p3}L_{p4},L_\infty L_{p3p4})=(P_3P_4,OP_\infty)=-1 \tag{4}$$

As shown in Fig. 1b, because l_{p3}, l_{p4} are the image of the tangents about circle, l_{p3}, l_{p4} intersect at the vanishing point p, which means pp_3, pp_4 are in the direction of l_{p3}, l_{p4}, respectively. Let the image of the vanishing point P_∞ in the direction of p_3p_4 be p_m, according to the correspondence of the corresponding point and the invariance of cross-ratio in the projective transformation, we obtain:

$$(p_3 p_4, op_m) = -1 \tag{5}$$

And because the projective transformation keeps the polarity relationship constant, from Corollary 1 we have:

$$(l_{p3} l_{p4}, l_\infty l_{p1p2}) = (p_3 p_4, op_m) = -1 \tag{6}$$

The next step is to obtain the vanishing line according to Theorem 3. From Theorem 2, if the coordinates of three lines l_{p3}, l_{p4}, l_{p1p2} are gotten, the coordinate of l_∞ can be determined uniquely, which is the vanishing line of the plane. Let the matrix of the image of circle C be c, l_{p3} can be solved from $S_{p3} = 0$, which is the tangent of point p_3 about C, that is:

$$(u_{p3}, v_{p3}, 1)c(x_1, x_2, x_3) = 0 \tag{7}$$

The extended representations of Eq. (7) can get the coordinate of $l_{p3,}$ can be gotten and is denoted by l_{p3}. Similarly, the tangent l_{p4} of point p_4 about C can be solved from $S_{p4} = 0$, that is:

$$(u_{p4}, v_{p4}, 1)c(x_1, x_2, x_3) = 0 \tag{8}$$

From Eq. (8), the coordinate of l_{p4} can be gotten, which is denoted by l_{p4}. From the coordinates of points p_1, p_2, the coordinate of l_{p1p2} can be solved, which is denoted by l_{p1p2}. So, we have:

$$x_1 l_{p1p2} = l_{p3} + x_2 l_{p4} \tag{9}$$

$$y_1 l_\infty = l_{p3} + y_2 l_{p4} \tag{10}$$

Because there is (l_{p3}, l_{p4}, l_∞ l_{p1p2}) - 1, have $y_2/x_2 = -1$, that is:

$$y_2 = -x_2 \tag{11}$$

So substitute the coordinates of l_{p3}, l_{p4}, l_{p4}, l_{p1p2} into Eq. (9) and solve it for x_2. Next, from Eq. (11), y_2 can be also calculated. And then the coordinate of vanishing line l_∞ can be obtained, Proof is completed.

Calculating the camera intrinsic parameters: Because any circle in a plane and the vanishing line must intersect at the two circular points, according to the nature of perspective transformation, the points of intersection between the image of circle and the image of the line at infinity (vanishing line) are the image of circular points (Meng and Hu, 2003). In the template, if the vanishing line is known, by calculating the intersection point between the vanishing line and the circle, the circular points can be obtained. Then making use of the method (Meng and Hu, 2003), the intrinsic parameter matrix for the camera can be gotten the matrix of the camera intrinsic parameter, so that the self-calibration can be realized. In the end, the self-calibration approach can be summarized as follows:

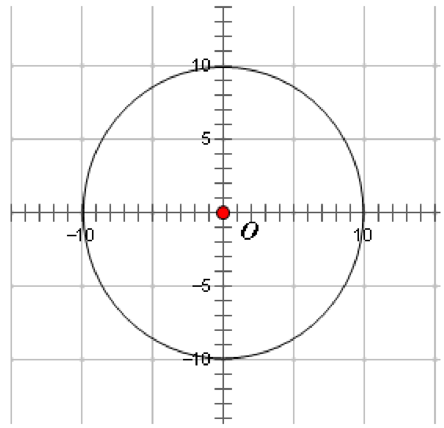

Fig. 2: Simulation picture

Step 1: Print a plane template containing a quartered circle and paste it on a hard surface.

Step 2: Change the relative position between the camera and the template and take 3 or more different pictures.

Step 3: Input these pictures and extract the ellipses and the two diameters.

Step 4: Solve the vanishing line from Eq. (9) and Eq. (11).

Step 5: Get the circular points and the camera intrinsic parameters to complete the self-calibration.

EXPERIMENTS

Simulation experiments: In the simulation experiments, the template is shown as Fig. 2, where the radius of this circle is 10 and the center coordinates are (0, 0).The camera intrinsic parameters are set to $\begin{bmatrix} 2000 & 0.2 & 800 \\ 0 & 2000 & 650 \\ 0 & 0 & 1 \end{bmatrix}$. The experiments were taken 3 simulation pictures, of which corresponding rotation matrixes were:

$$R_1 = \begin{bmatrix} 0.0080953 & 0.96861 & -0.24845 \\ 0.97668 & -0.060969 & -0.20587 \\ -0.21455 & -0.24099 & -0.94651 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} -0.97668 & 0.99932 & -0.02043 \\ 0.98461 & 0.026778 & -0.17269 \\ -0.17202 & -0.025428 & -0.98476 \end{bmatrix}$$

And

$$R_3 = \begin{bmatrix} -0.42245656512663 & -0.28102183151253 \\ -0.62381212869549 & -0.59958188366975 \\ -0.657562833249117 & 0.7935191665090 \end{bmatrix}$$

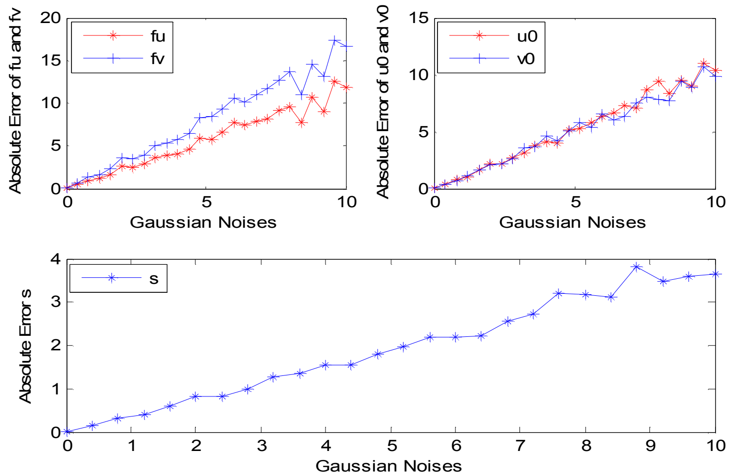

Fig. 3: The curves of the absolute error of the five camera intrinsic parameters (a) f_u, f_v(b) u_0, v_0(c) s under different noises levels

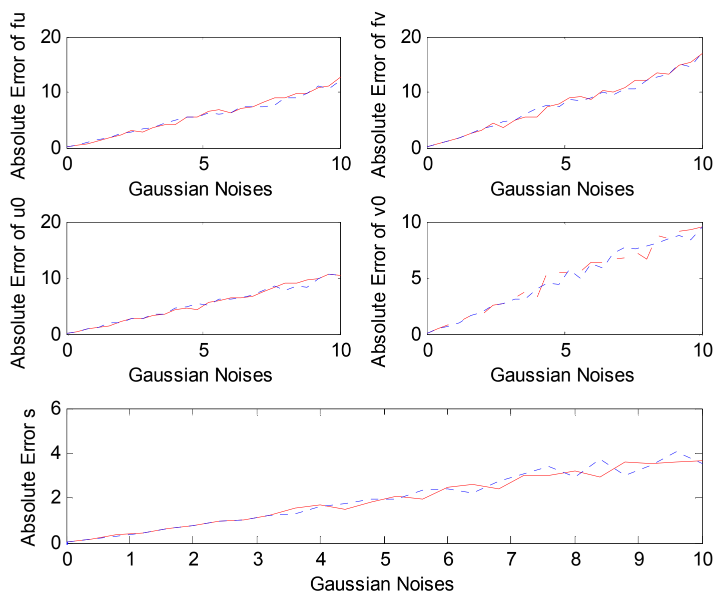

Fig. 4: The comparison of the results (a) f_u, f_v(b) u_0, v_0(c) s between the methods in this study and the literature (Meng and Hu, 2003) in the simulation experiments

$$\begin{bmatrix} -0.86171757600432 \\ 0.50135814830063 \\ 0.07799247616154 \end{bmatrix}$$

The transformation vectors of the camera were:

$$T_1 = [-115.05; -65.925; 431.2]$$
$$T_2 = [-101; -77.591; 381.17]$$
$$T_3 = [-142.56; -34.891; 521.456]$$

To verify the robustness of the approach in the study, a Gaussian noise with 0 mean and σ standard deviation was added to each projected image points. The noise varied from 0 to 10 pixels. All the results in the experiments were the average value of 100 independent trials. The standard deviations of the five intrinsic parameters at each different noise level were computed, which were shown in Fig. 3.

We compared the approach in the study with in the literature (Meng and Hu, 2003). All the parameters were the same set in the process of calibration of the two approaches, such as the intrinsic and extrinsic parameters, the numbers of pictures and the feature points and so on, so as to ensure the comparability of the experiment results. The absolute errors of the five intrinsic parameters gained by the two approaches are list in Fig. 4, where the solid line represents the results of the approach in this study and the dashed line represents the results of the approach in the literature (Meng and Hu, 2003). What we can know from the curve is that the two calibration results are similar linearly increasing with the

(a) (b) (c)

Fig. 5: Taking three pictures (a) (b) and (c) from different directions

Table 1: The comparison of the results of the two approaches

Calibration method	5 intrinsic parameters
Our approach	$\begin{bmatrix} f_u = 546.237, & f_v = 540.774 \\ s = 3.2543, u_0 = 330.7321, & v_0 = 270.5743 \end{bmatrix}$
The approach in Zhang (2000)	$\begin{bmatrix} f_u = 527.493, & f_v = 529.325 \\ s = 0, \quad u_0 = 321.857, & v_0 = 225.382 \end{bmatrix}$

strength of noise increasing and the curves of the two results are basically coincident, which is said that the results accuracy of this study and the literature (Meng and Hu, 2003) are consistent.

Experiments with real data: In this study, put a CD on a checkerboard, which can form a quartered circle. Paste the template on the wall and take three pictures from different directions, which are shown as Fig. 5. The image resolution is 640×480. Using these three pictures to calibrate the camera, the method in this study was compared with the method in the literature (Zhang, 2000). And the results of the two approaches are shown in Table 1.

CONCLUSION

In this study, if there is a quartered circle detected, the vanishing line can be obtained by two diameters perpendicular to each other. Based on this, we propose a camera self-calibration approach using a quartered circle. The simulation and experiments with real data show that the calibration theory is simple and has a high accuracy. In the study, the calibration accuracy depends critically on the image edge extraction and the linear fitting. So, in practical application process, we should pay attention to select appropriate methods for edge detection and linear fitting to improve the accuracy of calculating results.

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