Research Journal of Applied Sciences, Engineering and Technology 6(3): 382-386, 2013
DOI:10.19026/rjaset.6.4090
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: July 12, 2012 Accepted: September 08, 2012 Published: June 15, 2013

Research Article

Evaluation of Mode Field Diameter of Step-Index Fibers and Comparison Analysis

[1, 2]Bo Yang, [1]Jian Duan, [1]Zheng Xie and [2]Hongfeng Xiao
[1]State Key Laboratory of High Performance Complex Manufactory, Central South University, Changsha, 410083, China
[2]Institute of Image Recognition and Computer Vision, Hunan Normal University, Changsha 410081, China

Abstract: After detailed reviews of the exact mode field distribution of Single Mode Step-Index (SMSI) fibers and relevant definitions of Mode Field Diameter (MFD) and a careful comparison between them, a new approximate equation to calculate the mode field diameter is presented in this study. This equation is more accurate and flexible to determine MFD of Peterman I than Gaussian spot size with that of 1/e and Marcuse empirical equation, and what's more, have a analytic solution of its inverse problem that can be used to directly calculate normalized frequency, Numerical Aperture (NA) and the cut-off wavelength. In order to evaluate the new equation, a beam propagation method that simulates the distribution of fundamental mode field is adopted. Numerical simulation results indicate that the new equation is in good agreement with the theoretical predictions. The new approximation function of MFD is a high level functional equation for the theoretical study of the characteristics of the single-mode fibers and construction of new special fibers.

Keywords: Beam propagation method, mode field diameter, numerical aperture, **s**ingle mode fiber

INTRODUCTION

The MFD is an important transmission characteristic of single-mode step-index optical fibers, which controls substantial splice loss, disperses loss, and micro bending loss that is sensitive to fiber bend (Li *et al*., 2009; Chen, 2007; Artiglia *et al*., 1989). The basic approaches to measure MFD are Transmitted Near Field (TNF), Variable Aperture (VA), Knife Edge (KE), and Far-Field (FF) technologies recommended by CCITT and IEC international committees (N .Gisin *et al*., 1993; Marcuse, 1977). The standard deviation of TNF to measure conventional fibers is typically 0.2-0.4μm (Gloge, 1971). The Numerical Aperture (NA), related to the ability of optical fibers to receive light, is another important characteristic of single-mode fibers and is determined by the ratio of Refractive Index (RI) of the core to RI of cladding. TNF and FF can be used to measure NA and RI profile as well. The deviation of RI, which is obtained for step-index fibers by Gisin *et al*. (1993) and Peterman (1976), is less than 2.0×10^{-4}. Theoretically, the distribution of fundamental mode field LP_{01} determines the MFD of Single-Mode Fibers (SMF) of various sizes and many equations have been proposed to reveal their relations, but description of MFD with the RI profile in a simply and accurate way is still very difficult. An empirical equation of calculating MFD of single mode fibers, proposed by Marcuse (1977), was ever used to calculate Gaussian beam waist and to determine NA and cut-off wavelength (Gambling and Matsμmura, 1977). However, Marcuse (1977) empirical equation is arbitrary and inaccuracy for theoretical analysis and too complicated to obtain analytic solutions for inverse problem. Therefore, a more appropriate and flexible model is need.

This study attempted to establish a new simply analytic equation between the MFD and NA, which confirms the corresponding optical propagation theory. Firstly, we reviewed model field distribution and 3 MFD definitions used for circularly symmetric fibers. Secondly, we made a careful comparison between three definitions by numerical calculation; thirdly, based on the above results, we constructed a new exponent equation and further presented an analytic solution for the inverse problem. Last, we used Beam Propagation Method (BPM) to compute the mode field distribution of SMSI fibers and MFD of 2 kinds of definitions, Petermann I and 1/e. The simulation showed that the exponent model had good level of fitting the Petermann I diameter and the deviation of that was less than 1% in the range of $1.5\leq V\leq 2.404$.

NUMERICAL APERTURE OF FIBERS

SMSI fibers, commonly used in telecom, are typically circular symmetric waveguide. NA of them is

Corresponding Author: Bo Yang, State Key Laboratory of High Performance Complex Manufactory, Central South University, Changsha, 410083, China

given by $NA = \sqrt{n_1^2 - n_2^2}$, where n_1 and n_2 are the refractive indices of the core and the cladding, respectively. Those fibers, with the relative refractive index difference $\Delta = (n_1\text{-}n_2)/n_1 << 1$, typically $\Delta = 0.003\sim0.008$, are weakly guiding optical waveguides. Normalized frequency of them is defined as:

$$V = \frac{2\pi}{\lambda} aNA \tag{1}$$

where, λ is the wavelength of a light in vacuum and the parameter a is the radius of the core. According to the refractive index profile of those fibers, cut-off wavelength λ_c is given by:

$$\lambda_c = \frac{2\pi}{2.4048} aNA = 2.613aNA \tag{2}$$

NA is an important parameter of fibers, as it directly determines propagation characteristic parameters, i.e., cut-off wavelength and model field diameter. However, NA of commercial fibers, measured by experiments, is slightly large to make theoretical analysis because of approximate theoretical model and Rayleigh-Romman scattering of fiber end. For example, core diameter and NA of a kind of commercial fiber SMF-28 are respectively 8.2 and 0.14μm according to its preference, and thus the normalized frequency V, calculated by Eq.(1) to 2.753, is significantly larger than cut-off frequency of a single-mode fiber, V = 2.4082.

It is known that the RI profiles of a fiber and wavelength of incident light co-determine mode field distribution of an ideal single mode fiber and its mode field diameter. Marcuse (1977) ever gave an empirical formula describing the changes of w_m with V, which is:

$$\frac{w_m}{a} \approx 0.65 + 1.619/V^{3/2} + 2.879/V^6 \tag{3}$$

However, the Marcuse empirical equation is not too accuracy to calculate mode field diameter on basis of V and NA of a fiber, and it is still difficult to obtain value of NA from a measured or known mode field diameter. Hence, a more accurate and appropriate equation is need to rebuild relations of them.

MODE FIELD DISTRIBUTION OF SMSI FIBERS

In practical communication application, only LP_{01} mode exists in ideal single mode fibers to realize a long-distance transmission. Considering about y polarization in Cartesian coordinates, 2 transverse components E_y, E_x and a longitude component E_z of the LP_{01} mode, presented ever by Gloge (1971) and Hussey and Martinez (1985) are:

$$E_y = A\begin{cases} \dfrac{J_0(U\sqrt{y^2}/a)}{J_0(U)}, & |y| \le a \\ \dfrac{K_0(W\sqrt{y^2}/a)}{K_0(W)}, & |y| > a \end{cases} \tag{4}$$

$$E_z = iB\begin{cases} \dfrac{UJ_1(U\sqrt{y^2}/a)\cos\phi}{J_0(U)} & |y| \le a \\ \dfrac{WK_1(W\sqrt{y^2}/a)\cos\phi}{K_0(W)}, & |y| > a \end{cases} \tag{5}$$

$$E_x \approx 0 \tag{6}$$

In the above, A and B are 2 undetermined amplitude constants, and parameter y is a distance away from the centre of the core, J_0 , J_1 are Bessel functions, and K_0 and K_1 are the modified Hankel functions.

When the boundary condition $y = a$ is imposed on the light propagation, the Eigen value equation. of SMSI fibers is determined by:

$$\frac{J_0(U)}{UJ_1(U)} = \frac{K_0(W)}{WK_1(W)} \tag{7}$$

where, the parameter$U = \sqrt{K_0^2 n_1^2 - \beta^2 \alpha}$ relates to the propagation constant β in the fiber core, and the parameter $W = \sqrt{\beta^2 - K_0^2 n_2^2 \alpha}$ is written similarly as U with constant β in the fiber cladding. W and U are related to the V number by the following equation:

$$V^2 = U^2 + W^2 \tag{8}$$

Depending on the Eigen value equation of an ideal step-index fiber and its boundary condition, the frequency V number is limited in the range of $0 \le V \le 2.4048$ to ensure that only LP_{01} mode transmits in the fiber for communication applications.

Practically, the fundamental mode of single-mode fibers is often approximated by a Gaussian distribution and the transverse electric field is simplified to:

$$E_g(r) = Ae^{-\frac{r^2}{w_g^2}} \tag{9}$$

where, $A = \sqrt{\frac{2}{\pi}} \frac{1}{w_g}$ and w_g is the spot size of Gaussian beam waist, which determined by:

$$w_g = \sqrt{2} J_0(U) \frac{Va}{U} \frac{K_1(W)}{K_0(W)} \tag{10}$$

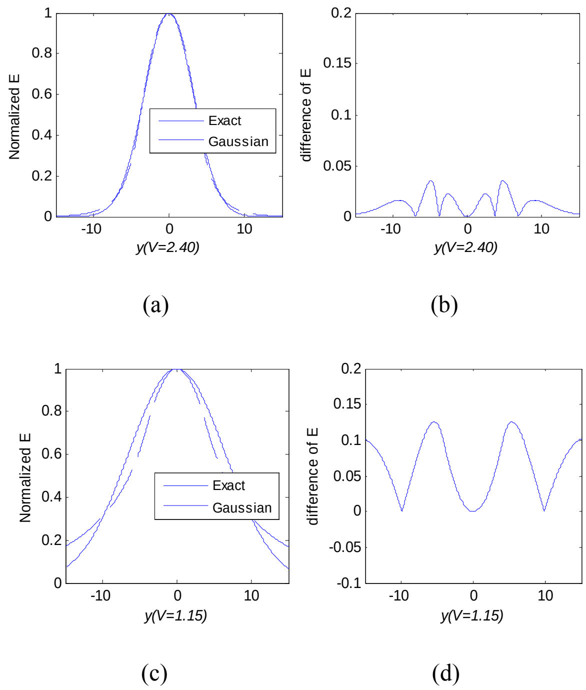

Fig. 1: Deviations of the electric field E distribution between the exact solution and Gaussian approximation with different V number, V = 2.40 and V = 1.15, respectively

Based on Eq.(4) and (9), the exact distribution and Gaussian approximation of electric field of a fiber is denoted in Fig. 1 by a numerical compute method, where the deviation of field mode distributions of a single mode fiber with different V number, V = 1.15 and V = 2.4. It is clear that the Gaussian distribution has a good level to approximate the field E with the max error less than 4% when V = 2.40 denoted in Fig. 1a and b. Nevertheless when V number decreases to V = 1.15, Gaussian approximation of the field E is poor and the max error of that increases to about 13% in Fig. 1c and d.

MODE FIELD DIAMETER CALCULATION

The MFD is a very important parameter of a step-index fiber and directly determine its propagation characteristic. However, the express of mode field distribution of a fiber is so complicated and the analytic solution for MFD is still difficult to obtain. Several kinds of definition of MFD have been proposed for mode field in a lot of literatures (Scarmozzino *et al*., 2000; Ling, 2011; Hu *et al*., 2011) and yet those definitions are not strict equivalent.

There are 3 kinds of MFD widely used in practice, which are Gaussian beam waist $2w_g$ (also known as 1/e, the width of the field when it decreases to 1/e) [8], and Petermann I diameter and Petermann II diameter[10], named as, $2w_{PI}$ and $2w_{PII}$. Marcuse (1977) proposed an empirical equation to evaluate the mode field diameter as the Gaussian beam waist size based on numerical results by Eq.(3), so the Marcuse diameter w_m is still an approximation of w_g. According to the mode field itself, Petermann ever proposed two mode field diameter $2w_{PI}$ and $2w_{PII}$.

On the basis of the near mode field, Petermann gave the definition of Petermann I diameter $2w_{PI}$ of a fiber as:

$$2w_{PI} = 2\sqrt{2\frac{\int_0^\infty r^3 E^2(r)dr}{\int_0^\infty rE^2(r)dr}} \tag{11}$$

where, $E(r)$ is the electric field of near mode field.

Related to far mode field E_{FF}, the mode field diameter of the second Petermann $2w_{PII}$ is:

$$2w_{PII} = 2\sqrt{2\frac{\int_0^\infty E_{FF}^2(q)qdq}{\int_0^\infty E_{FF}^2(q)q^3dq}} \tag{12}$$

Substituting Eq. (4) to Eq. (11), Gambling deduced the following Eq.11:

$$\frac{w_{PI}}{a} = \left[\frac{4}{3}\left(\frac{J_0(U)}{UJ_1(U)} + \frac{1}{2} + \frac{1}{W^2} - \frac{1}{U^2}\right)\right]^{\frac{1}{2}}. \tag{13}$$

After integrals of Eq. (12), the closed form of w_{PII} is obtained[12]:

$$\frac{w_{PII}}{a} = \frac{\sqrt{2}}{W}\frac{J_1(U)}{J_0(U)}. \tag{14}$$

Although above 2 equations. Which are composed of V number and many Bessel functions have exact solutions for mode field diameter, it is still complicated to calculate MFD by them.

According to the Eigen value equation of a single mode fiber and definitions of MFD, we calculated values of different definitions of MFD in the range of $0.5 \le V \le 2.4048$ by numerical methods, which were depicted in Fig. 2. Those real points, plus signs denote w_{PI}, w_{PII} respectively, which are obtained by Eq. (13) and (14). Similarly, circles and star signs describe w_g and w_m. The differences of other 3 kinds of diameter with w_{PI} is shown in Fig. 2b. The w_m is most close to Peterman I diameter, followed by Petermann II. In

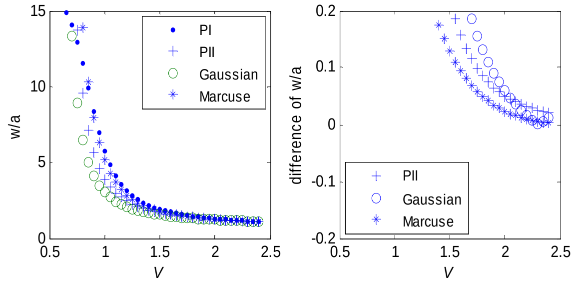

Fig. 2: w_{PI}/a, w_{PII}/a, w_g/a, w_m/a about V and their differences with w_{PI}/a where w_m is most close to w_{PI}, (a) Numerical results of many definitions of MFD, (b) Differences of them with w_{PI}/a

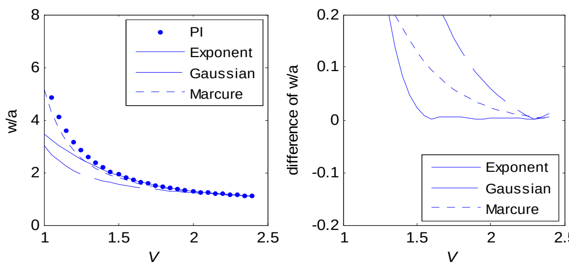

Fig. 3: Results of w_{PI}/a, w_E/a, w_g/a, w_m/a and their differences, (a) Numerical results of w_{PI}/a, w_g/a, w_m/a and w_E/a, (b) differences of w_E/a, w_g/a, w_m/a with w_{PI}/a

short, Fig. 2 shows that the value of those definitions is not completely same.

When $V \to 2.4048$, according to Eq. (7) and (11), the smallest mode field radius is obtained and $w/a \approx 1.10$. Based on the points calculated by Eq. (11) and numerical method, mode field radius, labeled as w_E, can be approximated in an exponent form by the Gauss-Newton fitting method as:

$$\frac{w_E}{a} \approx 172.04\exp(-\frac{(V+3.412)^2}{2.141^2})+1 \tag{15}$$

Figure 3 shows numerical results of the w_{PI}/a and w_E/a in the range of $1.0<V\leq 2.4048$, and the fitting difference of w_E with w_{PI} is less than 1% and less than that of w_m with w_{PI}.

Another significant advantage of Eq. (15) is that we can easily obtain its inverse analytic function:

$$V(w_E/a) = 2.141\sqrt{-\ln(\frac{w_E/a-1}{172.04})} - 3.412\,. \tag{16}$$

Using Eq. (16) and Eq. (1), Numerical Aperture is rewritten in an analytic form as:

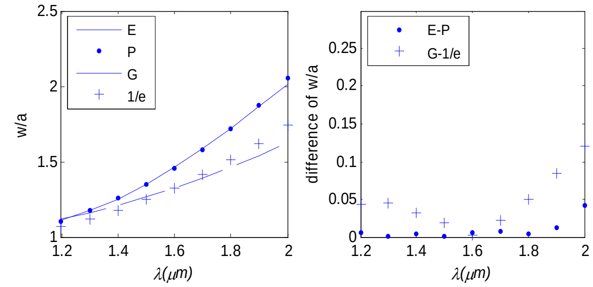

Fig. 4: Simulation results of mode field radius with different λ when NA = 0.1117, a = 4.1, (a) w_E/a, w_g/a by comparison with w_{PI}/a and that of 1/e, (b) differences of w_E/a with w_{PI}/a and w_g/a with that of 1/e

$$NA\,(w_E/a) = V\,(w_E/a)\frac{\lambda}{2\pi a}\,. \tag{17}$$

Similarly, the cut-off of a fiber is:

$$\lambda_c = \frac{\lambda}{2.4048}V\,(w_\lambda/a) \tag{18}$$

where, $2w_\lambda$ is a known MFD correspondent with wavelength λ.

SIMULATION OF FIELD MODE DIAMETER

In order to verify Eq. (15), we used the Finite Difference Beam Propagation Method (FDBPM) to simulate mode field distribution of single-mode step-index fibers, which is a well-known parabolic or paraxial approximation of Helmholtz equation with transparent boundary condition and widely used for propagation simulation of modeling fiber-optic devices. In our experiments, a single-mode step-index fiber was used, which is characterized by NA = 0.1117 and $2a$ = 8.2 μm, and the wavelength of an incident laser into the fiber is 1.3 or 2.0 μm. The mode fields diameters of Petermann I are 9.63 and 16.87μm by numerical compute methods, while the mode field diameters obtained from our exponent approximation are very close to that of Petermann I, 9.62 and 16.52μm respectively.

Generally speaking, the wavelength λ of lasers in communication is set to [1.2, 2.0], and then number V obtained as $1.44\leq V\leq 2.40$. Simulation results of Petermann I diameter and Gaussian (1/e) are shown in Fig. 4.

We selected intentionally some wavelengths of an incident light as listed in the Table 1 and simulated the MFD after calculated the V number. Based on Eq. (15),

Table 1: Simulated MFD and inverse results of *NA* for a fiber with *NA* = 0.1117 and a = 4.1μm

Λ (μm)	V	MFD (w_E, μm)	MFD (w_{PI}, μm)	*NA* simulated	Δ of *NA*
1.2	2.3989	9.094	9.05	0.1127	0.0043
1.3	2.2135	9.616	9.63	0.1115	0.0002
1.5	1.9183	11.068	11.07	0.1117	0.0000
1.6	1.7984	11.978	11.93	0.1120	0.0003
1.7	1.6927	12.994	12.93	0.1121	0.0004
1.8	1.5986	14.099	14.07	0.1119	0.0002
2.0	1.4388	16.520	16.87	0.1102	0.0015

we computed *NA* and their differences inversely which kept very small when those wavelengths below 1.8μm.

CONCLUSION

On the basis of the numerical solution of the mode field diameter of Petermann I with normalized frequency V number, a new simply equation of MFD with parameter V that had a good level of approximation in the range of $1.5 \le V \le 2.40$ was established by nonlinear numerical fitting methods. Since the new approximation equation has a more functional form for the theoretical study of the characteristics of the single-mode fibers, which is adopted to obtain an analytic Eq. about inverse problem conveniently, the mode field diameter can be calculated directly by the fiber geometric parameters, size of the core and refractive index difference, and vice versa. Comparison analyses by numerical calculations and simulations show that the new equation to evaluate Petermann I diameter had a more accuracy than the Marcuse diameter and the Gaussian diameter.

ACKNOWLEDGMENT

This study was supported in part by NSFC under Grant Nos. 51005074.

REFERENCES

Artiglia, M., G. Coppa, P.D. Vita, M. Potenza and A. Sharma, 1989. Mode field diameter measurements in single-mode optical fibers [J]. J. Light Wave Technol., 7(8): 1139-1152.

Chen, C.L., 2007. Foundations for Guided-Wave Optics [M]. John Wiley and Sons Inc., Hoboken, pp: 25-249.

Gambling, W.A. and H. Matsμmura, 1977. Simple characterization factor for practical single-mode fibres. Electron. Lett., 13(23): 691-693.

Gisin, N., R. Passy and B. Perny, 1993. Optical fiber characterization by simultaneous measurement of the transmitted and refracted near field [J]. J. Light Wave Technol, .11(11): 1875-1883.

Gloge, D., 1971. Weakly guiding fibers. Appl. Opt., 10(10): 2252.

Hu, Y., P. Xiong and X. Yang, 2011. Synthesis and characterization of surface-modified tourmaline with aluminic ester. IEIT J. Adapt. Dyn. Comput., (3): 5-9, DOI = 10.5813/www.ieit-web.org/IJADC/.3.2.

Hussey, C.D. and F. Martinez, 1985. Approximate analytic forms for the propagation characteristics of single-mode optical fibers. Electron. Lett., 21(23): 1103-1104.

Li, M.J., P. Tandon, D.C. Bookbinder, S.R. Bickham, M.A. McDermott, R.B. Desorcie, D.A. Nolan, J.J. Johnson, K.A. Lewis and J.J. Englebert, 2009. Ultra-low bending loss single-mode fiber for ftth. J. Light Wave Technol. (27): 376-382.

Ling, P.W., 2011. The needed optimal cycle for prediction accuracy of single stock price behavior in Taiwan by fractals theory. IEIT J. Adapt. Dyn. Comput., (3): 1-4. DOI=10.5813/www.ieit-web.org/IJADC/.3.1.

Marcuse, D., 1977. Loss analysis of single-mode fiber splices [J]. Bell Sys. Tec. J. 56(5): 703-18.

Peterman, K., 1976. Microbending loss in monomode fibers. Electron. Lett., 12: 107-109.

Scarmozzino, R., A. Gopinath, R. Pregla and S. Helfert, 2000. Numerical techniques for modeling guided-wave photonic devices. J. Selected Top. Quantμm Elec., 6: 150.