

## Research Article

### Wideband Radar Spread Targets Detection in Compound Gaussian Clutter

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**Abstract:** The aim of this study is to analyze the influence of neglecting compound Gaussian clutter texture on wideband radar targets detection. The texture of compound-Gaussian clutter is researched, the Probability Density Function (PDF) expression of G0 distributed clutter is proposed. The optimal detection statistics treats the texture of G0 distributed clutter as a certain function is derived. As contrast, another detector neglects the clutter texture is derived. The numerical results are presented by means of Monte Carlo simulation strategy. Assume that cells of signal components are available. Those secondary data are supposed to possess either the same covariance matrix or the same structure of the covariance matrix of the cells under test. The simulation results highlight that the performance loss of the two detectors in different roughness parameter, the result shows the loss less than 2dB due to the texture is neglected and adaptively estimating the covariance matrix.

**Keywords:** G0 distributed clutter texture, GLRT, Monte Carlo methods, wideband radar distributed targets detection

## INTRODUCTION

The problem of detecting spread targets has received great attention recently. It naturally arises that the detecting performance of High Resolution Radars (HRR). The clutter is very complex and it cannot be considered as Gaussian distributed (Yang, 2007; Kay, 1988). Much work has been studied towards the clutter model in compound-Gaussian (Conte *et al.*, 2000, 2002; Gini and Farina, 2002).

Various distributed detection algorithms in non-Gaussian background have been studied (Conte *et al.*, 2000, 2002; Gini and Farina, 2002; Vicen-Bueno *et al.*, 2008; Robey *et al.*, 1992; Miao and Iommelli, 2008; Shuai, 2011; Pascal *et al.*, 2008; Bon *et al.*, 2008). The spikiness of clutter is usually modeled as a compound-Gaussian vector (Conte *et al.*, 2000, 2002; Gini and Farina, 2002; Vicen-Bueno *et al.*, 2008; Miao and Iommelli, 2008; Shuai, 2011; Bon *et al.*, 2008). The Probability Density Function (PDF) information of clutter texture is always modeled as an unknown deterministic parameter in every range cell (Yang, 2007), there out the detectors needless to estimate some parameters and have a simple form, which made it easily to be operated. However, it needs to deeply research that whether the facilitation leads a large performance loss to these detectors.

We mainly concern the detection performance influence of detector when the clutter texture is neglected. In the study, we derive detectors in two different situations. One detector treats the texture of clutter as an unknown determinate parameter; another

treats the texture as a function obeying certain PDF. Finally the performance analyses of the tests are carried out via Monte Carlo simulations and the influence of clutter texture is discussed.

**Problem statements:** Assume that a coherent train of N pulses is transmitted by the radar and that the incoming waveform of the receiver is properly demodulated, filtered and sampled. The binary hypothesis can be written as:

$$\begin{cases} H_0 : \mathbf{z}_t = \mathbf{c}_t & t = 1, 2 \dots L, L+1 \dots L+K \\ H_1 : \begin{cases} \mathbf{z}_t = \alpha_t \mathbf{p} + \mathbf{c}_t & t = 1, 2 \dots L \\ \mathbf{z}_t = \mathbf{c}_t & t = L+1, \dots, L+K \end{cases} \end{cases} \quad (1)$$

where,

T : The transpose operator

P : The steering vector

$\alpha_t$  : Unknown deterministic parameter which account for the channel propagation effects and the target reflectivity

$z_1, z_2 \dots z_L$ : Collected from cells under test that are referred as primary data

$z_{L+1}, z_{L+2} \dots z_{L+K}$ : Secondary data which not contain any useful target echo and exhibit the same structure of the covariance matrix as the primary data

$\mathbf{z} = [z(0) \dots z(N-1)]^T$ : An N dimensional vector and N are the numbers of complex samples

Here the received data vectors are assumed to be independent between each range cell.

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The expression of compound-Gaussian clutter can be written as  $c_t = \sqrt{\tau_t} g_t$  where  $\tau_t$  is the texture of clutter and it obeys certain Probability Density Function (PDF). The speckle  $g_t$  is modeled as a zero mean complex Gaussian vector with covariance matrix  $M$ ,  $E\{xx^H\} = M$ , where,  $E\{\cdot\}$  denotes statistical expectation operator. Texture  $\tau_t$  is a positive random variable with an unknown PDF. The amplitude of clutter can be defined as:

$$q(c_t) = c_t^H M^{-1} c_t \quad (2)$$

$$p(c_t) = |M|^{-1} \pi^{-N} \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{2\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \quad (3)$$

Let:

$$h_N^G(q(c_t)) = \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \quad (4)$$

From the formulas (3) and (4), we can see the PDF of clutter is related with the PDF of texture in every range cell. So, in order to research the influence of texture to detection performance of detector, it needs only to study the PDF form of each texture. As all we known, the compound-Gaussian clutter includes Weibull distributed clutter, K distributed clutter and G0 distributed clutter and so on.

In this study, we research G0-distributed clutter deeply. The PDF of G0 clutter amplitude  $r = |c_t|$  can be written as:

$$p_R(r) = \frac{2\alpha\beta^\alpha r}{(\beta + r^2)^{1+\alpha}} \quad (5)$$

where,

$\alpha$  : Roughness parameter

$\beta$  : Scaling parameter

The clutter texture  $\tau_t$  is inverse Gamma distributed:

$$p_\tau(\tau) = \frac{1}{\Gamma(\alpha)} (\beta)^\alpha \tau^{-\alpha-1} e^{-\frac{\beta}{\tau}} > 0 \quad (6)$$

The amplitude of clutter can be written as:

$$p(c_t) = |M|^{-1} \pi^{-N} h_N^G(q(c_t)) \quad (7)$$

$$h_N^G(q(c_t)) = \int_0^\infty \tau_t^{-N} \exp\left(-\frac{q(c_t)}{\tau_t}\right) p_{\tau_t}(\tau_t) d\tau_t \quad (8)$$

$$= \beta^\alpha (\beta + q(c_t))^{-N-\alpha} \Gamma(N + \alpha) / \Gamma(\alpha)$$

### Detector design:

#### Optimal detector design under G0 clutter (NP-G0):

According to Neyman-Pearson (NP) criterion, the optimal detection statistics is the Likelihood Ratio Test

(LRT), under G0 clutter background can be expressed as Shuai (2011):

$$\frac{\max_{\alpha_t} p(\mathbf{z}_{1:L} | \alpha_t; H_1)}{p(\mathbf{z}_{1:L} | H_0)} \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (9)$$

Then we can obtain the PDF of target echo under  $H_0, H_1$  which can be expressed as:

$$p(\mathbf{z}_{1:L} | H_0) = |M|^{-L} \pi^{-NL} \prod_{t=1}^L h_N^G(q_0(\mathbf{z}_t)) \quad (10)$$

$$= |M|^{-L} \pi^{-NL} \beta^{L\alpha} \Gamma^L(N + \alpha) / \Gamma^L(\alpha) \prod_{t=1}^L (\beta + q_0(\mathbf{z}_t))^{-N-\alpha}$$

$$p(\mathbf{z}_{1:L} | H_1) = |M|^{-L} \pi^{-NL} \prod_{t=1}^L h_N^G(q_1(\mathbf{z}_t)) \quad (11)$$

$$= |M|^{-L} \pi^{-NL} \beta^{L\alpha} \Gamma^L(N + \alpha) / \Gamma^L(\alpha) \prod_{t=1}^L (\beta + q_1(\mathbf{z}_t))^{-N-\alpha}$$

where,  $q_1(\mathbf{z}_t) = (\mathbf{z}_t - \alpha_t p)^H M^{-1} (\mathbf{z}_t - \alpha_t p)$  and  $q_0(\mathbf{z}_t) = \mathbf{z}_t^H M^{-1} \mathbf{z}_t$ , substitute (10) and (11) into (9) and estimation targets information  $\alpha_t$ , because the PDF in (11) is monotonically decreasing function of  $q_1(\mathbf{z}_t)$ ,  $\max_{\alpha_t} p(\mathbf{z}_{1:L} | \alpha_t; H_1)$  equals to  $\min_{\alpha_t} q_1(\mathbf{z}_t)$ , that is the Maxima Likelihood Estimation (MLE) of  $\alpha_t$  can be expressed as  $\hat{\alpha}_t = \arg \min_{\alpha_t} q_1(\mathbf{z}_t)$ , by simple calculating, it can rewritten as  $\hat{\alpha}_t = \frac{p^H M^{-1} \mathbf{z}_t}{p^H M^{-1} p}$ , substitute  $\hat{\alpha}_t$  into  $q_1(\mathbf{z}_t)$  and let  $\hat{q}_1(\mathbf{z}_t) = (\mathbf{z}_t - \hat{\alpha}_t p)^H M^{-1} (\mathbf{z}_t - \hat{\alpha}_t p)$ , after simple calculating, the equation with G0 distributed clutter can be written as follows:

$$\frac{\prod_{t=1}^L \left( \frac{\beta + \hat{q}_1(\mathbf{z}_t)}{\hat{q}_1^2(\mathbf{z}_t)} \right)^{N+\alpha}}{\prod_{t=1}^L \left( \frac{\beta + q_0(\mathbf{z}_t)}{q_0^2(\mathbf{z}_t)} \right)^{N+\alpha}} \underset{H_0}{\overset{H_1}{\geq}} \eta \quad (12)$$

where, both parameter  $\alpha$  and  $\beta$  are known:

**Generalized Likelihood Ratio Test (GLRT) designs under G0 clutter:** As contrast, the detection statistics of neglecting the clutter texture is given, the expression of GLRT can be written as Robey *et al.* (1992) and Shuai (2011):

$$T_G = -N \sum_{t=1}^L \ln \left[ 1 - \frac{|p^H M^{-1} \mathbf{z}_t|^2}{(p^H M^{-1} p)(\mathbf{z}_t^H M^{-1} \mathbf{z}_t)} \right] \quad (13)$$

**Covariance matrix estimation:** In Eq. (12) and (13), the covariance matrix  $M$  is unknown; we need to estimate it via secondary data vector.

The estimator is obtained by maximizing the likelihood function by  $M$ , which can be written as:

$$\hat{M}_{MLE} = \arg \max_M p(\mathbf{z}_{1:K} | t_{1:K}, M) \quad (14)$$

In formula (14), to obtain  $\hat{M}_{MLE}$  via the likelihood function about covariance matrix partial differential equals to zero:

$$\sum_{t=1}^K \left[ \frac{\partial \ln|M|}{\partial(M)} - \frac{1}{\tau_t} \frac{\partial(z_t^H M^{-1} z_t)}{\partial(M)} \right] = 0 \quad (15)$$

Substitute (15) into (14),  $\hat{M}_{MLE}$  can be written as:

$$\hat{M}_{MLE} = \frac{1}{K} \sum_{t=1}^K \frac{z_t z_t^H}{\tau_t} \quad (16)$$

In formula (16), the estimator cannot be directly got because the unknown parameters  $\tau_t$ ,  $t = 1, 2, \dots, K$  we obtain the maximum likelihood estimation  $\hat{\tau}_t$  to replace  $\tau_t$ , which can be written as:

$$\hat{\tau}_t = \frac{z_t^H M^{-1} z_t}{N}, t = 1, 2, \dots, K \quad (17)$$

Substitute (17) into (16), we can get the MLE of covariance matrix, which can be written as:

$$\hat{M}_{MLE} = \frac{N}{K} \sum_{t=1}^K \frac{z_t z_t^H}{z_t^H M^{-1} z_t} \quad (18)$$

In formula (18), we can obtain the estimator by iterative method (Bon *et al.*, 2008; Conte *et al.*, 2002; Shuai, 2011):

$$\hat{M}_{MLE} (i) = \frac{N}{K} \sum_{t=1}^K \frac{z_t z_t^H}{z_t^H \hat{M}_{MLE} (i-1)^{-1} z_t} \quad (19)$$

where  $i = 1, \dots, N_{it}$ , in Eq. (19), set the initial value of  $\hat{M}_{MLE} (i)$  as:

$$\hat{M}_{MLE} (0) = \frac{N}{K} \sum_{k=1}^K \frac{z_k z_k^H}{z_k^H z_k}$$

Substitute  $\hat{M}_{MLE}$  in formula (18) into (12) and (13), we can obtain the NP-G0 detector and GLRT detector as follows:

$$\frac{\prod_{t=1}^L \left( \frac{\beta + \hat{q}_1(\mathbf{z}_t)}{\hat{q}_1^2(\mathbf{z}_t)} \right)^{N+\alpha}}{\prod_{t=1}^L \left( \frac{\beta + q_0(\mathbf{z}_t)}{q_0^2(\mathbf{z}_t)} \right)^{N+\alpha}} \underset{H_0}{\underset{H_1}{\geq}} \eta \quad (20)$$

where,

$$\hat{q}_1(\mathbf{z}_t) = (\mathbf{z}_t - \hat{\alpha}_t \mathbf{p})^H (\hat{M}_{MLE})^{-1} (\mathbf{z}_t - \hat{\alpha}_t \mathbf{p})$$

and

$$\hat{q}_0(\mathbf{z}_t) = \mathbf{z}_t^H (\hat{M}_{MLE})^{-1} \mathbf{z}_t$$

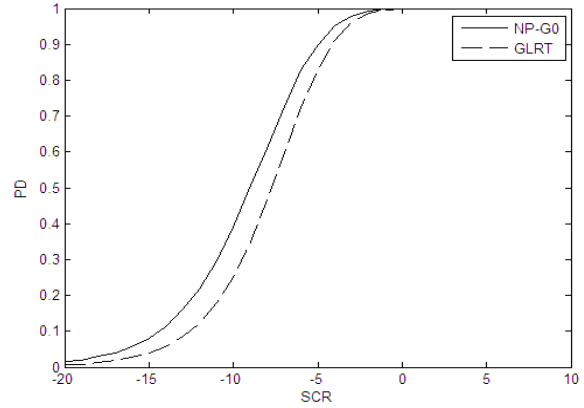


Fig. 1: Detection performance of detectors versus  $\alpha = 1.2$ ,  $\beta = 1$

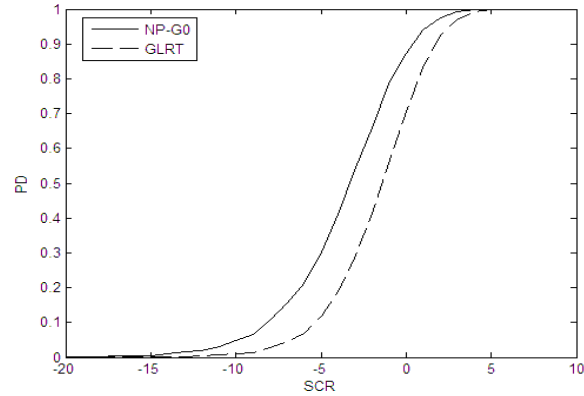


Fig. 2: Detection performance of detectors versus  $\alpha = 1.5$ ,  $\beta = 1$

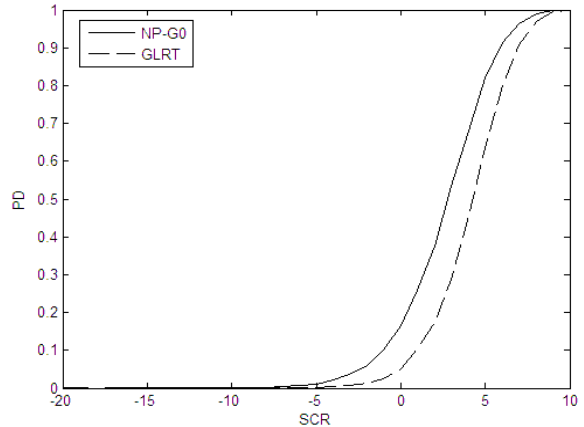


Fig. 3: Detection performance of detectors versus  $\alpha = 2$ ,  $\beta = 1$

$$T_G = -N \sum_{t=1}^L \ln \left[ 1 - \frac{|\mathbf{p}^H (\hat{M}_{MLE})^{-1} \mathbf{z}_t|^2}{(\mathbf{p}^H (\hat{M}_{MLE})^{-1} \mathbf{p})(\mathbf{z}_t^H (\hat{M}_{MLE})^{-1} \mathbf{z}_t)} \right] \quad (21)$$

**Performance assessments:** In the following simulation, both the Probability Detection (PD) and

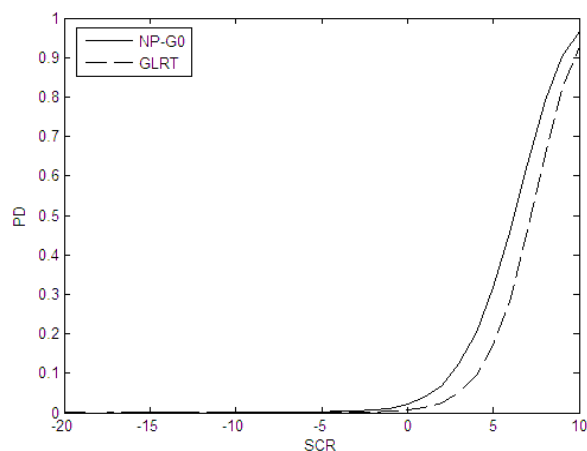


Fig. 4: Detection performance of detectors versus  $\alpha = 2.5$ ,  $\beta = 1$

Probability of False Alarm (PFA) of NP-G0 detector and GLRT detector are got by Monte Carlo methods. We mainly review the texture information lost of the K distributed clutter leads to the preference loss of NP-G0 detector. Assume  $PFA = 10^{-4}$ ,  $L = 4$ ,  $N = 8$  and  $\beta = 1$ , the detection performance of the NP-G0 detector and GLRT detector show in Fig. 1 to 4 as follows:

In Fig. 1 to 4, we can see that the detection performance difference between the two tests is increasing with the increasing of roughness parameter  $\alpha$ . Moreover, we discovery that the smaller of the roughness parameter  $\alpha$ , the better of the detection performance, especially when a low SCR, the reason is that the clutter is very acuity when roughness parameter is small and the energy of clutter mainly occupy several range cells of spread targets and the difference between GLRT and NP-G0 less than 2 db.

## CONCLUSION

In this study, we have addressed the problem of adaptive detection of range-spread targets in G0 distributed clutter. More precisely, we have analyzed the influence of neglecting the texture of clutter to the detection performance of test in G0 clutter.

In conclusion, we can state that: to compound Gaussian distributed clutter, it is reasonable to model the texture of every range cell as an unknown determinate parameter. It is feasible to use the generally compound Gaussian background instead of certain other complex clutter. Because this operates avoid estimating some special parameters and the form of detector is very simple with limit performance loss, especially when the clutter is very aculeate.

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