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Research Article

Evaluation for Confidence Interval of Reliability of Rolling Bearing Lifetime with Type I Censoring

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Abstract: In the lifetime test of rolling bearings under type I censoring with a small sample, the confidence interval of reliability needs to be evaluated to ensure safe and reliable operation of a system like an aerospace system. Thus the probability density function of Weibull distribution parameters must be attained. Owing to very few test data and for lack of prior knowledge, it is difficult to take it out for prevailing methods like the moment method, the maximum likelihood method and the Harris method. For this end, the bootstrap likelihood maximum-entropy method is proposed by fusing the bootstrap method, the maximum likelihood method and the Harris method. The lifetime test data with the small sample are made into the simulated parameter data with the large sample to obtain the probability density function on the parameters. The confidence intervals of the Weibull distribution parameters are estimated and the confidence interval of reliability is calculated. The tests of the complete large-sample data, the complete small-sample data and the incomplete small-sample data are produced to prove effectiveness of the proposed method. Results show that the proposed method can assess the confidence interval of the reliability without any prior information on the Weibull distribution parameters.

Keywords: Bearing failure, fatigue testing, life assessment, probability, reliability analysis

INTRODUCTION

In the lifetime test of rolling bearings, the confidence interval of the reliability needs to be evaluated to ensure safe and reliable operation of a system like an aerospace system, a nuclear reaction-diffusion system and a weapon system (Xia *et al*., 2009; Chowdhury and Adhikari, 2011). This is a new indicator of reliability analysis for the rolling bearing lifetime. Such a requirement, in theory, is justified in a performance test because, according to metrology, the estimated value of a parameter has uncertainty, along with a confidence level and interval. The value of the lifetime reliability is estimated by the parameters of a lifetime probability distribution and it has necessarily indirect uncertainty (Jean-Francois and Joseph, 2010; Radoslav *et al*., 2011; Luxhoj and Shyur, 1995; Joarder *et al*., 2011; Jiang *et al*., 2010; Chen *et al*., 2009; Fu *et al*., 2010; Liu *et al*., 2009a).

The two-parameter Weibull distribution is one of the most commonly used functions in reliability analysis for the rolling bearing lifetime. To assess the confidence interval of the reliability, the Weibull distribution parameters, the shape parameter β and the scale parameter η, should be estimated and the probability density functions of the two parameters should also be obtained.

According to Bayesian statistics, β and η can be regarded as two independent stochastic variables. Thus, the two should have their own probability density functions, denoted by γ (β) and ε (η), respectively. According to classical statistics, a large amount of the parameter data β_j and η_j, where β_j is the jth simulated value of β, η_j is the jth simulated value of η and j = 1, 2, …, is demanded in order to acquire γ (β) and ε (η). Under type I censoring with a small sample, the maximum likelihood method (Harris, 1991; Jiang and Zhou, 1999; Joarder *et al*., 2011), the moment method (Jiang and Zhou, 1999; Sirvanci and Yang, 1984; Tian and Liu, 2009) and Harris method (Harris, 1991) are usually employed to obtain β and η. However, these methods pose difficulty in estimating γ (β) and ε (η).

Recently, many new methods are developed, along with many findings. For example, depending on Bayes theorem, Chen *et al*. (2007) proposed the single and double variable sampling plans for the Weibull distribution and Fu *et al*. (2010) made a reliability assessment and a life prediction for very few failure data; by means of stochastic resampling methods, Heiermann *et al*. (2005) presented a strategy for the assessment of uncertainty in the estimation of the failure probability of ceramic components due to the scatter of

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material data; based on two independent normal random variables, Barbiero (2011) proposed the procedure to get confidence intervals for the reliability in stress-strength models; and with the help of a gamma function model, Kimura (2008) focused on the generalization of several software reliability models and the derivation of confidence intervals of reliability assessment measures. Nevertheless, available findings rely on the priori information of the parameters of a probability distribution. For type I censoring with a small sample, how to assess the confidence interval of the reliability of the rolling bearing lifetime is still a puzzle under the condition of the lack of the priori information about the Weibull distribution parameters.

As is well known, the bootstrap method (Kimura, 2008; Othman and Musirin, 2011; Xia *et al*., 2010) and the maximum entropy method (Yonamoto *et al*., 2011; Li and Zhang, 2011) are two of the prevailing methods for data analysis and information processing. Based on a small sample and via the maximum likelihood method, the bootstrap method can be adopted to imitate a large number of β_j and η_j, but it requires the priori knowledge of γ (β) and ε (η) in advance. If lack of the priori knowledge of γ (β) and ε (η), the estimated intervals of β and η and the confidence interval of the reliability can be calculated hardly. So far the priori knowledge of γ (β) and $\varepsilon(\eta)$, in fact, is reported rarely from the experimental investigation of the rolling bearing lifetime. For this reason, the maximum entropy method can be applied to structure the probability density functions γ (β) and ε (η), but it demands a large number of β_j and η_j.

Synthesizing the strong points of the bootstrap method, the maximum likelihood method and the maximum entropy method, a novel method called the bootstrap likelihood maximum-entropy method is proposed to evaluate the confidence interval of the reliability of rolling bearings under type I censoring with a small sample. The procedure is as follows:

- Based on the test data with a small sample of size n, a large number of the data t_{ji} (i = 1,2,…,n; j = 1,2,…,B) is generated by means of the bootstrap method, where t_{ji} stands for the ith data in the jth bootstrap sample, n for the number of the test data obtained in a lifetime test and B for the number of the bootstrap samples
- Based on t_{ji}, β_j and η_j are calculated with the help of the maximum likelihood method
- β_j and η_j are processed with the aid of the maximum entropy method and $\gamma(\beta)$ and ε (η) can accordingly be acquired
- The expected value β_{mean} of β and the expected value η_{mean} of η are computed via γ (β) and ε (η), respectively.
- Given a confidence level p, the estimated interval [β_L, β_U] of β and the estimated interval [η_L, η_U] of η are obtained by γ (β) and ε (η), respectively
- Given a failure probability q, the lifetime L and its reliability function R (t) are calculated by β and η, where t is the stochastic variable of the lifetime
- The interval [L_{Lq} , L_{Uq}] of L and the interval [R_L (t), R_U (t)] of R (t) are estimated by [β_L, β_U] and [η_L, η_U].

It is easy to see from the procedure that the bootstrap likelihood maximum-entropy method proposed in this study is characterized by theoretically fusing the strong points of the bootstrap method, the maximum likelihood method and the maximum entropy method. It follows that the probability density functions of the Weibull distribution parameters can be simulated, the confidence intervals of the shape parameter and the scale parameter can be obtained and then the confidence interval of the reliability of rolling bearings can be assessed. Clearly, only by the test data with small sample sizes the bootstrap likelihood maximum-entropy method is able to analyse the reliability without any priori information of the shape parameter and the scale parameter.

In this study, the tests of complete large-sample data, the complete small-sample data and the incomplete small-sample data are produced to prove the effectiveness and practicability of the bootstrap likelihood maximum-entropy method.

TWO-PARAMETER WEIBULL DISTRIBUTION

Suppose the lifetime of rolling bearings is of the two-parameter Weibull distribution with the probability density function:

$$f(t) = \beta\eta^{-\beta}t^{\beta-1}\exp(-(t/\eta)^{\beta}) \tag{1}$$

where,

$f(t)$	=	The probability density function of the two-parameter Weibull distribution
t	=	The stochastic variable of the lifetime
β	=	The shape parameter
η	=	The scale parameter.

The probability distribution function is given by:

$$F(t) = 1-\exp(-(t/\eta)^{\beta}) \tag{2}$$

where, $F(t)$ is the probability distribution function of the two-parameter Weibull distribution.

TEST DATA

Under type I censoring, the test data include the failure data and the truncated data. In terms of the reliability theory, the test data of the type may be classified as two categories, the complete data and the incomplete data. The former consists simply of the failure data and the latter consists of both the failure data and the truncated data.

Suppose n product units are randomly selected for a time censored test and the number of failure product units is r ($0<r\leq n$). The sample of the failure data is geven by:

$$T_{\mathrm{F}}=(t_1,t_2,...,t_r);t_1\leq t_2\leq ...\leq t_r \tag{3}$$

where,
T_{F} = The sample of the failure data
r = The number of the failure data

The sample of the truncated data is given by:

$$T_{\mathrm{C}}=(t_{r+1},t_{r+2},...,t_n)=(\underbrace{t_{\mathrm{C}},t_{\mathrm{C}},...,t_{\mathrm{C}}}_{\text{totaling } s=n-r}) \tag{4}$$

where,
T_{C} = The sample of the truncated data
n = The number of the test data
$s = n-r$ = The number of the truncated data

Bootstrapt method: To facilitate the description, β and η are noted by θ and γ (β) and ε (η) are noted by ξ (θ). The bootstrap method is to simulate the large sample Tj about the failure data with the bootstrap resampling from the sample TF, the maximum likelihood method is to calculate the simulated value θj of the parameter θ via the large sample Tj and the maximum entropy method is to establish the probability density function ξ (θ) by θ_j.

Assume the jth bootstrap sampling is conducted. A sampling datum t_{j1} can be obtained by an equiprobable sampling with replacement from T_{F}. Resampling so r times, the r sampling data which are regarded as a sample (t_{j1}, t_{j2} ,…, t_{jr}) can be obtained. Considering the s truncated data in equation (4), the jth bootstrap sample of the incomplete data in the time truncated test is structured, as follows:

$$T_j=(t_{j1},t_{j2},...,t_{ji},...,t_{jr},t_{j,r+1},t_{j,r+2},...,t_{j,n});\quad j=1,2,...,B \tag{5}$$

where, B is the number of the bootstrapt samples. In Eq. (5), the jth smaple of the truncated data is:

$$(t_{j,r+1},t_{j,r+2},...,t_{j,n})=(t_{r+1},t_{r+2},...,t_n) \tag{6}$$

MAXIMUM LIKELIHOOD METHOD

According to the maximum likelihood method for simulating β and η, there are:

$$\frac{1}{\beta_j}+\frac{\sum_{i=1}^{r}\ln t_{ji}}{r}-\frac{\sum_{i=1}^{n}t_{ji}^{\beta_j}\ln t_{ji}}{\sum_{i=1}^{n}t_{ji}^{\beta_j}}=0 \tag{7}$$

And

$$\eta_j=\left(\sum_{i=1}^{n}\frac{t_{ji}^{\beta_j}}{r}\right)^{1/\beta_j} \tag{8}$$

The jth simulated value β_j of β and the jth simulated value η_j of η can be calculated by Eq. (7) and (8), respectively and the B simulated results can hence be obtained, which are noted by:

$$\Theta=(\theta_1,\theta_2,...,\theta_j,...,\theta_B) \tag{9}$$

where, θ_j stands for the jth simulated value of θ and Θ for a sample of the simulated values of θ.

The sample Θ can be applied to establish ξ (θ) by means of the maximum entropy method deduced in detail below.

MAXIMUM ENTROPY METHOD

According to the information entropy theory, the information entropy H of ξ (θ) is defined as:

$$H=-\int_{\Omega}\xi(\theta)\ln\xi(\theta)\mathrm{d}\theta \tag{10}$$

where, H stands for the information entropy of ξ (θ) and $\Omega\in[\Omega_{\min},\Omega_{\max}]$ for the integral domain.

A basic idea of the maximum entropy method is that in all the feasible solutions to a problem, the solution of maximizing the information entropy is the most unbiased solution. Accordingly, let:

$$H\rightarrow \max \tag{11}$$

The constraint condition is:

$$\int_{\Omega} \theta^k \xi(\theta) \mathrm{d}\theta = m_k ; k = 0,1,\ldots,m; m_0 = 1 \tag{12}$$

where, k stands for the sequence number of the order of the origin moment, m_k for the kth origin moment and m for the highest order of the origin moment.

With the help of the histogram principle in statistics, θ_j is rearranged from small to large order and is divided into Z groups. As a result, the kth origin moment m_k is given by:

$$m_k = \sum_{z=1}^{Z} \theta_z^k \Xi_z ; k = 0,1,\ldots,m; m_0 = 1 \tag{13}$$

where,

θ_z = The median in the zth group

Ξ_z = The frequency at θ_z

According to the Lagrange method of multipliers, the probability density function of satisfying Eq. (10) to (13) is:

$$\xi(\theta) = \exp(\sum_{k=0}^{m} \lambda_k \theta^k) \tag{14}$$

where λ_k is the kth Lagrange multiplier and should meet Eq. (15):

$$1 - \frac{\int_{\Omega} \theta^k \exp(\sum_{j=1}^{m} \lambda_j \theta^j) \mathrm{d}\theta}{m_k \int_{\Omega} \exp(\lambda_k \lambda^k) \mathrm{d}\theta} = 0; k = 1,2,\ldots,m \tag{15}$$

The Lagrange multiplier λ_0 is given by:

$$\lambda_0 = -\ln(\int_{\Omega} \exp(\sum_{k=1}^{m} \lambda_k \theta^k) \mathrm{d}x) \tag{16}$$

Let the significance level $\alpha \in [0,1]$, then the confidence level p is:

$$p = (1 - \alpha) \times 100\% \tag{17}$$

Given the confidence level p, the estimated interval of θ can be obtained, as follows:

$$[\theta_{\mathrm{L}}, \theta_{\mathrm{U}}] = [\theta_{\alpha/2}, \theta_{1-\alpha/2}] \tag{18}$$

where, θ_{L} and θ_{U} are, respectively, the lower bound and the upper bound of θ and there are:

$$\frac{\alpha}{2} = \int_{\Omega_{\min}}^{\theta_{\alpha/2}} \xi(\theta) \mathrm{d}\theta \tag{19}$$

And

$$1 - \frac{\alpha}{2} = \int_{\Omega_{\min}}^{\theta_{1-\alpha/2}} \xi(\theta) \mathrm{d}\theta \tag{20}$$

The expected value θ_{mean} of θ is defined as:

$$\theta_{\mathrm{mean}} = \int_{\Omega} \theta \xi(\theta) \mathrm{d}\theta \tag{21}$$

The mid-value $\theta_{0.5}$ of θ is defined as:

$$0.5 = \int_{\Omega_{\min}}^{\theta_{0.5}} \xi(\theta) \mathrm{d}\theta \tag{22}$$

EVALUATION FOR RELIABILITY

Let the failure probability be q, then lifetime L_q is defined as Harris (1991):

$$L_q = \eta[-\ln(1-q)]^{1/\beta} \tag{23}$$

The reliability function $R(t)$ is defined as:

$$R(t) = \exp[-(t/\eta)^{\beta}] \tag{24}$$

In this study, the expected lifetime $L_{\mathrm{mean}q}$ based on the bootstrap likelihood maximum-entropy method is defined as:

$$L_{\mathrm{mean}q} = \eta_{\mathrm{mean}}[-\ln(1-q)]^{1/\beta_{\mathrm{mean}}} \tag{25}$$

where, β_{mean} is the expected value of β and η_{mean} is the expected value of η.

Let the lifetime interval be [$L_{\mathrm{L}q}$, $L_{\mathrm{U}q}$] based on the bootstrap likelihood maximum-entropy method, where $L_{\mathrm{L}q}$ is the lower bound of $L_{\mathrm{mean}q}$ and is given by:

$$L_{\mathrm{L}q} = \eta_{\mathrm{L}}[-\ln(1-q)]^{1/\beta_{\mathrm{L}}} \tag{26}$$

and $L_{\mathrm{U}q}$ is the upper bound of $L_{\mathrm{mean}q}$ and is given by:

$$L_{\mathrm{U}q} = \eta_{\mathrm{U}}[-\ln(1-q)]^{1/\beta_{\mathrm{U}}} \tag{27}$$

where, β_{L} and β_{U} are, respectively, the lower bound and the upper bound of β and η_{L} and η_{U} are, respectively, the lower bound and the upper bound of η.

The mid-value lifetime L_{mq} based on the bootstrap likelihood maximum-entropy method is defined as:

$$L_{mq} = \eta_{0.5}\left[-\ln(1-q)\right]^{1/\beta_{0.5}} \tag{28}$$

where $\beta_{0.5}$ is the estimated mid-value of β and $\eta_{0.5}$ is the estimated mid-value of η.

The expected reliability function $R_{\text{mean}}(t)$ based on the bootstrap likelihood maximum-entropy method is defined as:

$$R_{\text{mean}}(t) = \exp[-(t/\eta_{\text{mean}})^{\beta_{\text{mean}}}] \tag{29}$$

At the given confidence level p, let the estimated interval of R_{mean} (t) be [R_L (t), R_U (t)] based on the bootstrap likelihood maximum-entropy method, where R_L (t) is the lower bound which is given by:

$$R_L(t) = \min(\exp[-(t/\eta_L)^{\beta_L}], \exp[-(t/\eta_L)^{\beta_U}], \exp[-(t/\eta_U)^{\beta_L}], \exp[-(t/\eta_U)^{\beta_U}]) \tag{30}$$

and R_U (t) is the upper bound which is given by:

$$R_U(t) = \max(\exp[-(t/\eta_L)^{\beta_L}], \exp[-(t/\eta_L)^{\beta_U}], \exp[-(t/\eta_U)^{\beta_L}], \exp[-(t/\eta_U)^{\beta_U}]) \tag{31}$$

The mid-value reliability function R_m (t) based on the bootstrap likelihood maximum-entropy method is defined as:

$$R_m(t) = \exp[-(t/\eta_{0.5})^{\beta_{0.5}}] \tag{32}$$

EXPERIMENTS AND DISCUSSION

In this section, different types of experiments (three cases) are done to test the soundness and effectiveness of the bootstrap likelihood maximum-entropy method proposed in this study, along with a comparison with the existing methods.

Case study of complete large-sample data:

Case 1: This is a case of evaluation for the complete large-sample data, along with a comparison with the moment method, the maximum likelihood method and Harris method. Let β = 2.5 and η = 200, which can be regard as the true values of Weibull distribution parameters, then the failure data are imitated, as follows ($n = r = 50$)(Jiang and Zhou, 1999):

40 49 59 70 85 93 96 99 105 111 115 116 116 118 123 128 130 131 132 135 136 139 146 154 157 162 169 170 188 191 199 205 207 210 215 222 234 253 264 279 281 287 316 319 319 321 326 344 386 392

The results estimated for β and η are listed in Table 1. For β, the relative errors of the four methods are less than 5%, having a good effect. For η, the relative error of the proposed method is less than 10%, but the relative errors of the existing methods are more than 10%.

The results estimated for the lifetime are listed in Table 2. It can be seen that the results estimated using the four methods are less than the true value. By contrast the bootstrap likelihood maximum-entropy method achieves the smallest relative error, about 2.7% and the moment method achieves the largest relative error, about 9.4%. More significantly, the bootstrap likelihood maximum-entropy method is able to estimate

Table 1: Comparison between estimated results of Weibull distribution parameters in case 1

	Moment method		Maximum likelihood method and Harris method		Bootstrap likelihood maximum-entropy method (B =10000)	
Item	β	η	β	η	β	η
Expected value	2.161	208.716	2.198	209.487	2.306	209.893
Relative error between expected value and true value /%	13.56	4.36	12.08	4.74	7.76	4.95
Mid-value	-	-	-	-	2.293	209.531
Relative error between mid-value and true value /%	-	-	-	-	8.28	4.77
Estimated interval at 90% confidence level	-	-	-	-	[2.003,2.642]	[187.517,232.816]

Table 2: Comparison between results of lifetime in case 1 (q = 10%)

Item	Moment method	Maximum likelihood method and Harris method	Bootstrap likelihood maximum-entropy method	Remark
Estimated value of lifetime	73.6718	75.2516	79.1008	True value of lifetime is 81.3020
Relative error /%	9.3850	7.4419	2.7074	

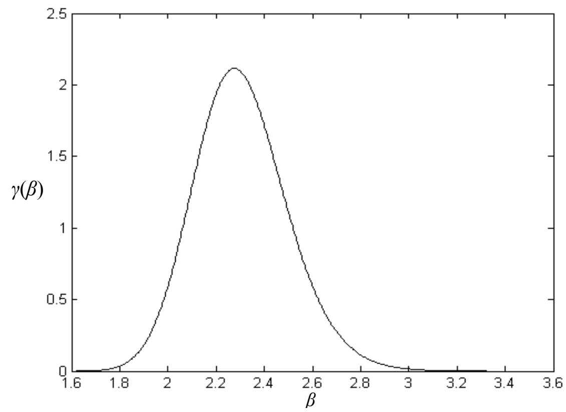

Fig. 1: Probability density function $\gamma(\beta)$ of shape parameter β in Case 1 (bootstrap likelihood maximum-entropy method)

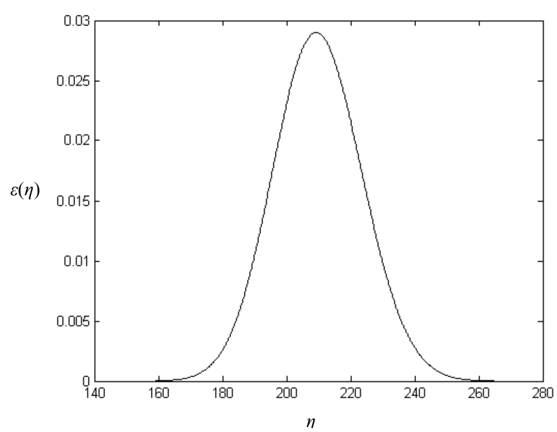

Fig. 2: Probability density function $\varepsilon(\eta)$ of scale parameter η in Case 1 (bootstrap likelihood maximum-entropy method)

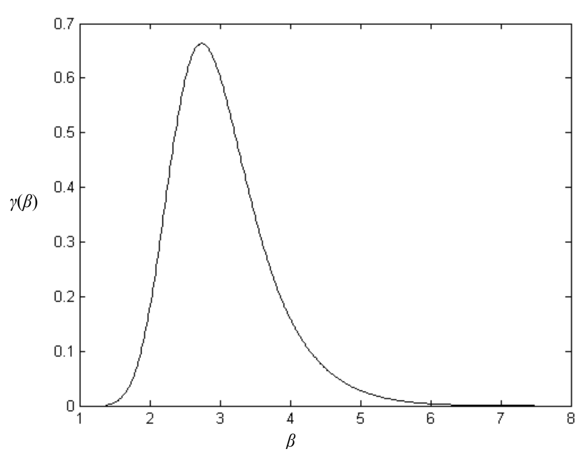

Fig. 3: Probability density function $\gamma(\beta)$ of shape parameter β in Case 2 (bootstrap likelihood maximum-entropy method)

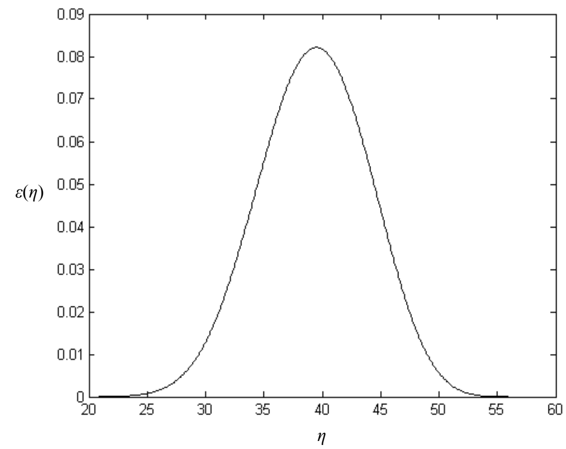

Fig. 4: Probability density function $\varepsilon(\eta)$ of scale parameter η in Case 2 (bootstrap likelihood maximum-entropy method)

$\gamma(\beta)$ and $\varepsilon(\eta)$, as shown in Fig. 1 and 2 and Table 1, but the moment method, the maximum likelihood method and the Harris method are unable to do those.

Case study of complete small-sample data:

Case 2: This is a case of evaluation for the complete small-sample data, along with a comparison with the Harris method employed frequently in reliability analysis of the rolling bearing lifetime test with a small sample. The failure data obtained by Harris (1991) are cited, as follows ($n = r = 10$):

14.01 15.38 20.94 29.44 31.15 36.72 40.32 48.61 56.42 56.97

With the help of the bootstrap likelihood maximum-entropy method, γ (β) and ε (η) are estimated, as shown in Fig. 3 and 4. The results of the Weibull distribution parameters and the rolling bearing lifetime are shown in Table 3 and 4.

It is easy to see from Table 3 and 4 that the bootstrap likelihood maximum-entropy method is able to obtain the estimated interval of the reliability, but the Harris method is unable to do that.

Case study of incomplete small-sample data:

Case 3: This is a case of evaluation for the incomplete small-sample data.

The accuracy life of bearing units for gyro motors is tested under type I censoring with a small sample and the truncated time is 4000h. The test is done with eight bearing units and the result is that the five bearing units lose the accuracy. The failure data, in h, are obtained as Liu *et al*. (2009b):

1313 2288 2472 2506 3382 and the truncated data, in h, are 4000 4000 4000.

Table 3: Estimated result of Weibull distribution parameter in case 2 (p = 90%)

	Harris method		Bootstrap likelihood maximum-entropy method (B = 10000)	
Item	β	η	β	η
Estimated mid-value	2.15	-	2.9204	39.3103
Expected value	2.5835	39.5578	3.0285	39.2979
Estimated interval	[1.41, 3.51]	-	[2.0875,4.3290]	[31.4256, 46.8431]

Table 4: Estimated result of lifetime in Case 2 (p = 90% and q = 10%)

Item	Harris method	Bootstrap likelihood maximum-entropy method
Estimated mid-value lifetime	15.3	18.1908
Estimated expected-lifetime	16.5553	18.6922
Estimated lifetime interval	[7.25, 23.3]	[10.6931, 27.8538]
Reliability of lifetime, R/%	90	90
Estimated interval of reliability of median lifetime at 90% confidence level /%	-	[72.66, 98.35]
Estimated interval of reliability of expected lifetime at 90% confidence level /%	-	[71.31, 98.14]

Table 5: Result estimated by bootstrap likelihood maximum-entropy method in case 3 (B = 10000)

	Confidence level, p%		
Item	90	95	99
Failure probability, q/%	10	5	1
Expected value of shape parameter, β_{mean}	2.5976	2.5976	2.5976
Expected value of scale parameter, η_{mean}/h	3823.7	3823.7	3823.7
Estimated interval of shape parameter, [β_L, β_U]	[1.7419, 3.5825]	[1.6212, 3.7736]	[1.4099, 4.1759]
Estimated interval of scale parameter, [η_L, η_U]/h	[3739.3, 3931.9]	[3726.5, 3950.1]	[3701.9, 3977.9]
Expected lifetime, L_{meanq}/h	1607.8	1218.7	650.70
Estimated lifetime interval, [L_{Lq}, L_{Uq}]/h	[1027.4, 2.098.0]	[596.53, 1797.9]	[141.72, 1322.0]
Reliability of expected lifetime, $R_{mean}(L_{meanq})$/%	90	95	99
Estimated interval of reliability of expected lifetime, [$R_L(L_{meanq})$, $R_U(L_{meanq})$]/%	[79.46, 96.02]	[84.93, 98.82]	[91.74, 99.95]

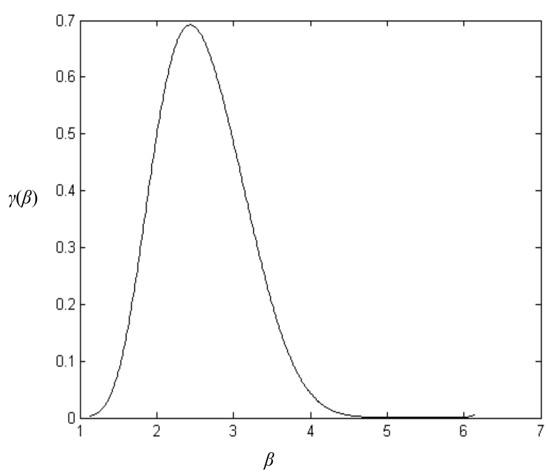

Fig. 5: Probability density function $\gamma(\beta)$ of shape parameter β in Case 3 (bootstrap likelihood maximum-entropy method)

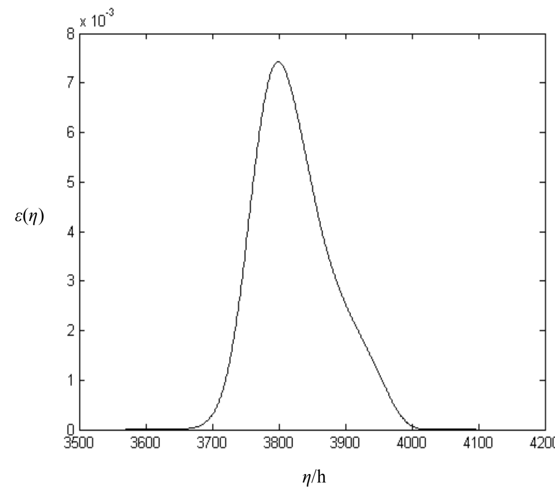

Fig. 6: Probability density function $\varepsilon(\eta)$ of scale parameter η in Case 3 (bootstrap likelihood maximum-entropy method)

The results estimated by the bootstrap likelihood maximum-entropy method are shown in Table 5 and Fig. 5 to 7. Let $t = L_{meanq}$ = 1023h, then the reliability of the expected lifetime of the bearing units takes value R_{mean} (1023) = 96.8% and the estimated interval of the reliability is [R_L (1023), R_U (1023)] = [90.08, 99.20] at the p = 96.8% confidence level.

Feature of probability density function of weibull distribution parameter: It can be seen from Fig. 1 to 3 that γ (β) is a curve with one left peak and ε (η) is a curve with one approximatively symmetrical peak.

In order to study γ (β) and ε (η), in Case 3, their change with s, the number of the truncated data, is presented In Fig. 8 and 9. As s increases, their widths

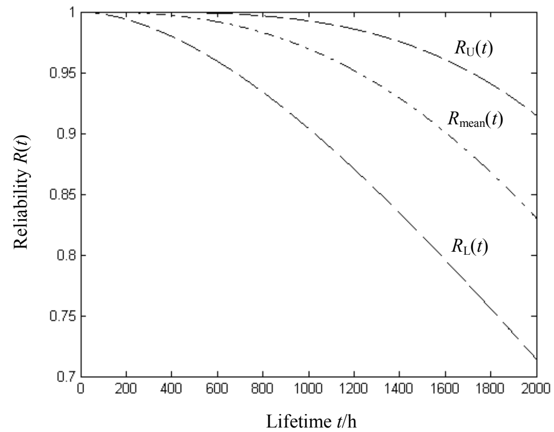

Fig. 7: Evaluation and prediction for reliability of bearing lifetime (bootstrap likelihood maximum-entropy method)

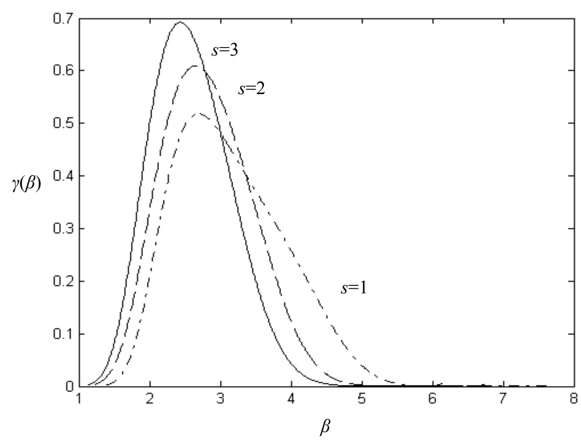

Fig. 8: Influence of s on $\gamma(\beta)$

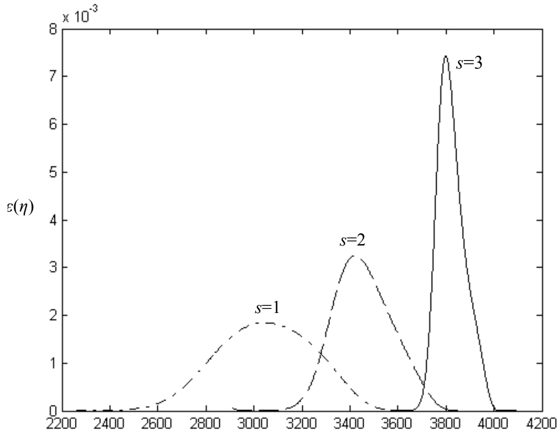

Fig. 9: Influence of s on $\varepsilon(\eta)$

decrease, their heights increase and their peaks move to the left and to the right, respectively. This indicates that the number of the truncated data has an effect upon the probability density functions of the Weibull distribution parameters. In addition, the feature of the left peak of γ (β) and of the approximatively symmetrical peak of ε (η) is irrelative to the number of the truncated data according to Fig. 8 and 9 and is also irrelative to the number of the failure data and the category of the test data according to Fig. 1 to 6.

CONCLUSION

Under type I censoring, without any priori information beyond the test data to be dealt with, the bootstrap likelihood maximum-entropy method is able to calculate the estimated interval of the reliability of the rolling bearing lifetime based on the two-parameter Weibull distribution.

The tests of the complete large-sample data, the complete small-sample data and the incomplete small-sample data prove the effectiveness of the bootstrap likelihood maximum-entropy method.

The probability density function of the shape parameter is a curve with one left peak and the probability density function of the scale parameter is a curve with one approximatively symmetrical peak. The feature of the two curves is irrelative to the number of the truncated data, the number of the failure data and the category of the test data.

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