The Study on the Value of Information Sharing with Correlated Market Demand and Cost of Information Sharing in Supply Chain

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Abstract: In a two-echelon supply chain system with 1 manufacture and N retailers, the two-stage stochastic dynamic program of whether manufacture to share information with n (0≤n ≤ N) retailers is constructed when the market demand is correlated with AR (1) module and information sharing is costly. The value of information shared with n retailers is analyzed analytically. One numerical case is simulated to illustrate the magnitude of the value of information shared with n retailers in the supply chain at last.

Keywords: AR (1), demand correlation, information shared, order-up-to S level, supply chain

INTRODUCTION

Information flow is the base of supply chain management. As the development of IT, the technological barriers are gradually disappearing. Today more and more companies are sharing the related information with their partners through Internet or other IT technology, because information sharing can help to decrease the cost of inventory, upgrade custom service level and at last strengthen the core competence capacity of the company. For example, Wal-Mart store, Inc. and P&G Inc. both decreased their inventory cost by 70% and increased the custom service level from 96 to 99% through Retail Link Program (Poirier and Reiter, 1996).

Even that, the mechanism of value of information sharing is not very clear. What kinds of information should be share, how to share, how much the value of the information sharing, by now there have not had same and clear answers for these questions. Some studies showed that information sharing can decrease the negative effects which are result from the information distortion and bull-whip (Lee and Whang, 1998; Chen et al., 2000). Lee et al. (2000) found that market demand and inventory information sharing can decrease inventory cost when market demand is the AR(1) distribution, but Raghynathan (2001) studied the same market demand module and found that there is not much value if only shared market demand information. Some papers (Milgrom and Robert, 1998; Kaijie, 2002; Schwarz et al., 1998) also studied the value of future market demand information sharing.

Most of above papers supposed that the market demand is independent and sharing information is no cost. Actually, the market demands are often correlated at retailers’ sites, such as some fashionable and cyclical commodities and sharing information has some fee, such as the cost to construct the information sharing platform. In this study, we set up a two-stage stochastic dynamic model of supply chain information sharing with 1 manufacture and N retailers when the market demand is correlated with AR (1) module and information sharing is costly. Then value of information sharing with N retailers is analyzed analytically. At last a numerical simulation example is given.

BASIC MODEL

We consider a two-echelon supply chain system which has 1 manufacture and N retailers and only retailers have the market demand with AR (1) distribution. Many papers (Lee et al., 2000; Raghynathan, 2001; Knhn, 1987) have the same market demand pattern. Suppose $D_{it}$ is the prediction of market demand at t time cycle made by retailer i based on the real market demand at (t-1) time cycle, then we have:

$$D_{it} = d + \rho D_{it-1} + \varepsilon_{it}$$

Here, $d>0$, $-1<\rho<1$, $i = 1, 2, 3, ..., N$. Random error $\varepsilon_{it}$ is the normal distribution with 0 mean and $\sigma^2$ variance and correlation coefficient is $\rho$ between $\varepsilon_{it}$ and $\varepsilon_{it}$, $i \neq j$. If fix i, $\varepsilon_{it}$ is independent identical distribution at different time cycle. To prevent retailer’s order being negative, we assume $\sigma<<d$.

We suppose manufacture and retailers’ order process is as same as that given in the study (Raghynathan, 2003). At first, retailer i calculates its own storage when market demand $D_{it}$ has come out at each time cycle t, $t = 1, 2, 3, ...$ Then retailer i send its
order $Y_i$ to manufacture and this order will arrive to retailer at the beginning of time cycle (t+1). Suppose manufacture has enough inventory to fill out all retailers’ orders and if not it will afford all cost to replenish the inventory shortage immediately. When manufacture fill out retailers’ orders, it will calculate its own inventory and send out its own order and let manufacture fill out retailers’ orders, it will calculate its own inventory shortage immediately. When manufacture has enough inventory to fill out all retailers’ orders and if not it will afford all cost to make up the shortage.

When manufacture and retailers have the information sharing, beside the order $Y_i$ is determined through Eq. (1) at the end of time cycle t. We assume the cost of information sharing is a constant $k$ at each time cycle. Suppose manufacture and retailers are all using the same model’s residual errors, it is a normal distribution. Therefore, the information of order $Y_i$ and shortage cost per unit at each time cycle.

At first stage, manufacture wants to decide the optimal order level $T^*$ at time cycle t. We assume the cost of information sharing is a constant $k$ at each time cycle. Due to the cost of information sharing, manufacture’s goal is to minimum total running cost through information sharing with right number of retailers. Based on the above assumption, the number $n(0 \leq n \leq N)$ of retailers with which manufacture will share information can be gotten through the following two-stage stochastic dynamic program:

$$\min_{0 \leq n \leq N} \{kn + E(\varepsilon_{11}, \ldots, \varepsilon_{nn}) \min_{T_1} E(\varepsilon_{11}, \ldots, \varepsilon_{nn}) \}$$

$$= \min_{0 \leq n \leq N} \{kn + E(\varepsilon_{11}, \ldots, \varepsilon_{nn}) \min_{T_1} E(\varepsilon_{11}, \ldots, \varepsilon_{nn}) \}$$

Expressed by Eq. (4), the optimal order level of manufacture is:

$$T^* = d + \rho D_h$$

If we change the subscript t to (t+1) in the Eq. (1), then we get that $D^*_h = d + \rho D_h + \varepsilon_{it+1}$. If $D_h$ is known, we can get conditional mean and variance of market demand $D^*_{it+1}$ at time cycle (t+1):

$$m_a = d + \rho D_h$$

$$v_a = \sigma^2$$

Based on expression (4) and (5), it is easy to get the order-up-to level $S_i$, which is determined as follows:

$$S_i = m_a + l\sqrt{v_a} = d + \rho D_h + l\sigma$$

Here $I = \Phi^{-1}[p/(p+h)]$, $\Phi$ is the standard normal distribution function.

According to Eq. (6), information sharing doesn’t have any affect on retailers’ order-up-to level $S_i$, because retailers can always get goods of their orders.

**MANUFACTURE’S ORDER DECISION**

Now, let’s study manufacture’s order decision. Manufacture has to give its own order to reach order-up-to level $T_i$ before the end of time cycle t, when it finished filling out all retailers’ orders at time cycle t. For calculating $T_i$, manufacture has to predict the amount of all N retailers’ orders $\sum_{i=1}^{N} Y_i$. It is required to know the distribution of $\sum_{i=1}^{N} Y_i$ which is related to sharing of the real market demand $D_h$ at retailer site.

According to (1), (3) and (6), we can get the order of retailer i at time cycle

$$Y_{it+1} = d + \rho Y_i + (1 + \rho)\varepsilon_{it+1} - \rho\varepsilon_{it}$$

So total amount of N retailers’ orders is:

$$\sum_{i=1}^{N} Y_{it+1} = Nd + \rho \sum_{i=1}^{N} Y_i + (1 + \rho) \sum_{i=1}^{N} \varepsilon_{it+1} - \rho \sum_{i=1}^{N} \varepsilon_{it}$$

When manufacture shares the information of $(\varepsilon_{i1}, \ldots, \varepsilon_{in})$ with retailers, 1, 2, …, n, then Eq. (8) can be adjusted as:

$$\sum_{i=1}^{N} Y_{it+1} = Nd + \rho \sum_{i=1}^{N} Y_i - \rho \sum_{i=1}^{N} \varepsilon_{it} + (1 + \rho) \sum_{i=1}^{N} \varepsilon_{it+1} - \rho \sum_{i=1}^{N} \varepsilon_{it}$$

Because Eq. (9) is a linear combination of AR (1) model’s residual errors, it is a normal distribution.

**Theorem 1:** At time cycle t, if manufacture and retailers 1, 2, …, n share the information $(\varepsilon_{i1}, \ldots, \varepsilon_{in})$, then:
Using Theorem 1, we can calculate the conditional mean and variance of expression (9) when \((e_{1s}, \ldots, e_{nt})\) is known:

\[
M_{\bar{u}} = Nd + \rho \sum_{i=1}^{n} Y_{\bar{u}} - \rho \\
\sum_{i=1}^{n} e_{\bar{u}} - \rho \cdot \rho \cdot (N - n) \sum_{j=1}^{n} e_{\bar{u}} \\
+ \rho^2 (N - n) \left[ (1 - \rho)^2 N \left(1 + (N - 1) \rho \right) \right]
\]

Since the total order amount of N retailers at \((t+1)\) time cycle is normal distribution, thence order decision of manufacture is a Newsvendor Problem when market demand is a normal distribution. So manufacture should use the order-up-to-S strategy to achieve the minimum running cost. Based on expression (10) and (11), we can get manufacture’s order level \(T_{ij}\):

\[
T_{ij} = M_{ij} + k \sqrt{V_{ij}} = Nd + \rho \sum_{i=1}^{n} Y_{ij} - \rho \sum_{i=1}^{n} e_{\bar{u}} - \rho \cdot \rho \cdot (N - n) \sum_{j=1}^{n} e_{\bar{u}} \\
+ K \sigma \sum_{(1 + \rho)^2 N \left(1 + (N - 1) \rho \right) + \rho^2 (N - n) \left[ (1 - \rho)^2 N \left(1 + (N - 1) \rho \right) \right]}
\]

Here \(K = \Phi^{-1}\) \[P/(P + H)\], \(\Phi\) is the standard normal distribution function.

**THE VALUE ANALYSIS OF MANUFACTURE AND N RETAILERS INFORMATION SHARING**

Through to solve the two-stage stochastic dynamic program (2), we can analyze the value of information sharing between manufacture and n (0 ≤ n ≤ N) retailers. Suppose manufacture has the information sharing with n (0 ≤ n ≤ N) retailers, it will use expression (12) as the order-up-to-level to achieve the minimum total sum cost of inventory and shortage. Hence the total cost of manufacture is:

\[
G_{\bar{u}} = k \sigma \left[ (H + P) \Phi (K) + HK \right] \Phi^{-1} \left[ (H + P) \Phi (K) + HK \right] \\
= k \sigma \left[ (1 + \rho)^2 N \left(1 + (N - 1) \rho \right) + \rho^2 (N - n) \left[ (1 - \rho)^2 N \left(1 + (N - 1) \rho \right) \right] \right]
\]

In which \(L\) (K) is the right lost function of standard normal distribution. Assume:

\[
a = \sigma \left[ (H + P) L(K) + HK \right] \\
b = (1 + \rho)^2 N \left(1 + (N - 1) \rho \right) \\
c(n) = \rho^2 (N - n) \left[ (1 - \rho)^2 N \left(1 + (N - 1) \rho \right) \right]
\]

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g(n) is the last part of G(n) except the part of kn, then we get that G(n) = kn + g(n) = kn + \alpha\sqrt{b + c(n)}.

Obviously, G(n) has two parts, one part is the cost of information sharing with n retailers which is the strictly monotone increasing function of n, the second part is the total cost of inventory and shortage which is caused by market demand uncertainty and it is the strictly monotone decreasing function of n. Through analyzing G(n), we can get the following conclusion.

**Theorem 2:** There are, \( \bar{k} > 0 \) and \( k > 0 \), let

- If \( k > \bar{k} \), G(n) (0 \leq n \leq N) has a minimum when n = 0
- If \( k < \bar{k} \), G(n) (0 \leq n \leq N) has a minimum when n = N

**Proof:** We loose n being discrete variable in G(n) and to get its first partial derivative:

\[
\frac{\partial G(n)}{\partial n} = k + \frac{\partial g(n)}{\partial n} = k - \frac{a}{2}(b + c(n))
\]

\[
\frac{1}{2} \rho^2(1 - \rho_s)^2 + N(\rho_s - \rho_s^2)(1 - \{(n - 1)\rho + 1\})
\]

Due to g(n) is strictly monotone decreasing function of n and its first partial derivative is a continuous function on \( [0,N] \), so the first partial derivative has a maximum M>0 and a minimum m<0.

Let \( \bar{k} = -m \), if \( k > \bar{k} \), then \( \partial G(n)/ \partial n > 0 \), so G(n) (0 \leq n \leq N) has a minimum when n = 0. Let \( \bar{k} = -M \), if \( k < \bar{k} \), then \( \partial G(n)/ \partial n < 0 \), so G(n) (0 \leq n \leq N) has a minimum when n = N.

**Theorem 2 shows three results:**

- Manufacture will share information with no retailers if the cost of information sharing is bigger than the marginal income
- Manufacture will share information with all retailers if the cost of information sharing is less than the marginal income
- Manufacture will share information with \( n^* \) (0 < \( n^* \) < N) retailers if the cost of information sharing is middle. About the size of \( n^* \), we have the following conclusion:

**Theorem 3:** There are \( \bar{\rho}_r > 0 \) and \( \bar{\rho}_s > 0 \), let:

- If \( \rho_r > \bar{\rho}_s \), G(n) is a convex function of n (0 \leq n \leq N)
- If \( \rho_r > \bar{\rho}_s \), G(n) is a concave of n(0 \leq n \leq N).

**Proof:** To make the second partial derivative of \( G(n) \) on n, we get that

\[
\frac{\partial^2 G(n)}{\partial n^2} = \frac{a}{4} \rho^2(1 - \rho_s)^2 + N(\rho_s - \rho_s^2)(1 - \{(n - 1)\rho + 1\})
\]

\[
\frac{1}{4} \rho^2(1 - \rho_s)^2 + N(\rho_s - \rho_s^2)(1 - \{(n - 1)\rho + 1\})
\]

Since \( -1 / (N - 1) < \rho_r < 1 \), so the part in front of brace is larger than 0, hence \( \partial^2 G(n)/ \partial n^2 \) has the same sign as the part in the brace in the expression (14). Assuming S(n, \( \rho_r \)) equals to the part in the brace in the expression (14).

Obviously, when \( \rho_r < 0 \), the first and the third parts are both less than 0 and the second part is larger than 0, so S(n, \( \rho_r \)) < 0. When \( \rho_r = 1 \), then S(n, \( \rho_r \)) = - \rho^2 \leq 0. Because S(n, \( \rho_r \)) is a continuous function, there exists one \( \bar{\rho}_r > 0 \) when \( |\rho_r| < \bar{\rho}_r \), when \( S(n, \rho) \leq 0 \). Combined S(n, \( \rho_r \))’s value range when \( \rho_r < 0 \), we know that there exists one \( \bar{\rho}_r > 0 \), when \( \rho_r > \bar{\rho}_r \), which \( \partial^2 G(n)/ \partial n^2 \leq 0 \). Hence G(n) is a concave of n (0 \leq n \leq N).

When \( \rho_r = 1 \), then S(n, \( \rho_r \)) = 4n(1 + \rho)^2 N^2 \geq 0.

Since S(n, \( \rho_r \)) is a continuous function, there exists one \( \bar{\rho}_s > 0 \), when \( \rho_s > \bar{\rho}_s \), then \( \partial^2 G(n)/ \partial n^2 \geq 0 \). Hence G(n) is a convex function of n (0 \leq n \leq N).

**Theorem 3:** Illustrates that manufacture’s cost is a convex function after information sharing with n retailers when the market demand is highly correlated. The practical significance is that manufacture’s marginal cost will increase if it shares information with more retailers when the market demand is highly correlated. Hence it will prefer to information sharing with fewer retailers. Contrary, when the market demand is slightly or negatively correlated, manufacture’s cost is a concave function. The practical significance is that manufacture’s marginal cost will decrease if it shares information with more retailers.

**NUMERICAL SIMULATION CASE**

Assuming there are 10 retailers (N = 10), the retailer side’s market demand d = 300 at each time cycle, moving average coefficient \( \rho = 0.7 \), mean square deviation \( \sigma = 80 \). The parameters for manufacture are: inventory cost per unit \( H = 1 \), shortage cost per unit \( P = 25 \), the cost for information sharing with each retailer at each time cycle \( k = 36 \). When \( \rho_r \) changes from 0 to 0.999 and \( n \) changes from 0 to 10. Manufacture’s cost at each time cycle is simulated and given in Table 1 and 2.

From Table 1 and 2 we know that the number of retailers that manufacture want to have information sharing for minimum its cost is decrease when \( \rho_r \) changes from 0 to 0.999. When \( \rho_r = 0 \) and cost of information sharing is modest, manufacture have
Table 1: The total cost of manufacture when \( \rho \) and \( n \) change

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( n )</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>6713.4</td>
<td>9368.5</td>
<td>11422.2</td>
<td>13159.2</td>
<td>15394.7</td>
</tr>
<tr>
<td>1</td>
<td>6656.3</td>
<td>9149.7</td>
<td>10993.4</td>
<td>12479.4</td>
<td>14272.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6615.2</td>
<td>9045.6</td>
<td>10870.0</td>
<td>12384.8</td>
<td>14307.7</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6586.2</td>
<td>8992.9</td>
<td>10870.0</td>
<td>12384.8</td>
<td>14307.7</td>
<td></td>
</tr>
<tr>
<td>4</td>
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<td>8967.7</td>
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<td>12384.8</td>
<td>14343.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td>8958.9</td>
<td>10816.2</td>
<td>12388.7</td>
<td>14415.3</td>
<td></td>
</tr>
<tr>
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<td>6548.8</td>
<td>8960.7</td>
<td>10828.9</td>
<td>12411.5</td>
<td>14451.3</td>
<td></td>
</tr>
<tr>
<td>7</td>
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<tr>
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<tr>
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<td>6558.0</td>
<td>9001.9</td>
<td>10921.8</td>
<td>12528.0</td>
<td>14595.2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6567.8</td>
<td>9022.8</td>
<td>10921.8</td>
<td>12528.0</td>
<td>14595.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The number of retailers that have information sharing with manufacture when \( \rho \) change from 0 to 0.99

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
<th>0.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
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</table>

This study studies the value of information sharing in a two-echelon supply chain system with 1 manufacture and \( N \) retailers. When market demand is independent and information sharing with no cost, manufacture will have the maximum value to have information sharing with all retailers (Lee et al., 2000; Cachon and Fisher, 2000). When market demand is dependent and information sharing with some cost, manufacture’s strategy of information sharing will change. When the information cost is very high, manufacture would have information with no retailers to minimize its cost; and when the information cost is very low, manufacture would have information with all retailers. When the information cost is modest, manufacture would have information with modest number of retailers. The number of retailers that manufacture will have information sharing with is related to the correlated coefficient of market demand \( S \rho \). When \( S \rho \) small, manufacture would have information sharing with big number of retailers and \( \rho \) is big, manufacture would have information sharing with small number of retailers. Of course, these questions need further study if the models of market demand at different retailers sites are different, retailers also afford some cost of information sharing and lead time aren’t 0.

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REFERENCES


