Research Journal of Applied Sciences, Engineering and Technology 6(5): 849-857, 2013
DOI:10.19026/rjaset.6.4130
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: September 30, 2012 Accepted: November 29, 2012 Published: June 25, 2013

Research Article

Dynamic Modeling Method of a Whole Structure with Joints

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Abstract: The aim of this study was to study an improved dynamic modeling method of a whole structure with joints by the Boundary Element Method (BEM). The dynamic model was composed of the elements such as the beam element, plate element, joint element, lumped mass and spring element by the BEM, joints characteristics were introduced to boundary dynamic equations by flexible constraint conditions on boundary. Finally, an improved dynamic model of a whole structure with joints was established based on plate-beam element system mainly. As a result, the dynamic characteristics of a whole structure with joints were analyzed and the comparison of computational and experimental results showed the modeling method was effective. The analyses indicate that the introduced method inaugurates a good way for analyzing dynamic characteristics of a whole structure with joints efficiently.

Keywords: Boundary element method; dynamic modeling; joints; a whole structure

INTRODUCTION

Finite Element Method (FEM) and Boundary Element Method (BEM) have been used to solve dynamic problems for many years (Barik and Mukhopadhyay, 1998; Zietsman, 2003; Gaul and Kogl, 1999; Shen *et al*., 2003; Providakis, 1997). At present, most methods of dynamic modeling of a whole structure include the FEM and BEM. Whereas, a whole structure consists of many parts and joints and the whole domain need to be meshed when solving issues by the FEM. Considering a joint is nonlinear, vast scale iterative operations exist in the process of FEM calculating in order for improving precision. Hence, the calculated efficiency can be affected greatly (Tony *et al*., 2007; Dhupia *et al*., 2007). The essential feature of the BEM is that the boundary is only discretized instead of the domain, which takes less CPU time due to the one-dimension reduction in mesh generations. So, the BEM has some main advantages of inputting less data and reducing computational dimensions when analyzing dynamic modeling of a whole structure with joints (Brebbia, 2002). In order to improve computational precision and efficiency, the BEM will be adapted to establish the dynamic model of a whole structure with joints in this study.

Recently, most of the aforementioned investigations are focused on a whole structure and the analytical and numerical basis for joints characteristics has not been fully established. Especially, relatively few dynamic modeling analyses of a whole structure with joints by the BEM can be found in Refs (Zhang *et al*., 2002, 2003). Zhang predicted dynamic behaviors of a CNC profile-machining centre with moveable column based on computer-aided engineering by the BEM (Zhang *et al*., 2003) and discussed the dynamic behaviors of guidway joint and its application in machine tools design, too (Zhang *et al*., 2002). It is noted that the dynamic model in literatures (Zhang *et al*., 2002, 2003) were established by adapting the beam element system mainly. So, the precision of modeling maybe be affected for those elements need to be equivalent as plate element. At present, some experts including the author discussed dynamic modeling and numerical value solutions for some plate structure problems by the BEM in details (Yuan and Dawe, 2004; Fang and Wu, 2007). These investigations bring important roles for dynamic modeling of the whole structure with joints by the BEM.

Different from a component, a whole structure is composed of many components connected with many joints commonly and the dynamic characteristics of joints can affect the dynamic performances of the whole structure considerably (Yoshimura, 1979; Damjan and Miha, 2008). So, the dynamic characteristics of joints should to be considered during establishing dynamic model of a whole structure by the BEM. Whereas, the computational scale is very heavy for the whole structure consists of many parts and joints and how to reasonable introduce joints characteristics is very important problem. In order to improve modeling precision and efficiency by the BEM, plate element will be also considered to establish the BEM model based on the previous researches (Zhang *et al*., 2002, 2003;

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Fang and Wu, 2007). Finally, the improved dynamic model will be composed of the elements such as the beam element, plate element, joint element, lumped mass and spring element, joints characteristics can also be introduced to boundary dynamic equations by flexible constraint conditions on boundary. Numerical simulation and dynamic experiments show that the introduced method has excellent precision and efficiency and can realize high efficiency computation under satisfying good accuracy conditions. Thereby, this study lays good foundation for developing computer-aided engineering commercial software.

PROBLEM DESCRIPTION AND ANALYSIS OF SOLUTION

Different from a single structure or a component, a whole structure is composed of many components connected with many joints and the dynamic characteristics of joints affect the dynamic behaviors of a whole structure considerably. The dynamic characteristics of joints are affected by many factors. Here, the dynamic characteristics of joints refer to the dynamic stiffness and damping of joints. Experiments show that dynamic characteristics of joints are intense nonlinearity (Li *et al*., 2010). For the intense non-linearity of dynamic characteristics of joints, the values of the dynamic stiffness and damping of joints, which was based upon the dynamic characteristic parameters of joint surfaces at unit area obtained by experiments, can be determined during the analysis of the dynamic behaviors of a whole structure. Hence, there would be many iterations process during this analysis. How to establish a reasonable dynamic model of a whole structure is a key to analyze the dynamic characteristics of a whole structure. This study solved this problem by establishing boundary dynamic equations of a whole structure with joints, which are introduced in "Modeling of a whole structure with joints" of this study. Especially, it is impossible to treat all of these factors by an analyzing method in determining the dynamic characteristics of joints. Joints cannot exist without the mechanical system. The values of the dynamic stiffness and damping of joints obtained directly by experiments are used in similar conditions in the experiment. Since there are so many values obtained by experiments and they are not general for variable conditions, we cannot use them easily to analyze dynamic characteristics of a new whole structure. This study solved this problem by use of the dynamic characteristics parameters of joint surfaces at unit area, which are introduced in "Dynamic characteristics of joints" of this study.

Because of the complexity of a whole structure with joints, a reasonable dynamic model should not only calculate efficiently but also treat the non-linearity behaviors of joints easily for a whole structure. The dynamic model of a whole structure proposed in this study can meet these two requirements. According to the former researches, a whole structure is equivalent as the structural element, joint element, spring element and lumped mass when establishing the dynamic model by the BEM in this study. For the structural element including plate and beam components, they had been earnest analyzed and the detailed papers can be found in the literature and see for example (Zhang *et al*., 2003; Fang and Wu, 2007; Fang *et al*., 2003). After appropriate elements are chosen depending upon different shapes of components in a whole structure, a reasonable dynamic model is established. Based on the model we can analyze the dynamic characteristics of a whole structure and the modeling and simulating are discussed in "Example computation of" this study.

Based on the above analysis, this study is organized as follows: In Modeling of a whole structure with joints, we begin with a dynamic modeling of a whole structure with joints by the BEM, discussion the introduced method of joints characteristics and educe the boundary dynamic equations. In Modeling of a whole structure with joints, the dynamic characteristics of a bolt joint are analyzed and the complex stiffness matrix of the bolt joint is deduced as an example. Numerical example is presented in Example computation to demonstrate the efficiency and validity of the BEM. Finally, a summary is given in Experiment verification to conclude this study.

MODELING OF A WHOLE STRUCTURE WITH JOINTS

Introducing of joints characteristics: There will discuss the introduced method of joints characteristics. For a whole structure, let the sum of domain points on joints surface is r, the total area of domain points is A and the formulas of stiffness and damping of the ith domain point can be written as follows. (i = 1-n)

$$K_{ni} = \frac{K_{nd}A}{r}, \quad K_{\tau i} = \frac{K_{\tau d}A}{r} \tag{1}$$

$$C_{ni} = \frac{C_{nd}A}{r}, \quad C_{\tau i} = \frac{C_{\tau d}A}{r} \tag{2}$$

where, K_{ni}, $K_{\tau i}$ denote normal stiffness and tangential stiffness of point i, C_{ni}, $C_{\tau i}$ denote normal damping and tangential damping of point i, k_{nd}, $k_{\tau d}$ denote the normal stiffness and tangential stiffness at unit joint area, c_{nd}, $c_{\tau d}$ denote the normal damping and tangential damping at unit joint area, respectively.

Joints information of stiffness and damp is included in Eq. (1) and (2). After confirming the size, pressure, excited frequency and relative displacement, the stiffness and damping of joints can be gotten. At last, the value of complex stiffness matrices can be obtained and these problems for dynamic characteristics analysis of joints will be discussed in Dynamic characteristics of joints. It can be seen from the above discussion that main factors of affecting joints dynamic behaviors are included in Eq. (1 and 2). Thereby, the

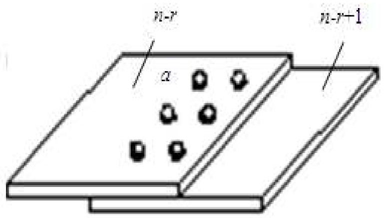

Fig. 1: The location *a*

above formulas have specific physical meaning and excellent universal property.

Relational expressions of joints conditions must be established, which is the modeling prerequisite of a whole structure. The dynamic equations of the whole structure can be educed effectively after all parts are assembled by joints conditions. A complex stiffness matrix can be used to denote joints conditions commonly. In order to illustrate easily, joints between parts No.*n-r*, No.*n-r*+1 are illustrated to establish the relational expressions of joints conditions.

It is easy to establish the equations for strength restraint and the following relational equations are satisfied.

$$(T_{n-r}{}^{i+1})^T = -S(T_{n-r+1}{}^{j})^T \tag{3}$$

where, $\boldsymbol{S}=(\boldsymbol{S}_a{}^{i+1})^{-1}(\boldsymbol{S}_a{}^{j})^{-1}$ denotes space coordination transforming matrices of location *a*, $\boldsymbol{T}_{n-r}{}^{i+1}$ denotes boundary strength matrix of part *n-r* at point *i*+1, $\boldsymbol{T}_{n-r+1}{}^{j}$ denotes boundary strength matrix of part *n-r*+1 at point *j*, the location *a* is shown in Fig. 1.

Based on the literature (Zhang *et al*., 2003; Fang and Wu, 2007), the restraint equations of the strength and displacement can be obtained after dealing with the matrices.

$$S_a{}^{j}KC_a(S_a{}^{i+1}U_{n-r}{}^{i+1} - S_a{}^{j}U_{n-r+1}{}^{j})^{\mathrm{T}} = -(T_{n-r+1}{}^{j})^{\mathrm{T}} \tag{4}$$

where, $\boldsymbol{U}_{n-r}{}^{i+1}$ is the boundary displacement matrix of part No.*n-r* at point *i*+1, $\boldsymbol{U}_{n-r+1}{}^{j}$ is the boundary displacement matrix of part No.*n-r*+1 at point *j*.

Modeling analysis of a whole structure with joints: In this section, dynamic modeling of a whole structure with joints will be established by the BEM. Let a whole structure is composed of *m* parts. As an example, the modeling process of parts No.*n-r*, No.*n-r*+1 are illustrated as follows.

According to the literatures (Fang and Wu, 2007), the governing boundary equations of parts No.*n-r*, No.*n-r*+1 are written:

$$\begin{bmatrix} EE_{n-r}{}^{11,i} & EE_{n-r}{}^{12,i+1} \\ EE_{n-r}{}^{21,i} & EE_{n-r}{}^{22,i+1} \end{bmatrix} \begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r}{}^{i+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r}{}^{i} \\ U_{n-r}{}^{i+1} \end{Bmatrix} \tag{5}$$

$$\begin{bmatrix} EE_{n-r+1}{}^{11,j} & EE_{n-r+1}{}^{12,j+1} \\ EE_{n-r+1}{}^{21,j} & EE_{n-r+1}{}^{22,j+1} \end{bmatrix} \begin{Bmatrix} T_{n-r+1}{}^{j} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r+1}{}^{j} \\ U_{n-r+1}{}^{j+1} \end{Bmatrix} \tag{6}$$

where,

EE = The coefficient matrix on boundary, the subscript sign of each matrix denotes serial number of parts.

U,T = The corresponding matrix and the subscript sign *g* denotes the joint surface elements in domain.

According to the relation of space coordination transforming matrix on boundary *a*, Eq. (4) can be denoted the following form after being dealt with.

$$\begin{bmatrix} \boldsymbol{0} & RR_{12} & RR_{13} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} U_{n-r}{}^{i} \\ U_{n-r}{}^{i+1} \\ U_{n-r+1}{}^{j} \\ U_{n-r+1}{}^{j+1} \end{Bmatrix} = T_{n-r+1}{}^{j} \tag{7}$$

where, RR denotes the joints complex stiffness matrix, RR_{12}=$-S_a{}^{j}KC_aS_a{}^{i+1}$, RR_{13}= $S_a{}^{j}KC_aS_a{}^{j}$.

Equation (5 and 6) is combined and the following equations can be gotten.

$$\begin{bmatrix} EE_{n-r}{}^{11,i} & EE_{n-r}{}^{12,i+1} & \boldsymbol{0} & \boldsymbol{0} \\ EE_{n-r}{}^{21,i} & EE_{n-r}{}^{22,i+1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & EE_{n-r+1}{}^{11,j} & EE_{n-r+1}{}^{12,j+1} \\ \boldsymbol{0} & \boldsymbol{0} & EE_{n-r+1}{}^{21,j} & EE_{n-r+1}{}^{22,j+1} \end{bmatrix} \begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r}{}^{i+1} \\ T_{n-r+1}{}^{j} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r}{}^{i} \\ U_{n-r}{}^{i+1} \\ U_{n-r+1}{}^{j} \\ U_{n-r+1}{}^{j+1} \end{Bmatrix} \tag{8}$$

Substitute Eq. (3) into Eq. (8) results in:

$$\begin{bmatrix} EE_{n-r}{}^{11,i} & -EE_{n-r}{}^{12,i+1}S & \boldsymbol{0} \\ EE_{n-r}{}^{21,i} & -EE_{n-r}{}^{22,i+1}S & \boldsymbol{0} \\ \boldsymbol{0} & EE_{n-r+1}{}^{11,j} & EE_{n-r+1}{}^{12,j+1} \\ \boldsymbol{0} & EE_{n-r+1}{}^{21,j} & EE_{n-r+1}{}^{22,j+1} \end{bmatrix} \begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r+1}{}^{j} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r}{}^{i} \\ U_{n-r}{}^{i+1} \\ U_{n-r+1}{}^{j} \\ U_{n-r+1}{}^{j+1} \end{Bmatrix} \tag{9}$$

Equation (9) is substituted into Eq. (7) and the following equations can be gotten after being dealt with.

$$\begin{aligned} T_{n-r+1}{}^{j} &= \left[I - RR_{13}EE_{n-r+1}{}^{11,j} + RR_{12}EE_{n-r}{}^{22,i+1}S\right]^{-1} \\ &\left[RR_{12}EE_{n-r}{}^{21,i} \quad RR_{13}EE_{n-r+1}{}^{12,j+1}\right] \begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} \\ &= \left[RE_{11} \quad RE_{22}\right] \begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} \end{aligned} \tag{10}$$

Equation (10) is substituted into Eq. (9) and the boundary dynamic equations of part No.*n-r*, No.*n-r*+1 are gotten after being dealt with.

$$\begin{bmatrix} EE_{n-r}{}^{11,i} - EE_{n-r}{}^{12,i+1}SRE_{11} & -EE_{n-r}{}^{12,i+1}SRE_{22} \\ EE_{n-r}{}^{21,i} - EE_{n-r}{}^{22,i+1}SRE_{11} & -EE_{n-r}{}^{22,i+1}SRE_{22} \\ EE_{n-r+1}{}^{11,j}RE_{11} & EE_{n-r+1}{}^{12,j+1} + EE_{n-r+1}{}^{12,j+1}RE_{22} \\ EE_{n-r+1}{}^{21,j}RE_{11} & EE_{n-r+1}{}^{22,j+1} + EE_{n-r+1}{}^{22,j+1}RE_{22} \end{bmatrix} \tag{11}$$

$$\begin{Bmatrix} T_{n-r}{}^{i} \\ T_{n-r+1}{}^{j+1} \end{Bmatrix} = \begin{Bmatrix} U_{n-r}{}^{i} \\ U_{n-r}{}^{i+1} \\ U_{n-r+1}{}^{j} \\ U_{n-r+1}{}^{j+1} \end{Bmatrix}$$

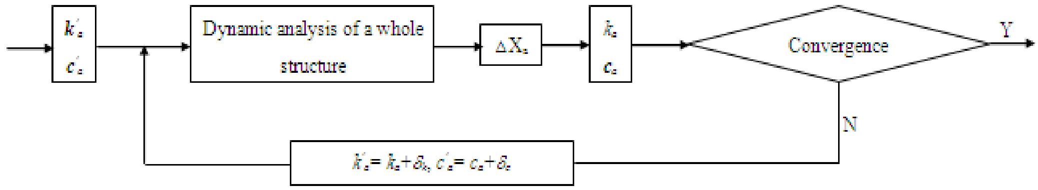

Fig. 2: Iterative calculating process

For a whole structure is composed of many components connected with many joints, Eq. (11) can be combined unceasingly according to the joints conditions. Repeating above process until finishing assembled process, the final dynamic equations of a whole structure can be established and the dynamic characteristics of the whole structure with joints can be gotten by solving Eq. (11).

Owing to joints characteristics had been introduced to boundary dynamic equations by flexible constraint conditions on boundary, the corresponding BEM dynamic model of the whole structure with joints was educed finally. Considering the flexible joints have nonlinear characteristics as well as the stiffness and damping are the functions of displacement, the values of stiffness and damping are unknown and they have only interdependent relations in the equation. Therefore, a large amount of computing exists in Eq. (14). In order to improve computational precision, joint's displacements must be repeated iteration to obtain the values. Namely, the known values of stiffness and damping are used to act as the iterative initial values firstly. As a result, the iterative calculating of the equations can be executed. The iterative convergence is judged by using the variation of front-to-back twice iterative values of stiffness and damping. Let analyze the joints on boundary *a*, the convergence can be judged when satisfying the relations of $|k_a - k'_a| < \varepsilon_k$ and $|c_a - c'_a| < \varepsilon_c$. The entire iterative calculating process can be given in Fig. 2, where δ_k and δ_c are the front-to-back error values of stiffness and damp, respectively. The detailed calculating approach of joints characteristics will be discussed in subsequent paper.

Dynamic characteristics of joints: Although the dynamic equations of a whole structure have been educed by the above discussion, the values of stiffness and damping of joints in Eq. (14) have still not been determined. As an example, the dynamic characteristics of a bolt joint will be analyzed.

Owing to joints surface between components produce small displacement and rotation, it can not only storage energy but also consume energy. So, analysis of joints characteristics is the foundation for discussing dynamic modeling of a whole structure with joints. Dynamic characteristics of joints in a whole structure are determined by many factors and the related theory of joints can be found in literatures (Chlebus and Dybala, 1999). These factors are categorized into two aspects. The first aspect is taken into account as the dynamic fundamental behavior parameters of joint surfaces at unit area and the other aspect is taken into account through analysis of dynamic behaviors of joints.

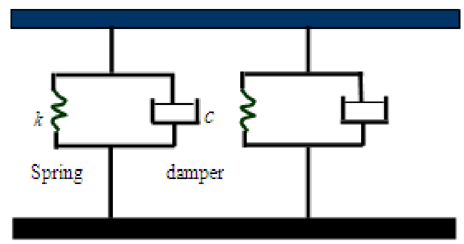

Fig. 3: The joint model

Joints model are equivalent as springs and dampers commonly, lots of scholars have agreed with the viewpoint for the joint model and the corresponding joint model is shown in Fig. 3.

According to some investigations (Tony *et al*., 2007; Dhupia *et al*., 2008), many kinds of dynamic fundamental behavior parameters of joint surfaces at unit joint area have been gotten by experimental method. At last, the dynamic fundamental parameters of joint surfaces at unit joint area, namely the universal behaviors formulas of stiffness and damp at unit joint area can be given as follows by experiments (Yoshimura, 1979; Fu *et al*., 1993; Dhupia *et al*., 2008; Guo *et al*., 2011):

$$k_{nd} = \alpha_n p_n^{\beta_n} \omega^{\gamma_n} x_n^{\eta_n} \quad k_{\tau d} = \alpha_\tau p_n^{\beta_\tau} \omega^{\gamma_\tau} x_\tau^{\eta_\tau} \qquad (12)$$

$$c_{nd} = \alpha_{nc} p_n^{\beta_{nc}} \omega^{\gamma_{nc}} x_n^{\eta_{nc}} \quad c_{\tau d} = \alpha_{\tau c} p_n^{\beta_{\tau c}} \omega^{\gamma_{\tau c}} x_\tau^{\eta_{\tau c}} \qquad (13)$$

where, α, β, γ, η denote the corresponding behaviors coefficient, p_n denotes the normal pressure and x_n, x_τ denote the normal and tangential amplitude.

A bolt joint is very common in a whole structure and the sectional drawing is presented in Fig. 4a. As an example, this section will discuss the dynamic modeling of a bolt joint. According to the former analysis, the equivalent model of a bolt joint can be established by equivalent as the number of *n* springs and dampers, as shown in Fig. 4b and c shows a coordinate system of dynamic analysis of a bolt joint.

In Fig. 4, *D* is the relative displacement matrix of joint in the whole coordinate system, $D = \{D_1, D_2, D_3,$

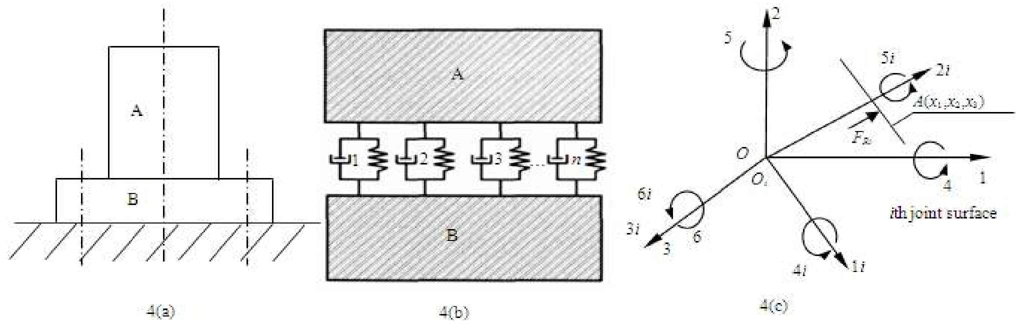

Fig. 4: The equivalent model of a bolt joint

D_4, D_5, $D_6\}$ denotes the amplitudes vector of translational displacement and rotational displacement, D_i is the relative displacement matrix of the ith joint surface, respectively. According to the selected coordinate system, the following equation can be given:

$$\begin{aligned}\{D_{ji}{}^{R}\} &= [T_i]^{-1}\cdot\{D_i\} \\ &= \{D_{1i}{}^{R}, D_{2i}{}^{R}, D_{3i}{}^{R}, D_{4i}{}^{R}, D_{5i}{}^{R}, D_{6i}{}^{R}\}\end{aligned} \tag{14}$$

where, T_i denotes the transform matrix, D_{ji}^{R} denotes the relative displacement matrix of the ith joint surface in the local coordinate system, respectively.

For a bolt joint, the relation of deformation and load is nonlinear. Assuming the influence of joint relative vibration displacement for stiffness is ignored, the displacement amplitudes at point $A(x_1$, $x2$ and $x3)$ on the ith joint surface are given as follows:

$$\begin{cases}\lambda_{1i} = \lambda_{P_{1i}} - D_{1i}{}^{R} + x_2 D_{6i}{}^{R} - x_3 D_{5i}{}^{R} \\ \lambda_{2i} = \lambda_{P_{2i}} - D_{2i}{}^{R} + x_1 D_{6i}{}^{R} - x_3 D_{4i}{}^{R} \\ \lambda_{3i} = \lambda_{P_{3i}} - D_{3i}{}^{R} + x_1 D_{5i}{}^{R} - x_2 D_{4i}{}^{R}\end{cases} \tag{15}$$

where, λ_{1i}, λ_{2i} and λ_{3i} are the displacement amplitudes at point $A(x_1,x_2,x_3)$ on the ith joint surface, $\lambda_{P_{1,i}}$, $\lambda_{P_{2,i}}$, and $\lambda_{P_{3,i}}$, are the initial displacement amplitudes on the ith joint surface.

From Eq. (12 to 13), the dynamic stiffness and damping per unit joint area at point $A(x_1,x_2,x_3)$ on the ith joint surface can be gotten:

Dynamic stiffness

$$\begin{cases}k_n = \alpha_n p_n{}^{\beta_n}\omega^{\gamma_n}|x_2|^{\eta_n} \\ k_\tau = \alpha_\tau p_n{}^{\beta_\tau}\omega^{\gamma_\tau}|x_1|^{\eta_\tau} \\ k_3 = \alpha_\tau p_n{}^{\beta_\tau}\omega^{\gamma_\tau}|x_3|^{\eta_\tau}\end{cases} \tag{16}$$

Damping

$$\begin{cases}c_n = \alpha_{nc} p_n{}^{\beta_{nc}}\omega^{\gamma_{nc}}|x_2|^{\eta_{nc}} \\ c_\tau = \alpha_{\tau c} p_n{}^{\beta_{\tau c}}\omega^{\gamma_{\tau c}}|x_1|^{\eta_{\tau c}} \\ c_3 = \alpha_\tau p_n{}^{\beta_{\tau c}}\omega^{\gamma_{\tau c}}|x_3|^{\eta_{\tau c}}\end{cases} \tag{17}$$

Based on the above equations, the unit force equations at point $A(x_1$, x_2 and $x_3)$ on the joint surface can be gotten.

$$\begin{cases}P_{1i} = (k_n + i\omega c_n)\lambda_{2i} \\ P_{2i} = (k_\tau + i\omega c_\tau)\lambda_{1i} \\ P_{3i} = (k_3 + i\omega c_3)\lambda_{3i}\end{cases} \tag{18}$$

Based on the Eq. (16 and 18), the forces on the ith joint surface are gotten by integration of the unit forces over the area of the joint surfaces as follows.

$$\begin{cases}F_{1i} = -\iint_{S_i} P_{1i}ds_i \\ F_{2i} = -\iint_{S_i} P_{2i}ds_i + \sum_{j=1}^{N_i}(P_{ji} + \Delta P_{ji}) \\ F_{3i} = -\iint_{S_i} P_{3i}ds_i \\ F_{4i} = -\iint_{S_i} P_{3i}x_2 ds_i + \iint_{S_i} P_{2i}x_3 ds_i - \sum_{j=1}^{N_i} D_{3i}{}^{R}(P_{ji} + \Delta P_{ji}) \\ F_{5i} = -\iint_{S_i} P_{1i}x_3 ds_i + \iint_{S_i} P_{3i}x_1 ds_i \\ F_{6i} = -\iint_{S_i} P_{2i}x_1 ds_i + \iint_{S_i} P_{1i}x_2 ds_i + \sum_{j=1}^{N_i} D_{1i}{}^{R}(P_{ji} + \Delta P_{ji})\end{cases} \tag{19}$$

where

$$\Delta P_{ji} = a\,\frac{EA}{l}\left(D_2 + D_{1i}{}^{R}D_6 - D_{3i}{}^{R}D_4\right)$$

$j = 1, 2,\ldots N_i$, N_i is the total number of contact surfaces on the ith bolt joint surface, P_{ji} is the pretightening force of the ith bolt joint, ΔP_{ji} is the force change of connective bolt, l, E, A, a is the length, basal area, longitudinal elastic ratio, influence coefficient of a bolt joint, respectively.

According to the definition of dynamic stiffness, the complex stiffness equations on the ith joint surface can be deduced. Owing to $\boldsymbol{D}_{ji}^{R}$ are included in λ_{1i}, λ_{2i}, λ_{3i}, the following equations is gotten:

$$K_{J_{ij}} = \frac{\partial F_{ji}}{\partial D_{ji}{}^{R}} = \frac{\partial F_{ji}}{\partial \lambda_{1i}}\cdot\frac{\partial \lambda_{1i}}{\partial D_{ji}{}^{R}} + \frac{\partial F_{ji}}{\partial \lambda_{2i}}\cdot\frac{\partial \lambda_{2i}}{\partial D_{ji}{}^{R}} + \frac{\partial F_{ji}}{\partial \lambda_{3i}}\cdot\frac{\partial \lambda_{3i}}{\partial D_{ji}{}^{R}} \tag{20}$$

Substitute Eq. (19 into Eq. 20), all elements expressions of complex stiffness matrix K_{Jij} can be

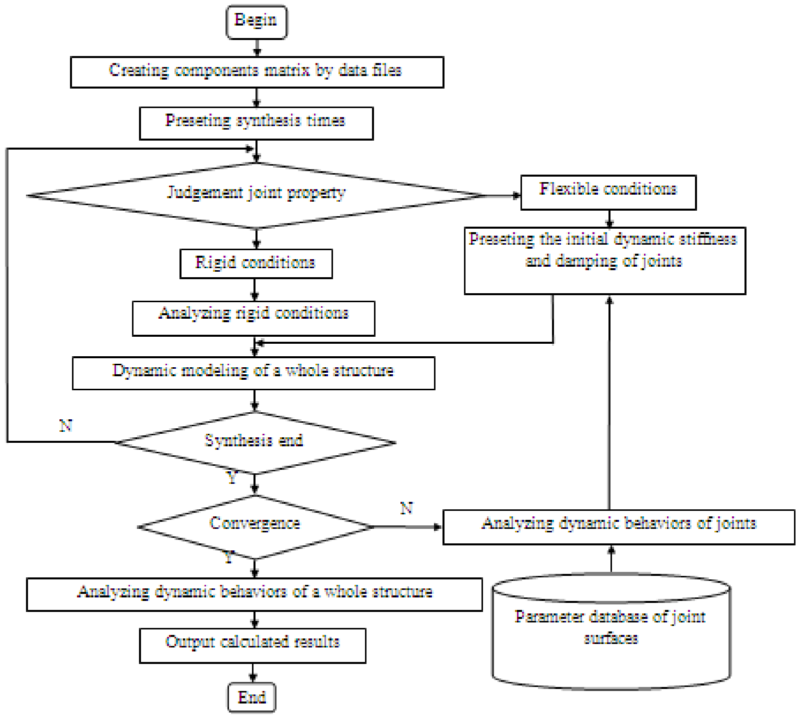

Fig. 5: The flowchart of calculating dynamic characteristics

Fig. 6: Entity picture of an oscillating table structure

educed. At last, the complex stiffness equation for the *i*th joint surface is expressed as:

$$\{F_{ji}\}=\left[K_{J_{ij}}\right]\{D_{ji}^{R}\} \tag{21}$$

The total force on all of bolt joint surfaces can be gotten as follows:

$$\{F_{j}\}=\left(\sum_{i=1}^{N}[T_i]\cdot[K_{J_{ij}}]\cdot[T_i]^{-1}\right)\cdot\{D_{j}^{R}\} \tag{22}$$

According to Eq. (22), the complex stiffness matrix of the bolt joint is deduced:

$$\left[KC_J\right]=\sum_{i=1}^{N}[T_i]\cdot[K_{J_{ij}}]\cdot[T_i]^{-1} \tag{23}$$

In Eq.23, $KC_J = K_J+i\omega C_J$. From the above discussion, it can be seen that the complex stiffness matrix of the bolt joint includes all the factors that affect the dynamic characteristics of the joint and it has very important physical meaning. Analysis of the dynamic characteristics of joints proposed in this study is also fit for other types of joints, such as cylinder joints, guide way joints, etc. So, the introduced approach can be used to analyze the dynamic characteristics of many joints by the similar modeling method.

EXAMPLE COMPUTATIONS

According to the above modeling process of a whole structure with joints, the corresponding procedure has been developed to analyze dynamic characteristics of the whole structure and the corresponding flowchart is shown in Fig. 5. In order to check the correctness of modeling method for a whole structure with joints, there analyzed dynamic characteristics of an oscillating table structure that include bolt and cylinder joints mainly and the entity picture of an oscillating table structure was shown in Fig. 6. Considering establish dynamic modeling based

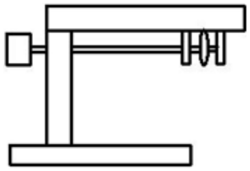

Fig. 7: The simplified model

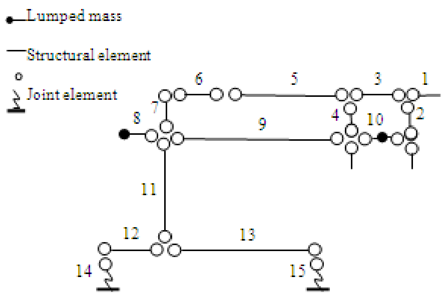

Fig. 8: The BEM calculating model

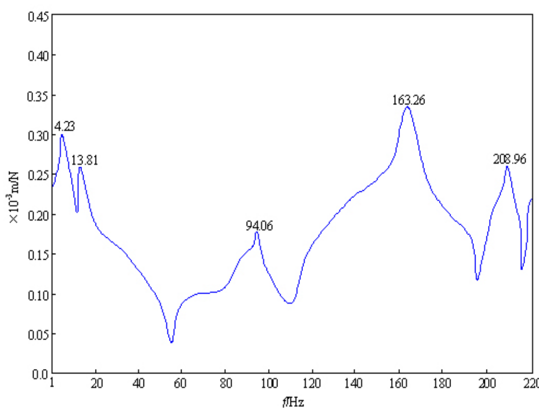

Fig. 9: The curve of frequency characteristics

Table 1: Comparison of computational and experimental values

Frequency (Hz)	Computational values	Experimental values	Relative error (%)
f_1	4.23	3.98	6.3
f_2	13.81	12.94	6.7
f_3	94.06	87.50	7.5
f_4	163.26	150.75	8.3
f_5	208.96	189.80	10.1

on the BEM, a simplified model of the whole structure was given as shown in Fig. 7 based on the principle of modeling method proposed in this study.

In Fig. 7, the electric engine and eccentric wheel are equivalent as the lumped mass, the upright column, bed plate and support plate are equivalent as plate parts, the bridging beam are equivalent as beam parts, the ground base is equivalent as springs. The bolt joints mainly exist in bolt pontes and the cylinder joints mainly exist in the bearing and support plate. At last, the calculating model of the whole structure with joints is established by the BEM, as shown in Fig. 8. The frequency curve is presented in Fig. 9 and the calculated values of first five natural frequencies are given in Table 1. It can be seen from Fig. 9 and Table 1 that the maximum difference between the calculated values and experiments' is 10.1%.

Fig. 10: Experimental scene of the test

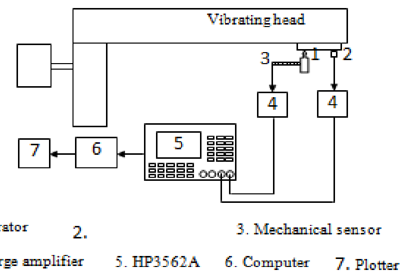

Fig. 11: The flow chart of dynamic experiments

Fig. 12: The curve of frequency responses

EXPERIMENT VERIFICATION

In order to verify the validity of the modeling method, the dynamic experiments was done by dynamic signal analyzer and the experimental scene and main flow chart were shown in Fig. 10 and 11.

The exciting force produced by the mechanical sensor was applied at the end of spindle, the accelerometer was also mounted at the end of spindle near the exciting point, all signals were transmitted to HP3562A dynamic signal analyzer by charge amplifiers and the frequency response curve was gotten by plotter. In order to reduce experimental error and improve signal to noise ratio, the average results of ten time self-

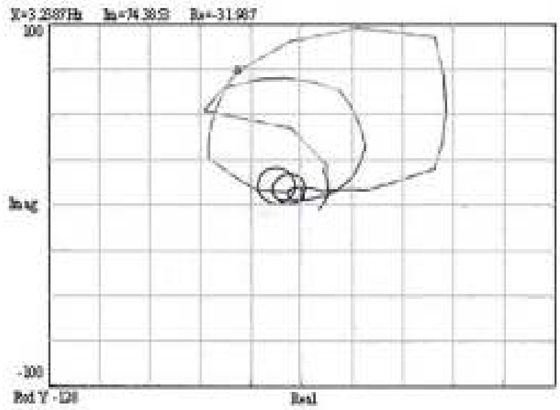

Fig. 13: The curve of amplitude responses

excited test are executed in the process of dynamic experiments. At last, the curves of frequency and amplitude responses of the whole structure were obtained by the experiments, as shown in Fig. 12 and 13 and the experimental results can be found in Table 1.

It can be seen from the Fig. 9, 12 and 13 that the computation results are close to experiments', the maximum difference between the computational and experimental values is less than 11%. So, the calculated model and method based on the BEM are effective. At the same time, it should be pointed out that the theoretical model used is based on the BEM, simplified dynamic model and processing joints can bring some influences for the calculated results. However, the trend observation should be valid by comparison of the computational and experimental results as shown in Table 1.

CONCLUSION

This study investigates the dynamic modeling method of a whole structure with joints, the proposed method is easy to introduce joints characteristics in the process of dynamic modeling and the process of modeling is succinct. The computed results and experiments' showed that the calculated model was effective and the maximum difference between the calculated values and experiments' is less than 11%. Thereby, it is excellent solve modeling issues of a whole structure with joints by the BEM. For instance, several observations have been obtained through the numerical examples:

- Flexible conditions are introduced to the dynamic model reasonably and it provides an efficient method for solving joints nonlinear problems.
- The modeling thought is legible and the introduced model is very convenient to analyze the dynamic characteristics of a whole structure with joints. Result shows the proposed method can solve BEM modeling issues of a whole structure with joints effectively. Hence, this study lays good foundation for developing computer-aided engineering commercial software.

Obviously, the proposed model is far from a mature approach. To make it more substantial, it is suggested that the future study focuses on the following areas:

- Determine all kinds of dynamic basic parameters of joints and then modify the proposed model with joints to improve the calculated precision.
- Extend the present model to explore the influence rules of structural modification for dynamic characteristics of a whole structure with joints based on the developed procedure and establish an analytic platform for structural modification and optimum design of a whole structure in future.

ACKNOWLEDGMENT

The authors gratefully acknowledge supports of the National Natural Science Foundation of China (No. 10972236), the Natural Science Foundation of Shaanxi Province, China (No. 2010JM6012) and the Science Foundation of China Postdoctor (No. 20100471835).

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