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Research Article

Implementation of Modified Conjugate Gradient Algorithm and Analysis of Convergence in Electromagnetic Tomography Lab System

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Abstract: Electromagnetic tomography technology is a new process tomography technology. The aim of this study is to develop a new image reconstruction algorithm suitable to electromagnetic tomography and verify its convergence. The advantages and development of electromagnetic tomography technology and image reconstruction algorithms are introduced briefly. Based on conjugate gradient algorithm, modified conjugate gradient algorithm for Electromagnetic Tomography (EMT) is proposed. Convergence of the modified conjugate gradient algorithm is analyzed. In the light of the lab electromagnetic tomography system, modified conjugate gradient algorithm for reconstructing images is verified. By evaluation of image error and the relevance, regularization algorithm, Landweber algorithm, conjugate gradient algorithm and modified conjugate gradient algorithm are compared. It can draw the conclusion that for different flow patterns, modified conjugate gradient algorithm is superior to other algorithms in the 8 coils electromagnetic tomography lab system.

Keywords: Convergence, electromagnetic tomography, modified conjugate gradient algorithm

INTRODUCTION

Electromagnetic Tomography (EMT) technology is one of process tomography technology developed in the 1990s based on the principle of electromagnetic induction (Peyton *et al*., 1996) and is suitable for testing and to reproduce available electrical conductivity or magnetic permeability material space. The same with other electrical tomography technology, the EMT system is use of more incentives and multi-detection measurement mode. Due to the volatility of the high-frequency electromagnetic fields, it has no need to contact, non-invasive and high-speed image capture, etc. EMT technology applied to many fields including medical field, mineral processing and separation, nondestructive testing, food processing, textiles and manufacturing industry (Korzhenevskii and Cherepenin, 1997; Korjenevsky *et al*., 2000). Of course, EMT is more suitable for the inspection of brain tissue (Li *et al*., 2002).

Al-Zeibak and Saunders (1993) first developed two-coil EMT hardware systems and applied to the biomedical field. Korzhenevskii and Cherepenin (1997) developed a very good two-coil EMT theoretical model and put forward the phase measurement to replace the amplitude measurement method. Yu *et al*. (1992, 1993, 1994) developed the parallel incentive EMT system, which greatly improved the accuracy and speed of the sampling rate. Scharfetter *et al*. (2008) proposed a parallel field excitation of an adjustable excitation frequency EMT system, whose excitation frequency bandwidth is 50 kHz-1.5 MHz.

EMT image reconstruction studied later. In 1996, the University of Averio developed heuristic algorithm and ART algorithm in EMT system (Peyton *et al*., 1996). Afterward, Korzhenevskii and Cherepenin (1997) proposed the back projection algorithm for image reconstruction of objects in a variety of media distribution. In 1999, Peyton *et al.* (1996) achieved a weighted linear back projection algorithm and sensitivity back projection algorithm (Peyton *et al*., 1996; Yin *et al*., 2008) in the parallel field excitation EMT system. In addition, many literatures described the Tikhonov regularization method (Hermann *et al*., 2006; Manuchehr and William, 2006) for process tomography.

However, current algorithms exist problems of multiple iterations and poor convergence. In order to improve the conjugate gradient algorithm, that is overcome not strictly convergence or long convergence time of the conjugate gradient algorithm, modified algorithm is derived. For lab electromagnetic tomography system, reconstructing ability and

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convergence of the modified method and other algorithms are evaluated.

MODIFIED CONJUGATE GRADIENT ALGORITHM

Conjugate gradient algorithm in EMT: In EMT, the relationship between the measured value the coil obtained and the space distribution of the object field is nonlinear, only be linearized by following formula:

$$A = SX \tag{1}$$

where,
A = m×1 dimensional detection voltage matrix
S = m×n dimensional sensitivity matrix
X = m×1 dimensional grayscale matrix

The optimization problem of the Eq. (1) is f = min (A-SX), whose nonlinear conjugate gradient method is:

$$x_{k+1} = x_k + \alpha_k p_k \qquad k = 0, 1, \ldots, \tag{2}$$

where, α_k = Step factor, which can be obtained by accurate linear search, Armijo linear search, or Wolfe linear search. Search direction p_k can be defined by:

$$p_k = \begin{cases} -r_k & k = 0 \\ -r_k + \beta_k p_{k-1} & k > 0 \end{cases} \tag{3}$$

where, β_k is determined by a variety of method and the most commonly used is the FR method, that is:

$$\beta_k = \frac{\|r_k\|^2}{\|r_{k-1}\|^2} \tag{4}$$

where, r_k is the gradient. The global convergence of FR method to solve non-convex optimization problem is proved in study (Zoutendijk, 1970), but in linear search sufficient decent condition:

$$-r_k^T p_k \geq c\|r_k\|^2 \quad c \in R^n, c > 0 \tag{5}$$

May not be valid (Zhang, 2006).

Modified FR method: In order to improve FR method, to produce the sufficient descent of f, from condition (5), the modified formula is deduced:

$$\frac{r_k^T p_k}{\|r_k\|^2} = \frac{r_{k-1}^T p_{k-1}}{\|r_{k-1}\|^2} h_k \leq -c \tag{6}$$

where, h_k>0, get h_k = 1, then:

$$p_k^T r_k = \frac{\|r_k\|^2}{\|r_{k-1}\|^2} p_{k-1}^T r_{k-1} \tag{7}$$

$$= -\frac{\|r_k\|^2}{\|r_{k-1}\|^2}\left[\frac{p_{k-1}^T(r_k - r_{k-1})}{\|r_{k-1}\|^2}\|r_{k-1}\|^2 - p_{k-1}^T r_k\right]$$

$$= -\frac{\|r_k\|^2}{\|r_{k-1}\|^2}\left[\frac{p_{k-1}^T(r_k - r_{k-1})}{\|r_{k-1}\|^2}\|r_{k-1}\|^2 - r_k^T p_{k-1}\right]$$

$$= -\frac{p_{k-1}^T(r_k - r_{k-1})}{\|r_{k-1}\|^2}\|r_k\|^2 + \frac{\|r_k\|^2}{\|r_{k-1}\|^2} r_k^T p_{k-1}$$

Get:

$$\lambda_k = \frac{p_{k-1(r_k - r_{l-1})}^T}{\|r_{k-1}\|^2}$$

then,

$$p_k^T r_k = -\lambda_k \|r_k\|^2 + \beta_k r_k^T p_{k-1}$$

and do the inner product on both sides, then:

$$p_k = -\lambda_k r_k + \beta_k p_{k-1} \tag{8}$$

Thus, Eq. (8) is introduced parameters λ_k to Eq. (3), that is, p_k can get the following form:

$$p_k = \begin{cases} -r_k & k = 0 \\ -\lambda_k r_k + \beta_k p_{k-1} & k > 0 \end{cases} \tag{9}$$

The sufficient decent direction p_k is ensured by the parameter λ_k in x_k, that is, $r_k^T p_{k<0}$ is satisfied. This method is defined as Modified Conjugate Gradient (MCG).

The Conjugate Gradient (CG) method gets λ_k = 1, but in MCG λ_k is determined by the search direction p_k, which is adaptive. When the search direction is not decreased, get λ_k>1, but when the search direction is decreased, get λ_k < 1, which can accelerate the convergence rate under the premise of the search direction sufficient descent.

CONVERGENCY

For step α_k, standard Wolfe line searches condition (3) and (4), then obtain:

$$f(x + \alpha p) - f(x) \leq \eta\alpha\lambda^T r^T p \tag{10}$$

where, $\alpha \in [0, \alpha_{max}]$ and:

$$\alpha_{\max} = \frac{2(\eta - 1)r(x)^T p}{L(x)\|p\|^2} \tag{11}$$

where,
L (x) = Lipschitz constant

From Eq. (10) and (5), we can get:

$$f(x_k) - f(x_{k-1}) \le \eta\alpha\lambda^T r_k^T p_k \le 0 \tag{12}$$

This means f (x_k) is a monotonically decreasing sequence.

Suppose (H_1): $x_k \in L$, L = {x∈ R^n; f(x) ≤f (x_1)} is bounded, where x_1 is the start point.

Lipschitz suppose (H_2): among a field U of L = {x∈ R^n; f(x) ≤f (x_1)}, f is continuously differentiable and its derivative g meets the Lipschitz condition, that is, there is a constant L>0 to meet $\|g(x) - g(y)\| \le L\|x - y\|, \forall x, y \in U$.

By the assumption (H_1) knowing, there is a constant f^* to meet $\lim_{k\to\infty} f(x_k) = f^*$, so:

$$\sum_{k=1}^{\infty}\big(f(x_k) - f(x_{k-1})\big) = \lim_{N\to\infty}(f(x_k) - f(x_1))) = f^* - f_1$$

that is:

$$\sum_{k=1}^{\infty}(f(x_k) - f(x_{k-1})) < \infty \tag{13}$$

According to (H_1) and (H_2), x_k is produced by the iterative Eq. (2), α_k satisfied Eq. (3) and (4) and p_k met $p_k^r r_{k<0}$. Therefore, for arbitrary k, there is a constant m to make:

$$f(x_k) - f(x_{k-1}) \ge m\|\alpha_k p_k\|^2 \tag{14}$$

By Eq. (13) and (14), we can obtain:

$$\infty > \frac{1}{m}\sum_{k=1}^{\infty}(f(x_k) - f(x_{k-1})) \ge \sum_{k=1}^{\infty}\|\alpha_k p_k\|^2 = \sum_{k=1}^{\infty}\|x_k - x_{k-1}\|^2 \tag{15}$$

where, the series $\sum_{k=1}^{\infty}\|x_k - x_{k-1}\|^2$ is convergence, so $\|x_k - x_{k-1}\|$⟶0, k⟶0. This shows that the MCG method has good convergence.

EMT system for 8 coils high frequency monolayer:

System design: The EMT system generally is constituted by excitation system, detection system and the PC. The excitation system includes excitation source and excitation coils to form the excitation magnetic field in object field. The detection system is constituted by the detection coils and the detection circuit to obtain different direction detecting value in order to achieve an image reconstruction. The PC is used for processing the detection values and reconstructing the images, including generating a sensitivity matrix, processing the detection value and developing image reconstruction algorithm and so on.

A diagram of the 8 coils HF monolayer EMT system is shown in Fig. 1. According to the EMT system circuit design principles, it can be divided into four parts, including the excitation signal processing circuit, the sensor array, the detection signal processing circuit and the PC. The excitation signal processing circuit is used for generating the adjustable RF signal, amplifying weak adjustable HF signal and exciting the excitation coil to form magnetic field. It includes adjustable frequency signal source circuit, the former power amplifier circuit and the end-stage power amplifier circuit. The sensors array includes designing the structure of coils, object field layout and electromagnetic shielding of the object field. The detection signal processing circuit sends the detected voltage signal obtained from the detection coil to a computer for image reconstruction, which is constituted by the ceramic filter circuit, instrumentation amplifier circuit, the valid values converter circuit and A/D converter. The ceramic filter is responsible for filtering

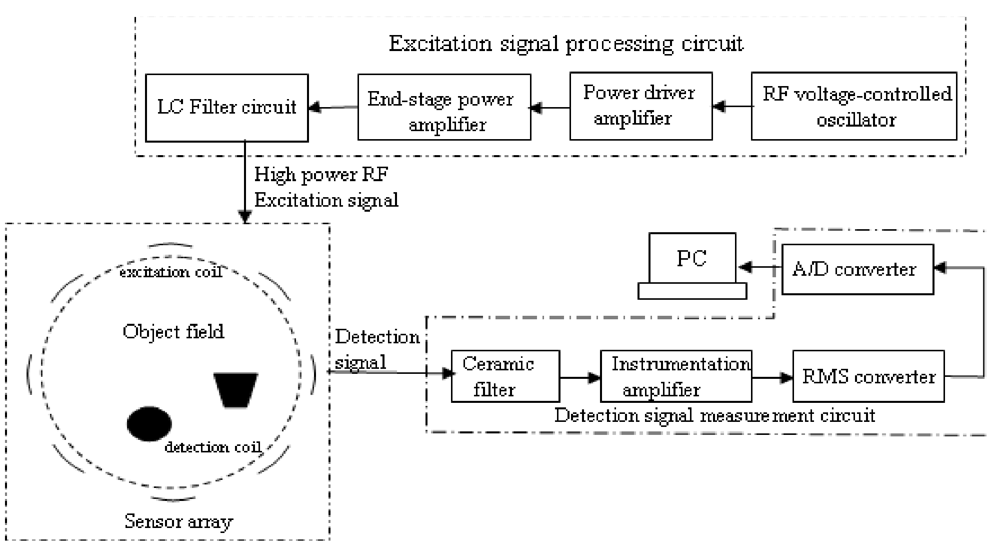

Fig. 1: Block diagram of 8 coils HF EMT system

Fig. 2: Eight coils EMT hardware system

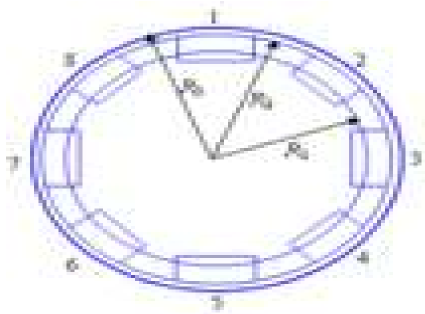

Fig. 3: Sensor array

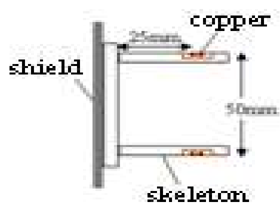

Fig. 4: Coil module

detection signal to obtain main frequency signal, which is the same with the excitation signal. Instrumentation amplifier is responsible for amplifying differential detection signal to get AC valid value, which can convert into a digital signal to send to the computer. The PC is used for processing the detection signal and image reconstruction.

Sensor array and structure parameters of the object field: For 8 coils monolayer HF EMT system, 8 coils act as the excitation coil or the detection coil alternately. Object field space is cylindrical and its wall, that is, the shield is made of 5 mm thick aluminum. Excitation signal is amplified to 10 MH 40 W sinusoidal AC signal by the preamp power driver amplifier and the last stage power amplifier and dual power supply, 27 V 8A switch power supply and 12 V 3A switch power supply, which is shown in Fig. 2 and 3 is a sectional view of the sensor array, where the space radius of the object field is R_1 = 140 mm, the inner diameter of the shield layer is R_2 = 165 mm and the shield outer diameter is R_3 = 170 mm. The 8 coils uniformly arrange on the same high cylindrical wall of the object field. A single coil sensor structure is shown in Fig. 4, which consists of 2 turns, ɸ0.5 mm solid copper and ɸ50 mm PVC insulation skeleton.

In the design of the hardware system, the impact of the external feedback on the stability of the circuit is the main problem. The parasitic feedback lied in the external amplifier circuit appears in the electromagnetic coupling. Due to HF excitation, interference source is inevitable, which is generally propagated by the capacitive coupling, electrical inductive coupling, public resistance coupling and radiation coupling, etc. During the design of these circuits, we take the way of the grounding and shielding to overcome it, for example, near multi-point connected to a common ground, lower ground impedance and coarse ground, to improve the stability of the current in the ground.

CONCLUSION

In order to test the sensitivity distribution and image quality of the EMT system, the measured object is placed in the lowest sensitivity point (Fig. 5a) and the highest sensitivity point (Fig. 5b), i.e., close to the coil ends. In addition, multi-object reconstructed images is shown in Fig. 5c and d, where the tested material is strong skin effect of copper medium, 15 mm diameter, permeability $\mu \approx 1$ and conductivity $\gamma = 1.7\times10^{-8}$ Ω.m. Landweber iterative algorithm, Tikhonov regularization algorithm, Conjugate Gradient algorithm (CG) and Modified Conjugate Gradient algorithm (MCG) are applied for image reconstruction in this system. Reconstruction images are shown in Fig. 5. To compare different algorithms, all algorithms are realized under the same standard, including the same sensitivity matrix generated by the simulation system, the same normalization algorithm (Li and Wang, 2012) and the same iteration times. Evaluate reconstructed images from image error and correlation in Table 1.

The following conclusions can be drawn based on the reconstructed images and image evaluation:

- As can be seen from Fig. 5, the reconstructed images can reflect the real position of the measured objects. Reconstruction of the model (b) is stronger than the others and single object reconstructed images are superior to multi-object, which are related to the distribution of the object field spatial sensitivity.
- As can be seen from Table 1, most of models have small image error and high correlation besides model (d). In particular, the smallest image error and the highest correlation are shown in the model (b). This indicates that this system has a higher quality of image reconstruction.
- Both from the visual comparison with the original image, or from the two evaluation, reconstructed images realized by Tikhonov regularization algorithm and modified conjugate gradient make a

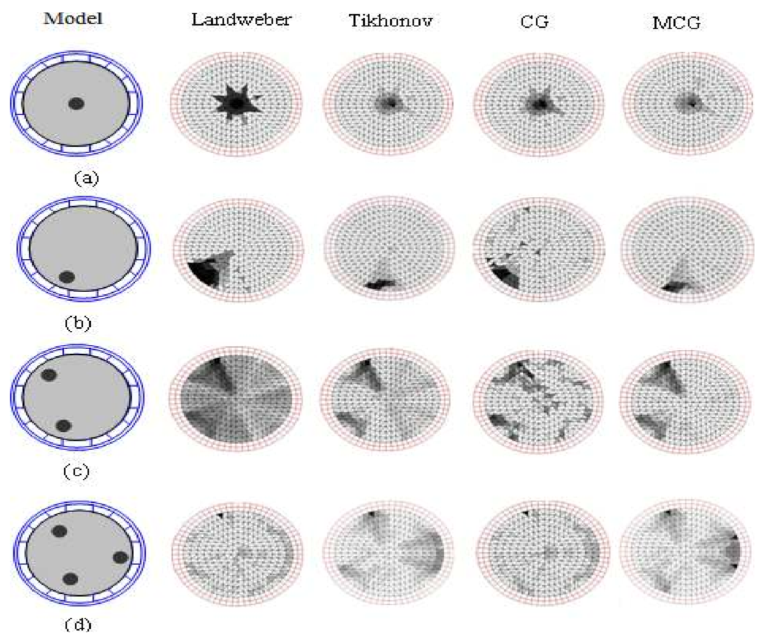

Fig. 5: Reconstruction images

Table 1: Image error and correlation in EMT system

Evaluation/model		Algorithm Landweber	Tikhonov	CG	MCG
Image error (%)	(a)	47.800	39.700	35.500	38.600
	(b)	110.800	35.900	34.300	30.300
	(c)	124.500	52.100	48.400	50.800
	(d)	122.800	54.500	51.300	42.300
Correlation	(a)	0.552	0.724	0.760	0.807
	(b)	0.738	0.752	0.782	0.801
	(c)	0.490	0.657	0.702	0.720
	(d)	0.500	0.453	0.525	0.550

good image results. Tikhonov regularization algorithm is a simple single-step algorithm, but the reconstructed image is too smooth. MCG algorithm is an iterative approach, for which the reconstructed image applied has the best image quality and better convergence.

- **The scope of the hardware system:**
  - Apply to the biomedical diagnostic. Since the internal organisms distribute low conductivity and non-conductor media, EMT system can visually reconstructs inter structure, which can provide the basis for the biological state.
  - By adjusting the frequency, the set can be applied to industrial process, especially to multiphase flow visual monitor process, which can provide protection for process control.
  - By changing the structure or parameters of the sensor, this system can be applied to the metal structure damage detection.

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