Research Journal of Applied Sciences, Engineering and Technology 6(4): 720-728, 2013
DOI:10.19026/rjaset.6.4187
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: October 12, 2012 Accepted: November 29, 2012 Published: June 20, 2013

Research Article

Analysis and Realization on Turbo Equalization based on 64-QAM in OFDM System

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Abstract: With the development of the mobile communication, the related technologies have gone through a rapid growing phase from 1G to 3G, until 4G. The technology evolution targets to improve the performance of overall system, and promote the related services and applications. In this paper, author makes analysis on Turbo equalization based on M-QAM modulation in OFDM system. Since Turbo equalization is an iterative receiver algorithm repeating equalization and channel decoding to improve the performance of the receiver, there researches on Turbo codes and turbo equalization have been emphasis, especially on the error-correction during these years. Due to the operations of equalization and decoding are repeated several times in one process, the computational complexity is increased in the receiver. Author aims at the cost function of CMA (Constant Modulus Algorithm) to change or recreate a new algorithm, and then as a contrast with the error functions, we can see the advantages about this change. Finally, author makes simulations on the Turbo equalization based on 64-QAM in OFDM system. Compared with the Turbo equalization based on 16-QAM, the performance of Turbo equalization based on 64-QAM is more efficient.

Keywords: CMA (Constant Modulus Algorithm), DFE (Decision Feedback Equalizer), MMSE (Minimum Mean Square Error), OFDM (Orthogonal Frequency Division Multiplexing), Turbo Equalization, 64-QAM

INTRODUCTION

As a high-speed digital transmission modulation technique, Orthogonal Frequency Division Multiplexing (OFDM) system has been widely used in wireless communication system, such as in a wireless LAN standard like IEEE 802.11a. In the OFDM system, the wideband channel is divided into N parallels of narrowband sub-channels that can be suppressed through between each time domain symbol prior to cyclic prefix (CP), the inter-symbol interference (ISI) and the inter-channel interference (ICI). Besides, in practical applications, not all of the sub-carrier frequency are used in OFDM systems, for example, in IEEE 802.11a only 52 of the 64 sub-carrier frequency are used, which further reduces the complexity of the proposed frequency domain methods. However, in order to increase the spectrum efficiency and shorten the CP or the unpredictable effect of channel, perhaps there some of the multipath components are delayed that is more than the CP length, which leads to the ISI and ICI affecting the whole system performance. Because of the channel changes, a lot of noise will be added to the signal in receiver. Therefore, it is important to apply equalization technology to the receiver as well as to make analysis on the equalization technology (Liu *et al*., 2002).

When the high-speed data is transmitted through band-limited channels in mobile communication system, the ISI caused by channel distortion and multipath propagation will bring decoding error. So it needs to adopt appropriate techniques to estimate or reduce such kind of interference. Equalization is the technique that is used to suppress ISI. With the research on error-correction has been emphasis. Turbo equalization is an iterative receiver algorithm repeating equalization and channel decoding to improve the performance. According to the previous research, the hard output decoding in Turbo equalization is proposed based on the below three kinds of algorithms: The first one uses the hard decision Viterbi decoding algorithm, the second one uses the soft decision Viterbi decoding algorithm and the third one uses the soft-in hard-out Viterbi decoding algorithm. The related simulations show that the complexity of the implementation is reduced significantly at negligible performance loss compared to the soft-in-soft-out decoding in Turbo equalization. And the complexity of these three kinds in Turbo equalization is similar, but the performance of the last one is the best. In general, turbo equalization algorithms are the combination between equalization and soft output channel decoding in an iterative process.

THE CONCEPTUAL FRAMEWORK

Equalization: The classifications of different kinds of equalization are shown in the Fig. 1.

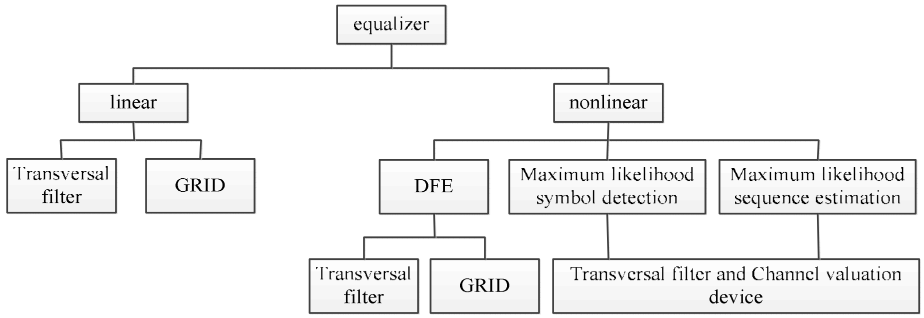


Fig. 1: Classification, structure and method of equalizer

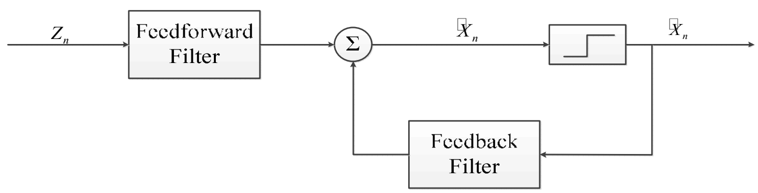


Fig. 2: Structure of DFE

In a communication system, the equalizer is necessary because it can enhance the transmission effect of the communication system. The equalizers can be classified into two types, the equalizers in the transmitter and the equalizers in the receiver. The main equalizer in communication system is decision feedback equalizer (DFE) (Bauch and Dhahir, 2010; Jianping *et al*., 2009), and the structure of DFE is shown in Fig. 2.

The output function of the equalizer obtained through the above transmission is shown in equation (1):

$$\hat{x}_n = \sum_{j=-k_1}^{0} c_j z_{n-j} + \sum_{j=1}^{k_1} c_j \tilde{x}_{n-j} \tag{1}$$

As a nonlinear equalizer, DFE is widely used in the communication system, and there have been also many other kind of equalizers. The proper equalizer needs to be chosen in accordance with the specific conditions.

Turbo: The concept of Turbo is firstly put forward by Berrou *et al*. (1993), in which the decoding results are close to the Shannon theoretical value. This kind of code has been widely used all the time in communication system since it was put forward (Michael and Ralf, 2011). The coding scheme for turbo is put forward by Berrou *et al*. (1993) and is named Recursive Systematic Convolutional (RSC). Turbo code can be formed by serial concatenated convolutional code or parallel concatenated convolutional code. For decoding process of turbo, many methods are invented, such as MAP, Viterbi, SOVA, Max-Log-MAP and others included.

So far, the performance of Turbo is better than other coding methods. However it still has disadvantages, the main disadvantage is that code can bring some delay in the transmission track. To solve the problem, some novel methods proposed are using of Turbo. Equalization is one of the methods for turbo processing in communication system. Therefore, people made the intensive research and developed many methods to solve this problem, among which the turbo equalization turns out to be the most useful and efficient one.

Turbo Equalization:

The theory on turbo equalization: Turbo equalization is a design project combining the decoding principle of turbo and equalization technology. It can effectively improve the performance of the turbo in communication system (Ouyang and Wang, 2002). The theory about turbo equalization can be shown in Fig. 3.

Binary code with length L_B come into coder, the data period before coder is T_B, and the data rate after coder is T_C, therefore, the system code rate will be as Eq. (2):

$$R = \frac{T_C}{T_B}, 0 \le R \le 1 \tag{2}$$

After coder, the binary data turns into the data c and after interleaving, the binary data turns into data x with length L_x.

For AWGN, the distribution of AWGN will be shown in Eq. (3):

$$f_w(x) = N\left(0, \sigma_w^2\right) \tag{3}$$

In above equation:

$$N\left(\mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \tag{4}$$

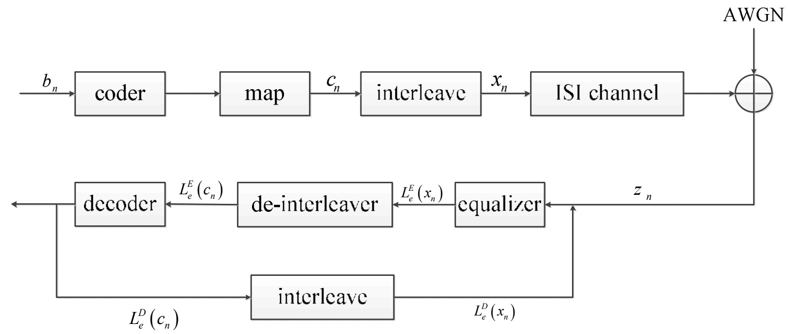


Fig. 3: Theory of turbo equalization

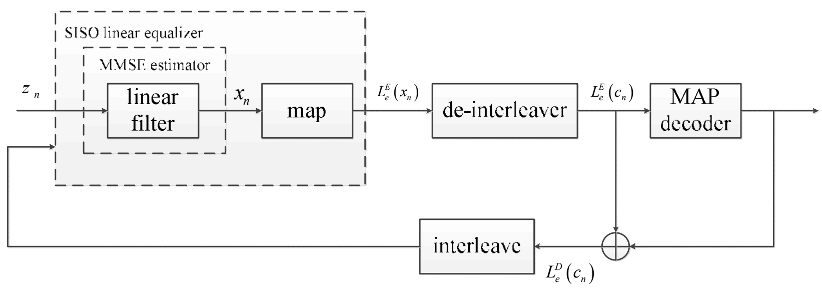


Fig. 4: Principle of MMSE linear turbo equalization

If the received sequence of signs is $z=\left[z_0, z_1, \cdots, z_{L-1}\right]^T$.

Then,

$$z_n = \sum_{k=-M_1}^{M_1} h_k x_{n-k} + w_k \tag{5}$$

Thus, the turbo equalization is shown:

$$L_e^E\left(x_n\right) = L_e^D\left(x_n\right) = \prod\left(L_e^D\left(c_n\right)\right) \tag{6}$$

$$L_e^E\left(x_n\right) = L_e^D\left(x_n\right) = \prod^{-1}\left(L_e^E\left(c_n\right)\right) \tag{7}$$

The former equation represent the process of interleave, while the latter one represents the process of de-interleave.

The algorithm on turbo equalization: There are many algorithms in extant researches, which are used in different communication system to meet the distinct users' needs. The frequently-used algorithms include SIC equalization, MAP equalization, MMSE equalization and others. In following, the MMSE algorithm will be illustrated in detail, since it is better performed in practice than other methods.

The principle of MMSE in turbo equalization is shown in Fig. 4.

The impulse response of filter goes as shown in Eq. (8):

$$c[n] = \sum_{k=-N_1}^{N_1} c_{k,n}\delta[n-k] \tag{8}$$

And the estimate value of x_n is expressed as the following:

$$\hat{x}_n = \sum_{k=-N_1}^{N_2} c_{k,n} z_{n-k} + d_n \tag{9}$$

Choose the right value of d_n, we can get the minimum expected value of x_n.

For SISO equalization, the formula below can be obtained:

$$L_e^E\left(x_n\right) = \frac{2\hat{x}_n}{1-s^H c_n} \tag{10}$$

And the variance can be replaced by the equation in the following:

$$\hat{\sigma}^2 = \frac{1}{L_c}\left(\sum_{\substack{n=1 \\ x_n \geq 0}}^{L_c}\left|\mu_n^{(+1)} - \hat{x}_n\right|^2 + \sum_{\substack{n=1 \\ x_n < 0}}^{L_c}\left|\mu_n^{(-1)} - \hat{x}_n\right|^2\right) \tag{11}$$

Therefore, the output of SISO equation can be change into the following equation:

$$L_e^E\left(x_n\right) = \frac{2\hat{x}_n \mu_n^{(+1)}}{\hat{\sigma}^2} \tag{12}$$

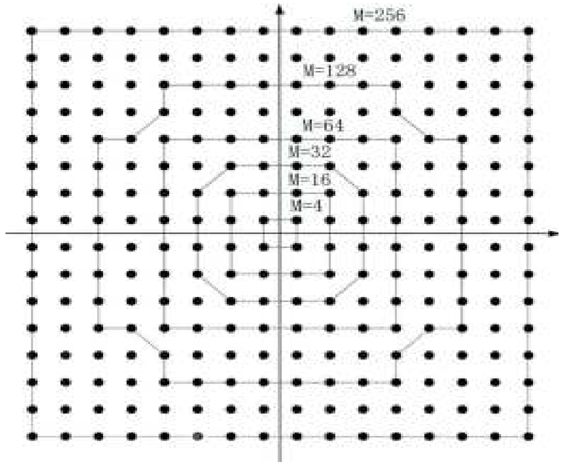


Fig. 5: Vector constellation of MQAM signal

Compared with many other algorithms, the MMSE algorithms will be a good choice due to its high efficiency, where the receiver is used as a blind receiving method. Afterwards, its related algorithm will be introduced.

QAM technology: The standard of DVB-C is established in 1994, which has been widely used in the communication system. The core technology of DVB-C is QAM, including 16-QAM, 32-QAM, 64-QAM, 128-QAM, and 256-QAM.

QAM system and QAM modulation technology: Different modulations of QAM are defined by different distance among different signals. The distance decreases when the value of M increases, at the same time when the error rate goes up. QAM modulation can overcome these problems, because it combines amplitude modulation and phase modulation (Benani and Gagnon, 2011). The QAM modulation can be expressed in Eq. (13).

$$e_0(t)=\left[\sum_n a_n g\left(t-nT_S\right)\right]\cos\left(2\pi f_c t+\psi_n\right) \tag{13}$$

In above equation:
g (t) represents rectangular pulse with height of 1
T_s represents the width of rectangular pulse
a_n represents the carrier with different electrical level
ψ_n represents the different phase position of carriers

The above equation can be changed into the following equation:

$$e_0(t)=\left[\sum_n \left(a_n\cos\psi_n\right)g\left(t-nTs\right)\right]\cos 2\pi f_c t+\left[\sum_n\left(-a_n\sin\psi_n\right)g\left(t-nTs\right)\right]\sin 2\pi f_c t \tag{14}$$

The performance of the QAM system can be explained more clearly by the vector constellation. In M-ary modulation system, the vector constellation is the good way to show the performance of the modulation system. The vector constellation of QAM is shown in Fig. 5, in which the performance of the modulation signal will be shown.

As stated in previous research, M-QAM system performs better than M-PSK modulation system. The distance between the modulation signal points is shown in following equations respectively:

$$d_{MPSK}=2A\sin\left(\frac{\pi}{M}\right) \tag{15}$$

$$d_{MQAM}=\frac{\sqrt{2}A}{L-1}=\frac{\sqrt{2}A}{\sqrt{M}-1} \tag{16}$$

The distance can be shown in the vector constellation of MQAM and MPSK. When M = 4, the distance of 4 QAM is equal to 4PSK, and the two systems have the function to resist noise. But when M>4, to resist noise, the performance of MQAM will be better (Xu and Du, 2007).

How to produce the M-QAM signal, since it has better performance, is naturally thought to be a key problem. The structure of M-QAM modulation system is shown in Fig. 6.

QAM Signal and 64-QAM: The features of QAM signal are to separate the signal into two carriers, of which the angle is 90°. And the two carriers are cos 2π f_ct and sin 2π f_ct, respectively. Therefore, the QAM signal goes like Eq. (17).

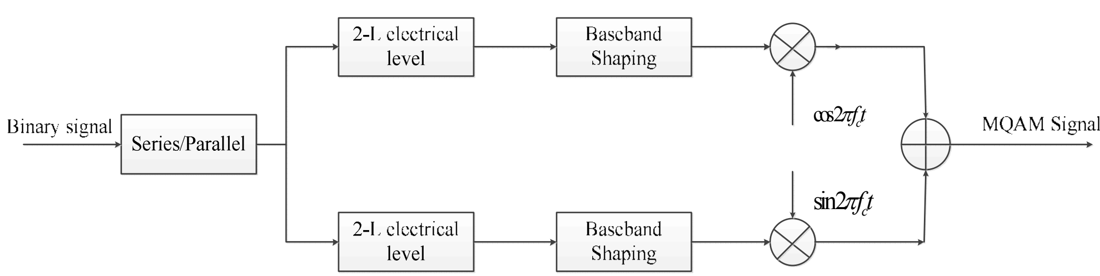


Fig. 6: Structure of M-QAM modulation system

$$u_m(t)=A_{mc}g_T(t)\cos 2\pi f_c t+A_{ms}g_T(t)\sin 2\pi f_c t,\quad m=1,2,\cdots,M \tag{17}$$

where,
$\{A_{mc}\}$ and $\{A_{ms}\}$ = Represent the set of amplitude
$g_T(t)$ = Represent the impulse of the signal

If $A_m=\sqrt{A_{mc}^2+A_{ms}^2}$ then the angle of the two carriers will be like in Eq. (18):

$$\theta_n=\arctan\left(\frac{A_{ms}}{A_{mc}}\right) \tag{18}$$

Therefore, the QAM will be like in Eq. (19):

$$\mu_{mn}(t)=A_m g_T(t)\cos(2\pi f_c t+\theta_n),\quad m=1,2,\cdots,M_1, n=1,2,\cdots,M_2 \tag{19}$$

From above equation, we can get the result. The QAM modulation can be seen as the combination of the PAM modulation and PSK modulation. If $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$, the transmission rate of the code will be $R_b/(k_1 + k_2)$ and $k_1 + k_2 = \log_2 M_1 M_2$. Then the signal can be also shown as the two dimension vectors forms like Eq. (20):

$$s_m=\left(\sqrt{\varepsilon_c}A_{mc},\sqrt{\varepsilon_s}A_{ms}\right), m=1,2,\cdots,M \tag{20}$$

In fact, 64-QAM signal is special compared with other signals, the distance among the signal points of 64-QAM signal is smaller but it is more easily to be affected by noises.

Adaptive equalization technology in QAM system: CMA is a kind of Godard algorithms with a parameter of (p = 2). The cost function is shown in Eq. (21):

$$J(n)=\frac{1}{4}E\left\{\left(|y(n)|^2-R_2\right)^2\right\} \tag{21}$$

The meaning of the symbol R_2 is shown in Eq. (22):

$$R_2=\frac{E\left\{|x(n)|^4\right\}}{E\left\{|x(n)|^2\right\}} \tag{22}$$

One nonlinear function is used in CMA algorithm, which is shown in Eq. (23):

$$g(y(n))=\frac{y(n)}{|y(n)|}\left[\left|y(n)+R_2|y(n)|-|y(n)|^3\right|\right] \tag{23}$$

We can get the iteration equation in Eq. (24):

$$w(n+1)=w(n)+\mu y(n)\left[R_2-|y(n)|^2\right]\dot{r}(n) \tag{24}$$

Although CMA algorithm is widely used in the communication system, but it also has some disadvantages, such as the slow convergence, lots of errors of convergence and others (Mendel, 2010). The effect factors include cost function, initialization of equalizer, and others. Therefore, it is imperative to find one new method instead of the original CMA algorithm.
The cost function can be shown in Eq. (25).

$$J_n(w)=E\left\{\frac{1}{4}\left(|y_k|^2-R_2\right)^2+\beta\left(g(y_{k\gamma})+g(y_{ki})\right)\right\} \tag{25}$$

Based on the Eq. (25), we can get the decomposed functions:

$$J_{nr}(k)=E\left[\frac{1}{4}\left(y_{kr}^2-R_{2,r}\right)^2+\beta g(y_{kr})\right] \tag{26}$$

$$J_{ni}(k)=E\left[\frac{1}{4}\left(y_{ki}^2-R_{2,i}\right)^2+\beta g(y_{ki})\right] \tag{27}$$

where,
$R_{2,r}=\dfrac{E\left[x_{kr}^4\right]}{E\left[x_{kr}^2\right]}$ and $R_{2,i}=\dfrac{E\left[x_{ki}^4\right]}{E\left[x_{ki}^2\right]}$

From above equations, we can also get the iteration equation in the following:

$$\nabla J_n(w)=E\left\{\left(y_k\left(|y_k|^2-R_2\right)+\beta\eta_k\right)^* r_k\right\} \tag{28}$$

$$w_{k+1}=w_k-\mu_n\dot{\varphi}_k r_k \tag{29}$$

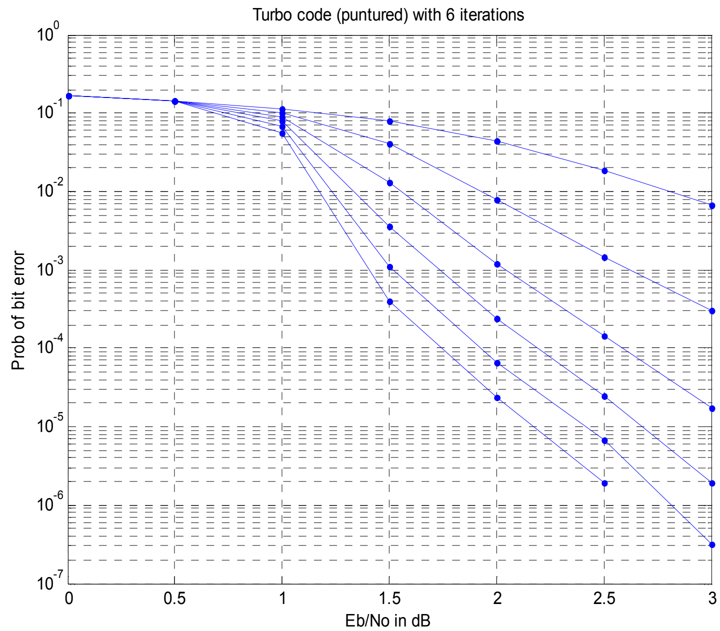
$$\varphi_k=y_k\left(|y_k|^2-R_2\right)+\beta\eta_k \tag{30}$$

And the advantages of this algorithm will be shown according to error function.

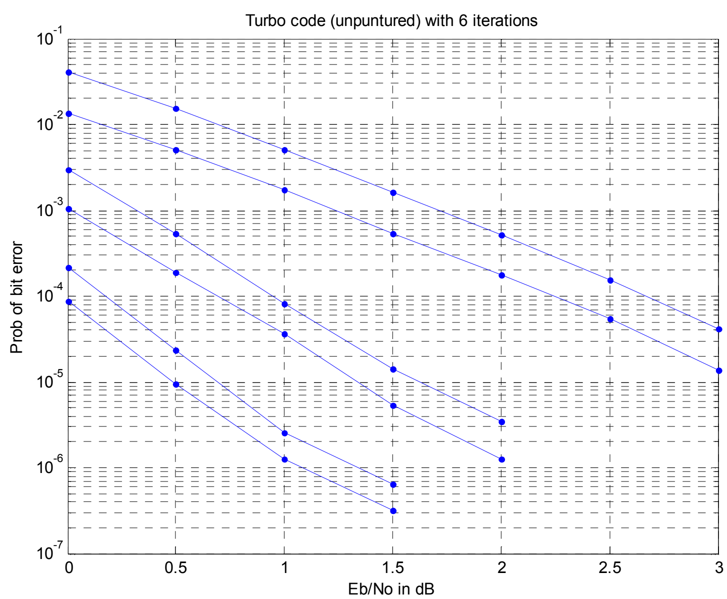
$$g_e(x)=1-\sin^{2n}\left(\frac{x}{2d}\pi\right) \tag{31}$$

SIMULATION ON TURBO EQUALIZATION BASED ON 64-QAM IN OFDM SYSTEM

The advantages of Turbo equalization based on 64-QAM in OFDM system can be shown in this section. The advantages of this new algorithm are shown in the simulation results. And the simulations include the simulation on turbo code and turbo equalization based on 16/64-QAM in OFDM system (Yumin, 2010).



(a)



(b)

Fig. 7: Turbo code (punctured/non-punctured) with 6 iterations

Simulation on turbo equalization: As one kind of turbo equalization algorithm, MMSE algorithm is invented based on the DFE algorithm. The structure of circuit to realize this algorithm is shown in above. In this section, the some simulations based on Simulink will be taken, and the simulation is to show the advantage of the new algorithm comparing with the original algorithm.

The performances of the different turbo code are shown in the different figures as follows (Fig. 7).

Compared with the right picture, the left one in Fig. 7 obviously has more advantages. And we also get the importance of the iterations in the turbo code. And we get a result, as algorithm to realize the turbo equalization, the MMSE is a good choice, but with the development of the communication technology, more and more better algorithm will be invented as well.

Simulation on Turbo Equalization based on 64-QAM: As mentioned above, the technology of turbo equalization is described as one key technology in the OFDM system. But when turbo equalization works on the different QAM modulations, the performance will be different. In this section, computer simulations are used to verify the effectiveness of two methods, and compare, and then we will get the simulation results

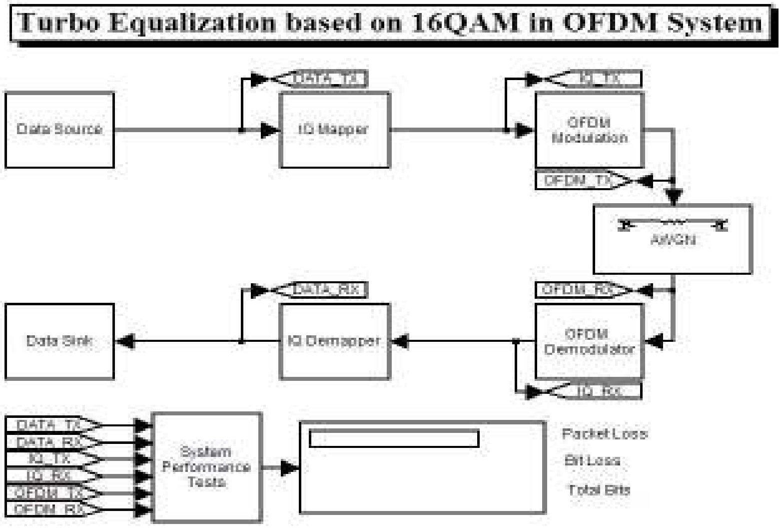
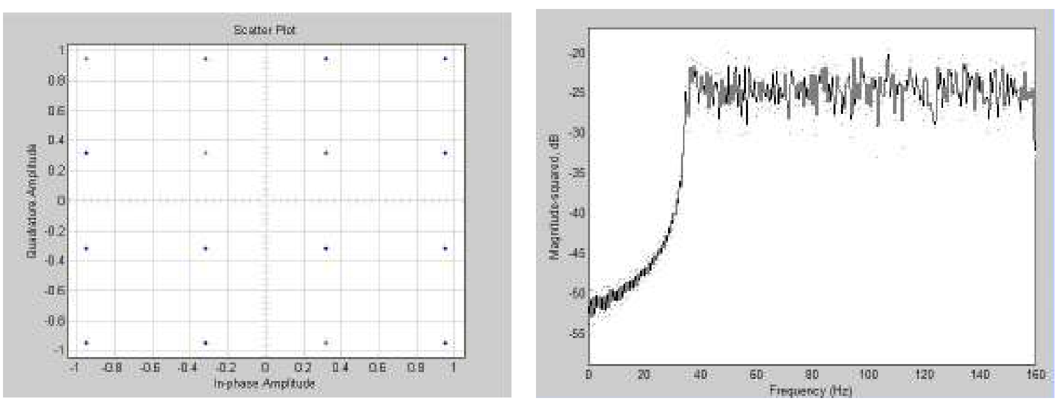


Fig. 8: Structure of turbo equalization based on 16QAM in OFDM system



(a) (b)

Fig. 9: Features of turbo equalization based on 16QAM in OFDM system

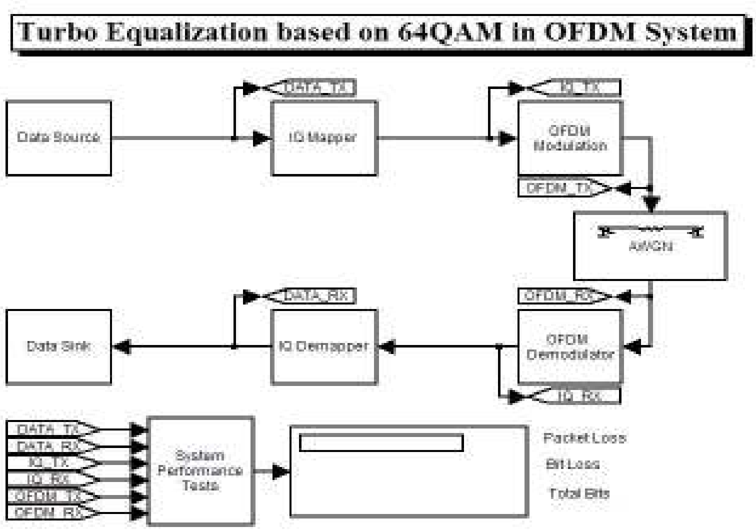
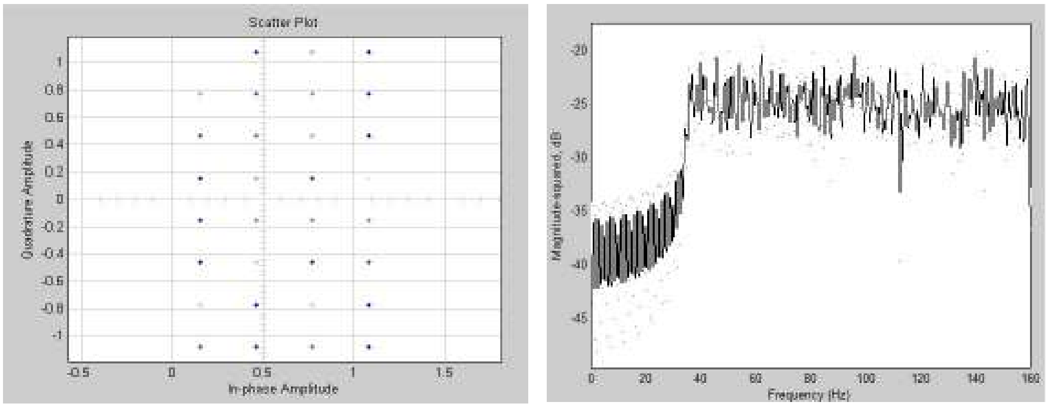


Fig. 10: Structure of turbo equalization based on 64QAM in OFDM system



(a) (b)

Fig. 11: Features of turbo equalization based on 64QAM in OFDM system

over comparing the turbo equalization based on 16-QAM and 64-QAM in OFDM system. The software named Simulink will be used in this simulation. In this simulation, the Simulink will be used to build the system circuit, and the MMSE algorithm will be added to this circuit.

The structures of 16-QAM and 64-QAM are shown in Fig. 8 and 10. In the two structures of OFDM system, the same algorithm is applied. This simulation is to show the different results of turbo equalization based on 16-QAM and 64-QAM.

The features of signal in receiver are shown in Fig. 9, from which we can see the distance of the points and the result of noise. The Fig. 10 is the real signal in the receiver.

The number of sub-carrier frequency used in the simulation is 64, and the modulation scheme is 64-QAM, the CP length is 8, and is used in the calculation of the lower bound of the sufficient long CP. The tap between independents and the energy changed to meet the exponential decay of the Rayleigh distribution. In the Fig. 10, compared with the above simulation method in 16-QAM, we start the simulation on turbo equalization based on 64-QAM in OFDM system.

The features of signal in receiver based on 64-QAM is shown in Fig. 11. The processing of two pictures in Fig. 10 goes nearly the same as in Fig. 8, but the results in the related Fig. 9 and 11 are different. Compared with the above picture of Fig. 9, the distance of points in the above picture of Fig. 11 is obviously smaller. That is to say, the signal will be easily influenced by the noise. From these figures, we can also get the same result, while it is more difficult to realize the OFDM system based on the 64-QAM.

From the simulation results about turbo equalization based on 16-QAM and 64-QAM in OFDM system, we find the disadvantages of 64-QAM. And we can also get the features of 64-QAM in OFDM system. We get the results that the distance between the points in scatter plot is smaller in 64-QAM than that in 16-QAM, and the 64-QAM will be easier to be affected by the noise. But from the figures listed above, we can see that the feature of signal based on 64-QAM can be accepted in spite of it has some disadvantages.

CONCLUSION

Due to the operations of equalization and decoding are repeated several time in one process, the computational complexity is increased in the receiver. In this study, one kind of reduced complexity turbo equalization is proposed, which used hard output channel decoding to instead soft output way. Turbo equalization based on 64-QAM will be widely used in communication system in the last decade. In the technology developing period, a variety of algorithms have been invented by many scholars. It is necessary to make analysis and realization on the turbo equalization based on 64-QAM in OFDM system since the turbo equalization technology has more advantages than other processing methods.

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