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Research Article

Dynamic Model of Urban Sports Infrastructure Supply and Demand Based on GDP Growth

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Abstract: For different city, the change rate of the supply and demand of the sports infrastructure is not the same and is subject to regional GDP growth constraints. Taking GDP growth as control variables, a dynamic model of multi-city sports infrastructure supply and demand system was established. According to Lyapunov stability theory, the system asymptotically stable condition was obtained. Using the linear matrix inequality method, the paper gets a control method that cities with different development level can be unified use for making their sports infrastructure system asymptotically stable and supply-demand equilibrium. The method can reduce the cities' sports infrastructure construction control complexity.

Keywords: Construction control, dynamic model, GDP growth, sports infrastructure, supply and demand

INTRODUCTION

At present, China's gross national product has risen to the second place in the world. In order to make economic developmental achievement share with the citizens, the government has stressed the significance and value of improving the social and public services, including culture and sports.

For sports public service, sports infrastructure is the material foundation and basic guarantee.

How to realize the balance of supply and demand of sports public service infrastructure construction for different city?

How to coordinate sports infrastructure construction progress with local GDP growth?

Can we build a universal asymptotic stability control scheme simultaneously coordinating multi city's GDP growth and sports infrastructure construction progress?

These problems are worthy of analyzing for city managers.

For the issues of sports public service infrastructure and regional economic, current research mainly focused on evaluation and management.

For example, through building the VEC model of sports public service, Yao and Ding (2012) adopts the primary components extracting method to simplify the empirical data in the field of sports public service. Then analyzed relevant data both qualitatively and quantitatively to study the effect of the input in the field of sports public service. The research hopes to promote the role of public sports organizations, distribute public sports investment more rationally, improve the level of sports public service effectively and to make the sports public service meet people's increasing requirements on it.

Qi *et al*. (2012) dealt with how to reasonably arrange the start time of these activities while meeting various constraint conditions to obtain the entire project scheduling and to minimize the total duration of the projects' occupancy for social sports resources. Combined with the simulated annealing algorithm, scheduling algorithm for the social sports project is designed to study the problem of social sports scheduling.

On the basis of Porter's Diamond Model, Li (2012) established the evaluation model of Chinese sports industry competitiveness. The competitiveness model of Chinese sports industry combines the characteristics of sports industry and the situation in China. Empirical result is consistent with the actual situation of Chinese sports industry in China. Mainly because some evaluation indicators are not available or cannot be measured, the deficiency of the research is that it is difficult to select the evaluation indicators of operation level in accordance with the ideal situation.

Fang (2012) investigated the Multiple Attribute Decision Making (MADM) problems for evaluating the sports management system with interval-valued intuitionistic fuzzy information.

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Wu (2012) proposed a new comprehensive evaluation model with combinational weight based on ant colony optimization. The use of evaluation model to evaluate the culture of people sports can analyze the comprehensive construction of sports culture, and the development of all factors of the sports culture, which is helpful to grasp the overall and partial situation and promote the development of people sports culture.

Through a series of research, Zheng and Wang (2008) and Zheng *et al*. (2008) proposed "the comprehensive benefits-net investment" national economic evaluation method and constructed the national economic comprehensive benefit index system in the framework of "the comprehensive benefits - net investment", provides a basis for large public sports facilities scientific and reasonable investment.

According to public sports construction situation, problems and influence factors in Shandong, China, Wang (2010) put forward the project management mode of government investment in public sports construction.

Wang *et al*. (2009) made function fitting with public sports investment and the GDP annual value in Yunnan Province, China, got the relationship between public sports investment and economic development and predicted the problems about public sports investment in the next few years.

Existing research has made some achievements for evaluation and management in sports public service infrastructure and regional economic relations.

While for a specific region, the control balance dynamic quantitative relationship between GDP growth and sports infrastructure construction, less involved in the previous study.

This study makes research on the multi city simultaneous stabilization, supply and demand equilibrium control method for GDP growth and sports infrastructure construction by using economic control theory, in order to reduce the sports infrastructure construction control complexity.

MODEL

For the cities with different development level of economy and society, the supply and demand change rates of the sports infrastructure are not the same.

Suppose there are m cities with different development level, the state equation of sports infrastructure demand function D (t) of city i is:

$$\dot{D}(t) = \alpha_i D(t) + \beta_i R(t), i = 1, \cdots, m \tag{1}$$

where, t is the time unit, R(t) is regional GDP growth rate, α_i , β_i are the system parameters.

The change rate of the urban residents' sports infrastructure demand $\dot{D}$(t) is affected by two factors.

One is the current level of demand, this is the state variable.

The other is the regional GDP growth rate R(t), this is the control variable.

Because of different values of α_i, β_i, the demand of urban sports infrastructure can show in the form of growth curve model (Jian and Hu, 2009).

The impact of sports infrastructure demand degree D (t) on $\dot{D}$(t) is: in the high-speed economic growth city, the residents' sports infrastructure demand will be rapid growth, α_i is a desirable larger value. In the stable developed city, most of residents' demand already has been met, α_i is a smaller value. In the less-developed city, residents' sports infrastructure demand growth is relatively weak, α_i is a desirable smaller value.

The impact of sports infrastructure demand degree R (t) on $\dot{D}$(t) is: Due to the economic and social development level is different, the same GDP growth, for the cities with different development level, sports infrastructure demand rate will be differently affected.

The developed cities are more willing to spending economic growth for sports infrastructure construction and promoting the demand growth, so β_i is larger.

In contrast, underdeveloped cities are more willing to sparing economic growth in expenditures for other economic construction, so β_i is smaller.

By means of different values of β_i, regional GDP growth rate R (t) can regulate the change rate of the sports infrastructure demand.

According to production theory of the economics (Takayama, 2001), fixed capital is the important factors of production, the sports infrastructure supply function S(t) of city i can be expressed as:

$$S(t) = \theta_i K(t), i = 1, \cdots, m \tag{2}$$

where, K (t) is fixed capital stock of sports infrastructure supply agency, θ_i is the output capital ratio.

To different development level city, the output capital ratio is not the same.

Relatively speaking, manufacturing level and capital utilization rate in the economic developed city are higher than that in the underdeveloped city.

By the investment theory of the economics (Takayama, 2001), the relation among sports infrastructure fixed capital stock changes, fixed capital investment and fixed capital depreciation can be used dynamic equation to express as:

$$\dot{K}(t) = I(t) - \delta_i K(t), i = 1, \cdots, m \tag{3}$$

where, $I(t)$ is fixed capital investment of sports infrastructure supply, δ_i is fixed capital depreciation rate.

Sports infrastructure investment is also constrained by the GDP growth in the region. Supposing investment is a certain percentage of the regional GDP growth, we have:

$$I(t) = \gamma_i R(t), i = 1, \cdots, m \tag{4}$$

where, γ_i is the proportion coefficient of investment to the regional GDP growth.

The economic and social different situation will have different impact on investment ratio coefficient.

The developed cities can spend more proportion of economic growth on sports infrastructure investment, so γ_i value is larger.

In contrast, underdeveloped cities are willing to spend more proportion of economic growth on other economic construction expenditures, so γ_i is smaller.

Next we calculate the state equation of sports infrastructure supply function.

First differentiate both sides of the formula (2) to time t, then substitute formula (3) and formula (4), we get:

$$\dot{S}(t)=\theta_i\dot{K}(t)=\theta_i\left(I(t)-\delta_iK(t)\right)=\theta_i\gamma_iR(t)-\delta_iS(t), i=1,\cdots,m \quad (5)$$

Formula (5) is the time varying state equation of the sports infrastructure supply function S(t) of city i.

Joining the formula (1) and formula (5) in a matrix form, we can get sports infrastructure dynamic supply and demand model of city i as follows:

$$\begin{pmatrix}\dot{D}(t)\\ \dot{S}(t)\end{pmatrix}=\begin{pmatrix}\alpha_i & 0\\ 0 & -\delta_i\end{pmatrix}\begin{pmatrix}D(t)\\ S(t)\end{pmatrix}+\begin{pmatrix}\beta_i\\ \theta_i\gamma_i\end{pmatrix}R(t), i=1,\cdots,m \quad (6)$$

Construction control goal is that, for the m cities with different economic and social development degree, to determine a unified control scheme for the control variable R (t).

Through the implementation of this scheme, can make the sports infrastructure dynamic supply and demand systems of the m cities with different development degree to be asymptotically stable and finally to achieve the balance of supply and demand.

The advantage of this implementation is that, an unified construction control scheme can be used for a number of cities with different development level and can reduce the complexity of the construction control.

CONSTRUCTION CONTROL SCHEME

Formula (6) sports infrastructure dynamic supply and demand model can be expressed as:

$$\dot{x}(t)=A_ix(t)+B_iu(t), i=1,\cdots,m \quad (7)$$

where,

$$x(t)=\left(D(t),S(t)\right)^T$$

is 2-dimensional state vector:

$$u(t)=R(t)$$

is a 1-dimensional control input:

$$A_i=\begin{pmatrix}\alpha_i & 0\\ 0 & -\delta_i\end{pmatrix},$$
$$B_i=\begin{pmatrix}\beta_i\\ \theta_i\gamma_i\end{pmatrix}, i=1,\cdots,m$$

are respectively 2×2 and 2×1 constant coefficient matrix.

Control scheme is designed to make use of the state feedback control, that is to let:

$$u(t)=Gx(t) \quad (8)$$

where G is 1×2 dimensional state feedback gain matrix. Substituting formula (8) into formula (7), we have:

$$\dot{x}(t)=\left(A_i+B_iG\right)x(t), i=1,\cdots,m \quad (9)$$

By the Lyapunov stability theory (Yu, 2011), the necessary and sufficient condition of the closed-loop system formula (9) asymptotic stability is to exist a symmetric positive definite matrix P, making:

$$\left(A_i+B_iG\right)^TP+P\left(A_i+B_iG\right)<0, i=1,\cdots,m \quad (10)$$

Therefore, the stabilizing controller design problem is reduce to determine the matrix G and the symmetric positive definite matrix P and to make the matrix inequality (10) hold.

For the matrix variables G and P, formula (10) is not linear matrix inequality.

In order to solved by linear matrix inequality method, we need try to transform it into linear matrix inequality. So we expand formula (10) as:

$$PA_i+A_i^TP+G^TB_i^TP+PB_iG<0, i=1,\cdots,m \quad (11)$$

Left multiplied and right multiplied P^{-1} on both sides of formula (11), we have:

$$A_iP^{-1}+P^{-1}A_i^T+\left(P^{-1}G^T\right)B_i^T+B_i\left(GP^{-1}\right)<0, i=1,\cdots,m \quad (12)$$

As P is a symmetric positive definite matrix, there is:

$$\left(P^{-1}\right)^T=P^{-1}$$

Let:

$$X=P^{-1},$$
$$Y=KP^{-1}$$

formula (12) can be transformed into:

$$A_iX+XA_i^T+Y^TB_i^T+B_iY<0, i=1,\cdots,m \quad (13)$$

For matrix variable X and Y, formula (13) is a linear matrix inequality group containing m inequalities.

Due to the positive definiteness of the matrix P and that of the matrix X is equivalent, if the linear matrix inequality group:

$$\begin{cases} A_i X + X A_i^T + Y^T B_i^T + B_i Y < 0, i = 1, \cdots, m \\ X > 0 \end{cases} \tag{14}$$

existing feasible solution, then formula (8) stabilization controller of formula (7) system also has feasible solution.

If X and Y are feasible solutions of formula (14) linear matrix inequality group, then $G = YX^{-1}$ is a stabilizing state feedback gain matrix of formula (7) system and $P = X^{-1}$ is a Lyapunov matrix for formula (9) corresponding closed-loop system.

Now the formula (8) control scheme can make the cities with different level of economic and social development reach sports infrastructure dynamic system asymptotically stable and supply and demand balance.

Example: Three cities A, B and C are, respectively the developed city with steady economic growth, the medium-developed city with rapid economic growth and the less developed city with slower economic growth.

To meet the public sports service needs of the city residents, the three cities all plan the construction of sports infrastructure.

The cities' sports infrastructure dynamic supply and demand system model parameters are as follows:
City A is:

$$\alpha = 0.05, \beta = 0.09, \theta = 0.8, \delta = 0.05, \gamma = 0.007$$

City B is:

$$\alpha = 0.08, \beta = 0.05, \theta = 0.07, \delta = 0.04, \gamma = 0.005$$

City C is:

$$\alpha = 0.02, \beta = 0.01, \theta = 0.06, \delta = 0.03, \gamma = 0.003$$

To reduce the complexity of the construction control, the system target is to determine a unified control scheme for GDP growth in various cities and can make the three cities with different level of economic and social development reach sports infrastructure dynamic system asymptotically stable and supply and demand balance.
Here we have:

$$m = 3,$$

$$A_1 = \begin{pmatrix} 0.05 & 0 \\ 0 & -0.05 \end{pmatrix}, B_1 = \begin{pmatrix} 0.09 \\ 0.0056 \end{pmatrix}, A_2 = \begin{pmatrix} 0.08 & 0 \\ 0 & -0.04 \end{pmatrix}, B_2 = \begin{pmatrix} 0.05 \\ 0.0035 \end{pmatrix},$$
$$A_3 = \begin{pmatrix} 0.02 & 0 \\ 0 & -0.03 \end{pmatrix}, B_3 = \begin{pmatrix} 0.01 \\ 0.0018 \end{pmatrix}$$

Substituting the system parameter into formula (14), using linear matrix inequality toolbox of MATLAB software (Xue and Chen, 2007), we can get:

$$X = \begin{pmatrix} 1.1093 & 0.0020 \\ 0.0020 & 1.4421 \end{pmatrix},$$
$$Y = \begin{pmatrix} -9.1862 & -0.4792 \end{pmatrix}$$

X, Y are feasible solution of linear matrix inequality group formula (14).
Substituting X, Y into:

$$P = X^{-1}$$

and:

$$G = YX^{-1}$$

we get:

$$P = \begin{pmatrix} 0.9015 & -0.0013 \\ -0.0013 & 0.6934 \end{pmatrix},$$
$$G = \begin{pmatrix} -8.2805 & -0.3206 \end{pmatrix}$$

The matrix P is the symmetric positive definite Lyapunov matrix and the matrix G is the stabilizing state feedback gain matrix.
Substituting G into formula (8), we get:

$$R(t) = \begin{pmatrix} -8.2805 & -0.3206 \end{pmatrix} \begin{pmatrix} D(t) \\ S(t) \end{pmatrix}$$

The above solved R (t) is the construction control design scheme.

The coefficient of stabilizing state feedback gain matrix G is negative, indicating that the control design scheme is the negative feedback control.

When the value of state variables D (t) or S (t) is less than its stable value, its value should be regulated to increase.

While when the value of state variables D (t) or S (t) is higher than its stable value, its value should be regulated to decrease.

By means of an unified negative feedback control design scheme, can make the A, B and C three cities with different level of economic and social development reach sports infrastructure dynamic system asymptotically stable and supply and demand balance.

The control design scheme can achieve system goals and reduce construction control complexity.

CONCLUSION

Different cities with different development level and different parameters of sports infrastructure dynamic supply and demand system, results in that the change rates of city sports infrastructure supply and demand function are not the same.

But for system control is concerned, the cities for the system asymptotically stable and equilibrium of supply and demand have the same requirements.

Regional GDP growth rate has important influence on both the supply and demand of city sports infrastructure.

Using Regional GDP growth rate as the system control variables, application of Lyapunov stability theory, can establish an unified control design scheme.

Using linear matrix inequality method, can achieve the target that sports infrastructure dynamic supply and demand system of the cities with different level of economic and social development are asymptotically stable and balanced supply and demand.

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