

Research Article

Stagnation-point Flow and Heat Transfer of a Nanofluid Adjacent to Linearly Stretching/Shrinking Sheet: A Numerical Study

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Abstract: In this study, the steady stagnation point flow and heat transfer of three different types of nanofluid over a linearly shrinking/stretching sheet is investigated numerically. A similarity transformation is used to reduce the governing system of partial differential equations to a set of nonlinear ordinary differential equations which are then solved numerically using the fourth-order Runge-Kutta method with shooting technique. The effects of the governing parameters on the nanofluid flow and heat transfer characteristics are analyzed and discussed. Numerical results for the local Nusselt number, skin friction coefficient, velocity profiles and temperature profiles are presented for different values of the solid volume fraction (ϕ) and for three different types of nanoparticles (Cu, Al₂O₃ and TiO₂) in stretching or shrinking cases. It is found that the skin friction coefficient and the heat transfer rate at the surface are highest for Cu-water nanofluid compared to the Al₂O₃-water and TiO₂-water nanofluids. Furthermore, it was seen that the effect of the solid volume fraction of nanoparticles on the heat transfer and fluid flow characteristics is more important compared to the type of the nanoparticles.

Keywords: Forced convection, nanofluid, numerical solution, similarity transform, stagnation point flow, stretching/shrinking sheet

INTRODUCTION

The viscous flow and heat transfer in the boundary layer region due to a stretching sheet has several theoretical and technical applications in industries such as the aerodynamics, food processing, extrusion and glass fiber production. Crane (1970) was the first to consider the steady boundary layer flow of a viscous fluid which is incompressible due to a linearly stretching plate. On the other hand, Hiemenz (1911) investigated the two-dimensional stagnation point flow over a plate. The local velocity of the fluid at the stagnation-point is zero. Stagnation flow, describing the fluid motion near the stagnation region, exists on all solid bodies moving in a fluid. The stagnation region encounters the highest pressure, the highest heat transfer rate and the highest rates of mass deposition. Chiam (1994) extended the works of Hiemenz (1911) and Crane (1970) by study the stagnation-point flow over a stretching sheet. The velocity of the stretching/shrinking sheet can vary linearly or nonlinearly. Some very important investigations regarding the stagnation-point flow over stretching sheet under different physical situations were made by Nazar *et al.* (2004), Ishak *et al.* (2006, 2009), Layek *et al.* (2007) and Nadeem *et al.* (2010). Recently an

analysis is carried out to investigate the stagnation-point flow and heat transfer over an exponentially shrinking sheet by Bhattacharyya and Vajravelu (2012). The viscous flow and heat transfer in the boundary layer region due to a shrinking sheet attracted the attention of researchers for its interesting physical character, for example, on a rising, shrinking balloon. The boundary layer flow caused by a shrinking sheet is quite different from the stretching case. It is also shown that mass suction is required to maintain the flow over the shrinking sheet. Wang (2008) studied the steady stagnation-point flow towards a shrinking sheet. The unsteady case of recent problem is investigated by Fan *et al.* (2010) with assumptions that the sheet is shrunk impulsively from rest and simultaneously the surface temperature is suddenly increased from that of surrounding fluid.

Nanofluids are a new class of nanotechnology-based heat transfer fluids engineered by dispersing nanometer-scale solid particles with typical length scales on the order of 1 to 100 nm in traditional heat transfer fluids (Das *et al.*, 2007). Nanoparticles have different shapes such as: spherical, rod-like or tubular shapes and so on. Choi (1995) was the first who introduced the term of nanofluids to describe this new class of fluid. The presence of the nanoparticles in the

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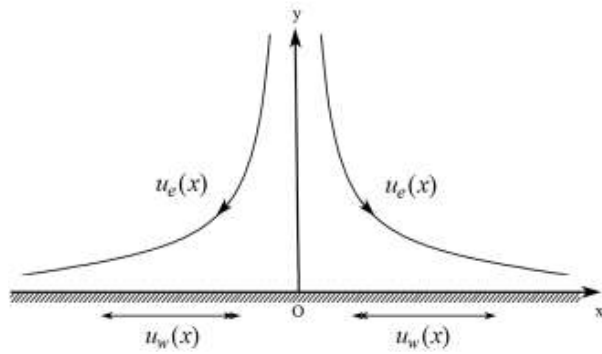


Fig. 1: The schematic diagram of two-dimensional stagnation point flow over a stretching/shrinking sheet

fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics. This fact has attracted many researchers such as Abu-Nada (2008), Wang and Mujumdar (2007), Tiwari and Das (2007), Oztop and Abu-Nada (2008), Maïga *et al.* (2005) and Nield and Kuznetsov (2009) and so on, to investigate the heat transfer characteristics in nanofluids.

The main goal of the present study is to investigate the stagnation-point flow and heat transfer of a nanofluid adjacent to a linearly stretching/shrinking sheet. The model introduced by Tiwari and Das (2007) has been used in the present study. Using a similarity transform, the Navier-Stokes equations have been reduced to a set of nonlinear ordinary differential equations. The resulting non-linear system has been solved numerically using the fourth-order Runge-Kutta method with shooting technique. Finally, the results are reported for three different types of nanoparticles namely alumina, titania and copper with water as the base fluid.

PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

Consider the steady two-dimensional stagnation-point flow of a nanofluid on a stretching/shrinking sheet as shown in Fig. 1. Cartesian coordinates x and y are taken with the origin O at the stagnation point and are defined such that the x -axis is measured along the stretching/shrinking sheet and the y -axis is measured normal to it. It is assumed that the velocity of the external flow is given by $u_e(x) = bx$, where $b > 0$ is the strength of the stagnation flow and the surface temperature T_w is a constant. Let $u_w(x) = ax$ be the velocity of the stretching/shrinking sheet, where a is the stretching/shrinking rate, with $a > 0$ for stretching and $a < 0$ for shrinking cases. The nanofluid is assumed incompressible and the flow is assumed to be laminar. It is also assumed that the base fluid (i.e., water) and the

nanoparticles are in thermal equilibrium and no slip occurs between them. Under the boundary layer approximations and using the nanofluid model proposed by Tiwari and Das (2007), the basic steady conservation of mass and momentum equations for a nanofluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2}, \tag{2}$$

subject to the boundary conditions:

$$\begin{aligned} u &= u_w(x) = ax & \text{at } y &= 0, \\ u &= u_e(x) = bx & \text{as } y &\rightarrow \infty, \end{aligned} \tag{3}$$

where,

u, v = The velocity components along the x -axes and y -axes respectively

μ_{nf} = The viscosity of the nanofluid

ρ_{nf} = The density of the nanofluid, which are given by:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{4}$$

where,

ϕ = The solid volume fraction of the nanofluid

ρ_f = The density of the base fluid

ρ_s = The density of the solid particle

μ_f = The viscosity of the base fluid

It is worth mentioning that the viscosity of the nanofluid can be approximated as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles and its expression has been given by Brinkman (1952).

To obtain similarity solutions of Eq. (1) and (2) with the boundary conditions (3), we introduce the following similarity variables (Cheng, 1977; Lai and Kulacki 1990):

$$\psi = (v_f x u_e)^{1/2} f(\eta), \quad \eta = \left(\frac{u_e x}{v_f} \right)^{1/2} \frac{y}{x}, \tag{5}$$

where α_f is the thermal diffusivity of the fluid and ψ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfy Eq. (1). Using the non-dimensional variables in Eq. (5), Eq. (2) reduce to the following ordinary differential equation:

$$\frac{1}{(1-\phi)^{2.5} \left(1-\phi + \phi \frac{\rho_s}{\rho_f}\right)} f'''(\eta) + f(\eta)f''(\eta) - (f'(\eta))^2 + 1 = 0, \quad (6)$$

subject to the boundary conditions:

$$\begin{aligned} f(0) &= 0, & f'(0) &= b/a = c, \\ f'(\eta) &\rightarrow 1 & \text{as } \eta &\rightarrow \infty, \end{aligned} \quad (7)$$

where primes denote differentiation with respect to η . The velocity ratio parameter defined as the ratio of stretching rate of the sheet and strength of the stagnation flow. $c > 0$, $c < 0$ and $c = 1$ correspond to stretching, shrinking sheets and the flow with no boundary layer ($u_w = u_e$), respectively, while $c = 0$ is the planar stagnation flow towards a stationary sheet.

The skin friction coefficient C_f is defined as:

$$C_f = \frac{\tau_w}{\rho_f u_e^2 / 2}, \quad (8)$$

where, τ_w is the surface shear stress which is given by:

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (9)$$

Substituting (5) into Eqs. (8) and (9) we obtain:

$$\frac{1}{2} C_f (\text{Re}_x)^{1/2} = \frac{f''(0)}{(1-\phi)^{2.5}}, \quad (10)$$

where, $\text{Re}_x = u_e x / \nu$ is the local Reynolds number.

The energy equation is (Tiwari and Das (2007):

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2}, \quad (11)$$

which is subjected to the boundary conditions:

$$\begin{aligned} T &= T_w & \text{at } & y = 0, \\ T &= T_\infty & \text{at } & y \rightarrow \infty, \end{aligned} \quad (12)$$

The thermal diffusivity of the nanofluid (α_{nf}) and the effective thermal conductivity of the nanofluid (k_{nf}) approximated by the Maxwell-Garnetts model (Oztop and Abu-Nada, 2008) and are given by:

$$\begin{aligned} \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}}, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \end{aligned} \quad (13)$$

Here k_f is the thermal conductivity of the fluid, k_s is the thermal conductivity of the solid and $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid. It should be mentioned that the Maxwell-Garnetts model is based on the assumption that the discontinuous phase is spherical in shape and the thermal conductivity of spherical particles, the base fluid and the particle volume fraction. It should be mentioned that the use of the approximation for k_{nf} is restricted to spherical nanoparticles and does not account for other shape nanoparticles. The interested reader can find several other property expressions for the thermophysical properties of the nanofluid in the review paper by Kakaç and Pramuanjaroenkij (2009).

Now we look for a similarity solution of Eq. (11) subjected to the boundary conditions (12) of the form:

$$\theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \quad (14)$$

By using (5), (13) and (14), we obtain the following ordinary differential equation:

$$\frac{1}{\text{Pr}} \cdot \frac{k_{nf}/k_f}{1-\phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}} \theta''(\eta) + f(\eta)\theta'(\eta) = 0, \quad (15)$$

subjected to the boundary conditions:

$$\theta(0) = 1, \quad \theta(\infty) = 0, \quad (16)$$

where, $\text{Pr} = \nu_f / \alpha_f$ is the prandtl number:

The local Nusselt number Nu_x is defined as:

$$Nu_x = \frac{xq_w}{k_f (T_w - T_\infty)}, \quad (17)$$

where, q_w is the surface heat flux which is given by:

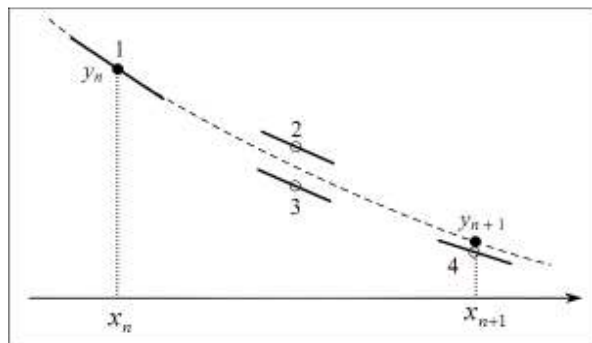


Fig. 2: Fourth-order Runge-Kutta method

Table 1: Thermophysical properties of the base fluid and the nanoparticles (Oztop and Abu-Nada, 2008)

Physical properties	Fluid phase			
	(water)	Cu	Al ₂ O ₃	TiO ₂
C_p (J/kg K)	4179	385	765	686.2
ρ (kg/m ³)	997.1	8933	3970	4250
k (W/mK)	0.613	400	40	8.9538
$\alpha \times 10^{-7}$ (m ² /s)	1.47	1163.1	131.7	30.7

$$q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \tag{18}$$

Using the non-dimensional variables in Eq. (5), (17) and (18) we obtain:

$$Nu_x (Re_x)^{-1/2} = -\frac{k_{nf}}{k_f} \theta'(0), \tag{19}$$

NUMERICAL ANALYSIS

Equation (6) and (15) subject to the boundary conditions (7) and (16), respectively, are solved numerically for some values of the solid volume fraction (ϕ), the three different types of nanofluid and as well as for the stretching or shrinking parameter c by the fourth-order Runge-Kutta method with shooting technique. The fourth-order Runge-Kutta method requires four evaluations of the right-hand side per step h shown in Fig. 2. In this method, in each step the derivative is evaluated four times: once at the initial point, twice at trial midpoints and once at a trial endpoint. From these derivatives the final function value is calculated (Press *et al.*, 1988). There are some advantages of foregoing procedure:

- One step method-global error is of the same order as local error
- Don't need to know derivatives of f
- Easy for "Automatic Error Control"

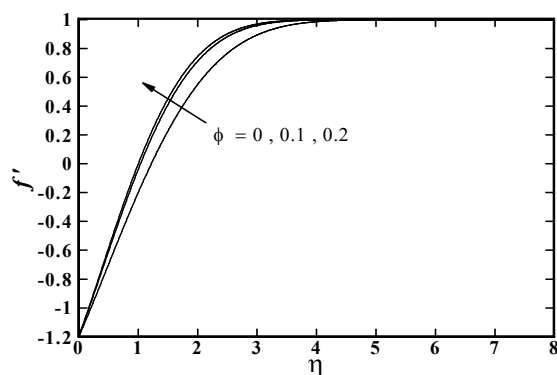


Fig. 3: Velocity profiles $f'(\eta)$ for various values of ϕ for Cu as nanoparticle in shrinking case ($c = -1.2$)

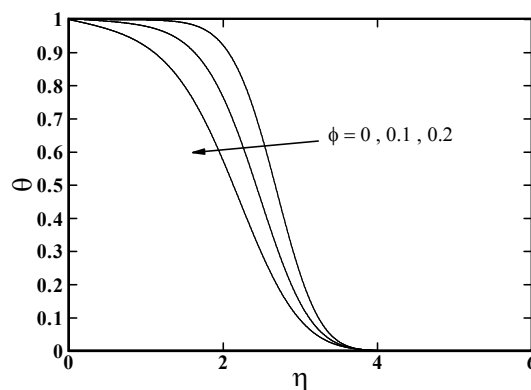


Fig. 4: Temperature profiles $\theta(\eta)$ for various values of ϕ for Cu as nanoparticle in shrinking case ($c = -1.2$)

RESULTS AND DISCUSSION

We consider three different types of nanoparticles, namely Cu, Al₂O₃ and TiO₂ with water as the base fluid. The thermophysical properties of the base fluid and the nanoparticles are listed in Table 1 (Oztop and Abu-Nada (2008)). We restrict our study to a common Prandtl number for nanofluids, taking $Pr = 6.2$. We expect our results are qualitatively similar with other values of Pr of $O(6.2)$. It is worth mentioning that the present study reduces to a viscous or regular fluid study when $\phi = 0$.

The considered values of the shrinking rate c are $+1.2$ and -1.2 for stretching sheet and shrinking sheet, respectively. To illustrate the velocity and temperature profiles, nanoparticle volume fraction parameter ϕ is considered 0, 0.1 and 0.2.

Figure 3 presents the velocity profiles $f'(\eta)$ for various values of ϕ for Cu as nanoparticle and the shrinking case ($c = -1.2$), with the corresponding temperature profiles being shown in Fig. 4. Also, the

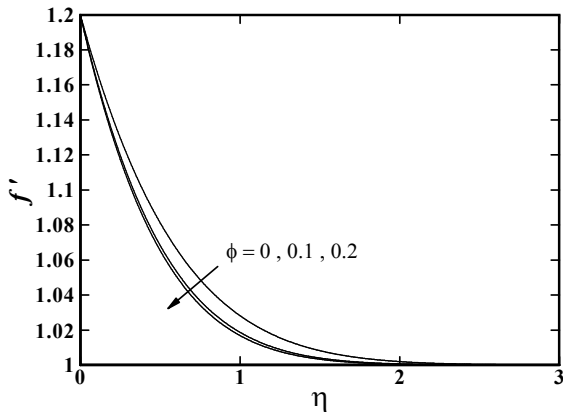


Fig. 5: Velocity profiles $f'(\eta)$ for various values of ϕ for Cu as nanoparticle in stretching case ($c = +1.2$)

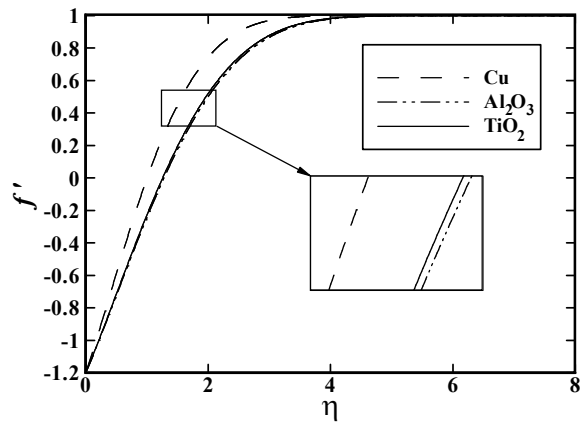


Fig. 7: Velocity profiles $f'(\eta)$ for various nanoparticles, when $\phi = 0.2$, in shrinking case ($c = -1.2$)

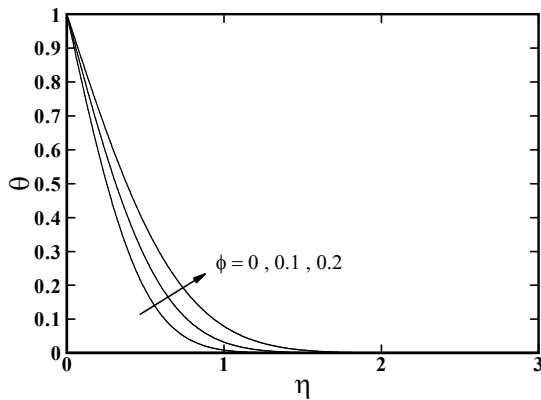


Fig. 6: Temperature profiles $\theta(\eta)$ for various values of ϕ for Cu as nanoparticle in stretching case ($c = +1.2$).

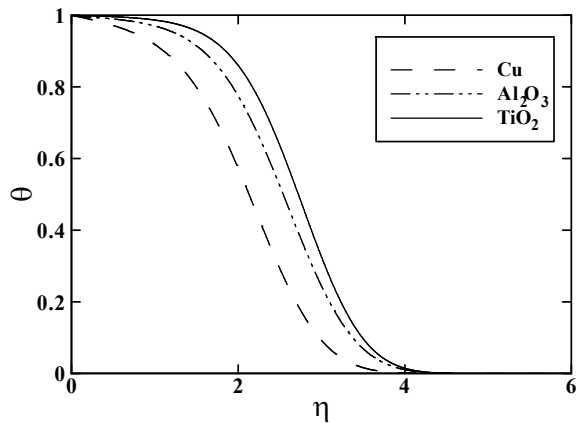


Fig. 8: Temperature profiles $\theta(\eta)$ for various nanoparticles, when $\phi = 0.2$, in shrinking case ($c = -1.2$)

velocity and temperature profiles of nanofluid with the same particle for the stretching case ($c = +1.2$), have been presented in Figs. 5 and 6, respectively. It is evident from these figures that numerical solution profiles satisfy the far field boundary conditions asymptotically, thus support the validity of the numerical results obtained. It worth to mentioning that the behavior of the velocity and temperature profiles in shrinking case ($c = -1.2$) is similar to that reported by Rosali *et al.* (2011) for regular fluid ($\phi = 0$). It is clear that from Fig. 4 and 6, the temperature profiles change smoother with the increase in the nanoparticles volume fraction ϕ . This is agrees with the physical behavior, when the volume of nanoparticles increases the thermal conductivity increases. Figures 3 and 5 show the velocity Profiles change abruptly with the increase in the nanoparticles volume fraction ϕ . This is due to the increase in the friction. It is evident from these figures that for a stretching sheet, the velocity profiles approach faster than for the shrinking sheet. Therefore,

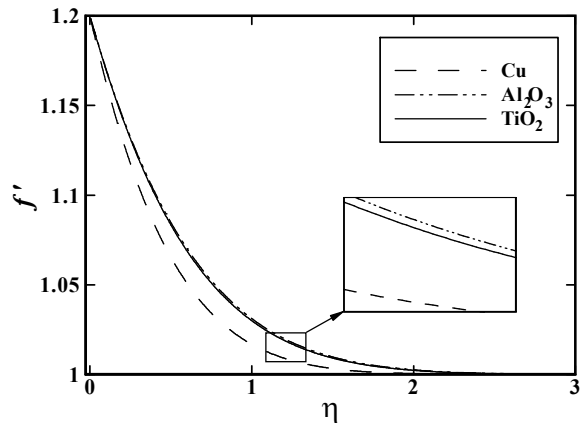


Fig. 9: Velocity profiles $f'(\eta)$ for various nanoparticles, when $\phi = 0.2$, in stretching case ($c = +1.2$)

the boundary layer thicknesses for shrinking sheets are higher than those for stretching sheets. Figures 7 to 10 display the behavior of the velocity and temperature

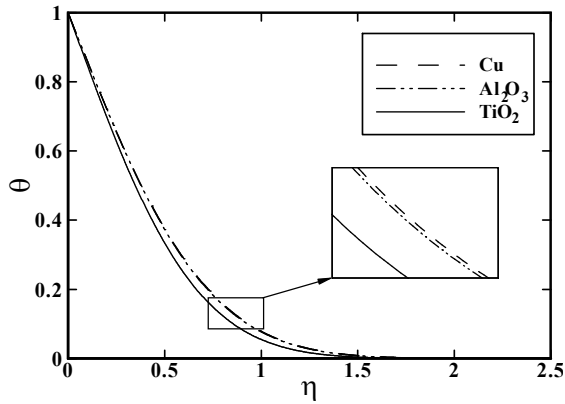


Fig.10: Temperature profiles $\theta(\eta)$ for various nanoparticles, when $\phi = 0.2$, in stretching case ($c = +1.2$).

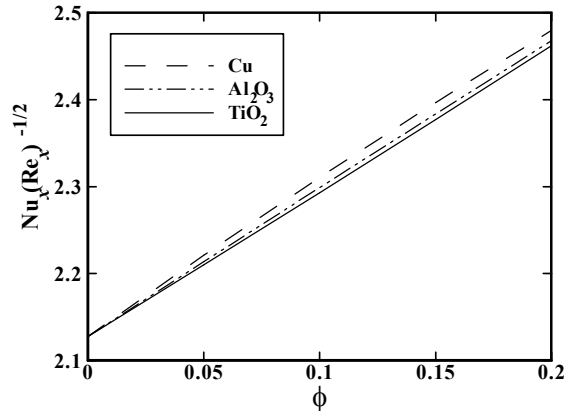


Fig. 13: Variation of the local Nusselt number with ϕ for different nanoparticles in stretching case ($c = +1.2$)

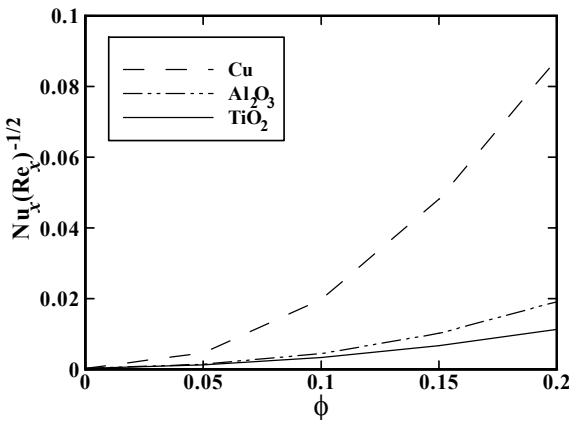


Fig. 11: Variation of the local Nusselt number with ϕ for different nanoparticles in shrinking case ($c = -1.2$).

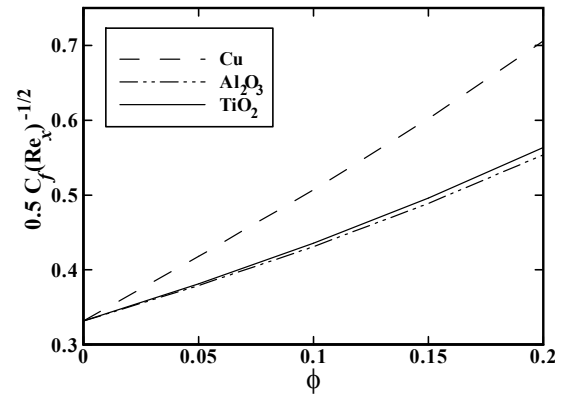


Fig. 14: Effects of the nanoparticle volume fraction ϕ on reduced skin friction coefficient in stretching case ($c = +1.2$).

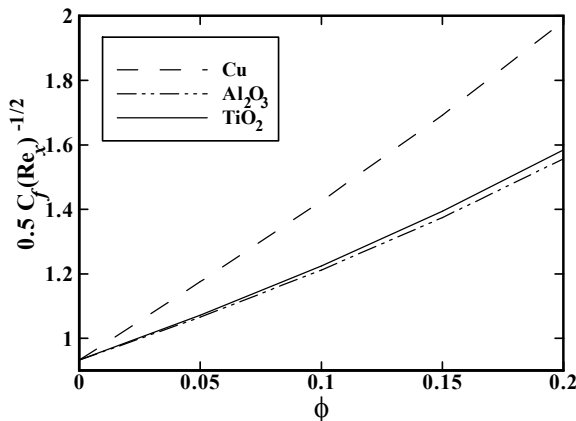


Fig. 12: Effects of the nanoparticle volume fraction ϕ on reduced skin friction coefficient in shrinking case ($c = -1.2$).

profiles for different types nanofluids for both shrinking/stretching cases when $\phi = 0.2$. From Fig. 7,

we can see that for shrinking case, Cu-water and Al_2O_3 -water nanofluids have the highest and lowest velocities components, respectively. Besides, Fig. 8 demonstrates Cu-water and TiO_2 -water nanofluids have the lowest and highest temperature in this case, respectively. A reverse behavior for stretching sheet is clear from Fig. 9 and 10.

Figures 11 and 12 show the variation of the local Nusselt number and the skin friction coefficient versus the values of the nanoparticles volume parameter ϕ for various nanoparticles in the case of shrinking surface. Figures 13 and 14 show the same variation in the case of stretching surface. It is clear that from Fig. 11 and 13, the heat transfer rates increase with the increase in the nanoparticles volume fraction ϕ . This is due to the increase in the volume of solid nanoparticles with relatively higher thermal conductivity. Also change in the local Nusselt number is found to be higher for higher values of the parameter ϕ . Figures 12 and 14 display the variation of the skin friction coefficient for

Table 2: The influence of the different nanoparticle volume fractions on the skin friction coefficient and the nusselt number for the different nanoparticles, when $c = -1.2$ and $+1.2$.

Nanoparticle	ϕ	Shrinking case ($c = -1.2$)		Stretching case ($c = +1.2$)	
		$Nu_x/Re_x^{1/2}$	$\frac{1}{2}C_f Re_x^{1/2}$	$Nu_x/Re_x^{1/2}$	$\frac{1}{2}C_f Re_x^{1/2}$
Regular fluid Rosali <i>et al.</i> (2011)	0	0.000326	0.932473	2.12762	3.31674E-01
Cu	0.05	0.004700	1.175513	2.22116	4.18121E-01
	0.10	0.019810	1.425522	2.31078	5.07048E-01
	0.15	0.048181	1.692255	2.39683	6.01922E-01
	0.20	0.087748	1.984149	2.47965	7.05747E-01
Al ₂ O ₃	0.05	0.001499	1.065753	2.21368	3.79080E-01
	0.10	0.004504	1.211983	2.29896	4.31093E-01
	0.15	0.010228	1.374459	2.38366	4.88885E-01
	0.20	0.019171	1.557157	2.46795	5.53869E-01
TiO ₂	0.05	0.001281	1.072244	2.20994	3.81389E-01
	0.10	0.003360	1.225021	2.29306	4.35731E-01
	0.15	0.006772	1.394318	2.37707	4.95948E-01
	0.20	0.011380	1.584313	2.46211	5.63528E-01

different values ϕ . It is noticed that the change in the skin friction coefficient to be higher for higher value of ϕ in these figures.

Table 2 shows the values of local Nusselt number and the skin friction coefficient for different types of nanofluids and various ϕ for both shrinking/stretching cases. It is found that the lowest heat transfer rate is obtained for the nanoparticles TiO₂. This is because TiO₂ has the lowest value of thermal conductivity compared to Cu and Al₂O₃, as can be seen from Table 1. This behavior of the local Nusselt number (heat transfer rate at the surface) is similar to that reported by Oztop and Abu-Nada (2008). The thermal conductivity of Al₂O₃ is approximately one tenth of Cu, as given in Table 1. However, a unique property of Al₂O₃ is its low thermal diffusivity. The reduced value of thermal diffusivity leads to higher temperature gradients and, therefore, higher enhancements in heat transfer. The Cu nanoparticles have higher values of thermal diffusivity and therefore reduce the temperature gradients. As volume fraction of nanoparticles increases, the local Nusselt number becomes larger specially in the case of stretching sheet. The highest value of the local Nusselt number is recorded for Cu-nanofluid for both cases of shrinking and stretching surface.

CONCLUSION

We have studied how the nanoparticle volume fraction parameter ϕ , influence the boundary layer flow and heat transfer characteristics in a nanofluid toward the shrinking/stretching sheet using three different types of nanoparticles: copper Cu, alumina Al₂O₃ and titania TiO₂. It is found that the heat transfer rates and skin friction coefficient increase as the nanoparticle volume fraction ϕ increases. Furthermore the highest value of the local Nusselt number is

recorded for Cu-nanofluid for both cases of shrinking and stretching surface. Furthermore, it is shown that the lowest heat transfer rate is obtained for the nanoparticles TiO₂. This is because TiO₂ has the lowest value of thermal conductivity compared to Cu and Al₂O₃. Therefore, the type of nanofluids is a key factor for heat transfer enhancement and using a mixture of nanoparticles and the base fluid is an effective technique to develop the advanced heat transfer fluids with substantially higher conductivities. It is worth mentioning here, that the study of nanofluids is still at its early stage so that complementary works are necessary to understand the heat transfer characteristics of nanofluids and identify new and unique applications for these fluids.

NOMENCLATURE

- a, b = onstant
- C_f = Skin friction coefficient
- k = Thermal conductivity
- Nu_x = Local Nusselt number
- Re_x = Local Reynolds number
- Pr = Prandtl number
- q_w = Surface heat flux
- T = Fluid temperature
- T_w = Surface temperature
- T_∞ = Ambient temperature
- u, v = Velocity components
- x, y = Cartesian coordinates
- u_e = Free stream velocity
- $f(\eta)$ = Dimensionless stream function

Greek symbols:

- α = Thermal diffusivity
- β = Thermal expansion coefficient
- ϕ = Nanoparticle volume fraction
- η = Similarity variable

$\theta(\eta)$ = Dimensionless temperature
 μ = Dynamic viscosity
 ν = Kinematic viscosity
 ρ = Fluid density
 τ_w = Wall shear stress
 ψ = Stream function

Subscripts:

w = Condition at the surface of the plate
 ∞ = Ambient condition
 f = Fluid
 nf = Nanofluid
 s = Solid

Superscripts:

' = Differentiation with respect to η

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