Research Journal of Applied Sciences, Engineering and Technology 7(3): 430-437, 2014
DOI:10.19026/rjaset.7.272
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: November 12, 2012 Accepted: January 17, 2013 Published: January 20, 2014

Research Article

Probabilistic Optimal Allocation and Sizing of Distributed Generation

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Abstract: The optimal allocation of Distributed Generation (DG) in distribution system is one of the important parts of DG research studies so as to maximize its benefits. For this purpose, a probabilistic approach is proposed in this study to consider time varying load demands as uncertain parameters of distribution system. It is assumed that each load point consists of three categories of voltage dependent loads: residential, industrial and commercial. The proposed algorithm is based on a probabilistic load flow solved by Point Estimate Method (PEM). The objective function is considered as a combination of active power loss, reactive power loss and voltage profiles indices. To solve the optimization problem, an Invasive Weed Optimization (IWO) technique is adopted and the optimal location and size of different types of DG are obtained. Examining on a test distribution system, the performance of the proposed approach is assessed and illustrated.

Keywords: DG allocation and sizing, Invasive Weed Optimization (IWO), Point Estimate Method (PEM), voltage dependent load

INTRODUCTION

Through the last decade, environmental issues along with the development of renewable energy technologies for connection to the network have increased the trend toward the Distributed Generation (DG) systems. In distribution system planning, in the presence of DG, several factors including the use of the best available technology, optimal number, capacity and location of DG units, network connection types, etc., are considered. Impacts of DG on system performance characteristics should properly be evaluated because the installation of DG units at non-optimal locations may have adverse effects such as an increase in power losses which would consequently lead to increase in operating costs (Borges and Falcão, 2006). Hence, finding out the optimal location and size of DG is of great importance and proper optimization methods should be employed for solving it.

So far, numerous methods have been suggested to determine the optimal size and location of DG in power networks. Abookazemi *et al*. (2010) briefly review these proposed methods. These approaches are various according to their objective functions and optimization methods. The various existing optimization approaches has categorized into five different major headings consist of: analytical approaches, meta-heuristics, artificial intelligence approaches, Genetic Algorithm (GA), hybrid approaches and other ones (Viral and Khatod, 2012). In order to find best location and size of DG, different network parameters can be considered such as power loss, network cost, voltage profile, reliability indexes and voltage stability index, etc. Gözel and Hocaoglu (2009) determined the optimal location and size of DG in order to minimize total power losses by an analytical method. Moeini-Aghtaie *et al*. (2011) scrutinized three main factors including network loss, the costs associated with the investment, operation and maintenance of the DGs and system reliability and solved multi objective optimization problem via a robust method of NSGAII. Kalantari and Kazemi (2011) used a genetic algorithm based method for DG unit and shunt capacitors placement for loss reduction and improvement in voltage profile. Wang and Singh (2008) considered a composite reliability index as the objective function in the optimization procedure and used ant colony system algorithm to obtain the optimal recloser and DG placements for radial distribution networks. El-Zonkoly (2011) and Maciel *et al*. (2012) proposed an optimization technique based on Particle Swarm Optimization (PSO) for optimally determine the size and location of multi-distributed generation units in distribution systems. Since the load types have important effects on the power system studies, in some papers, uncertainties in loads are overcome by fuzzy presentation of load in problem formulation (Haghifam and Malik, 2007) whereas in most of papers the constant power load model have been used for DG allocation and sizing (Kalantari and Kazemi, 2011).

In this study, different models of voltage dependent load are considered in each load point. Also the

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placement procedure is carried out taking into consideration time-varying loads through a probabilistic approach. In this approach, a probabilistic load flow based on the point estimate method is applied. The objective function is formed by combining the indices that show the effects of the presence of DG on active and reactive power losses and voltage profile. The probability distribution of objective function can be obtained by applying the probabilistic method. Then, the optimization problem is solved using Invasive Weed Optimization (IWO) algorithm which is a novel evolutionary optimization technique.

PROBLEM DESCRIPTION

Objective function and constraints: The considered objective function is defined as a combination of active and reactive power loss indices and voltage deviation index, according to relationships (1) to (4) Objective Function:

$$\alpha_1\ LPI + \alpha_2\ LQI + \alpha_3\ VDI \tag{1}$$

$$LPI = \frac{P_{loss}\ \text{with DG}}{P_{loss}\ \text{without DG}} \tag{2}$$

$$LQI = \frac{Q_{loss}\ \text{with DG}}{Q_{loss}\ \text{without DG}} \tag{3}$$

$$VDI = \sum_{i=1}^{NB} \left(\frac{|V_{rated}| - |V_i|}{|V_{rated}|} \right)^2 \tag{4}$$

V_{rated} is the magnitude of desired voltage (1 p.u.) for buses and VDI reduction indicates the improvement of voltage profile.

In (1), α_i is the weighting factor where $\Sigma\alpha_i = 1$ and is defined based on the impact rate of corresponding parameter on objective function. In this study, the values of these weights are considered 0.35, 0.35 and 0.3 for α_1, α_2 and α_3, respectively.

During power flow in power system some constraints must be satisfied. These constraints are as follow:

- Power flow equations
- Voltage limits at each bus

$$V^{\min} \le V_i \le V^{\max}$$

- DG's active and reactive power generation limits

$$P_{DG}^{\min} \le P_{DG} \le P_{DG}^{\max}$$

$$Q_{DG}^{\min} \le Q_{DG} \le Q_{DG}^{\max}$$

DG models: Distributed generations can be classified based on the basic principle of operation, control strategies and integration with the grid. Shao-Qiang and Sen-Mao (2010) and Teng (2008) have investigated different control strategies for various DGs and assigned a bus type to each kind of DGs according to its certain control strategy. Hence, in this study DG nodes are considered as three models (PV nodes, PQ (V) nodes and constant power factor nodes) in load flow calculation. The properties and equations of each type of DG nodes are showed as follows:

- **PV node:** This model is used for large-scale controllable DGs. The specified values of this DG node are the active power output and bus voltage magnitude and the value of reactive power is calculated for voltage regulating.
- **PQ (V) node:** The reactive power that wind energy generations absorb or export by means of asynchronous machines is uncertain. The reactive power absorbed by generator is related to terminal voltage and slip and relationship of reactive power and voltage can be obtained from (5). This type of DGs can be counted as PQ (V) nodes:

  $$Q = -\frac{V^2}{x_p} + \frac{-V^2 + \sqrt{V^4 - 4P^2x^2}}{2x} \tag{5}$$

  where, x_p and x are impedance parameters of the machine.
- **Constant power factor nodes:** The model can be used for controllable DGs, such as synchronous generator based DGs and power electronic based DGs. For this model, the specified values are the active power output and power factor of DG. The reactive power of DGs can be calculated by (6):

  $$Q = P\tan(\cos^{-1}(pf)) \tag{6}$$

Load model: The load in a distribution system generally consists of three main types, i.e., residential, industrial and commercial. The common static load models for active and reactive power are expressed in an exponential form (Qian *et al.*, 2011; Eminoglu and Hocaoglu, 2005). The characteristic of the exponential load models can be given as (7):

$$P = P_0\left(\frac{V}{V_0}\right)^{n_p},\ Q = Q_0\left(\frac{V}{V_0}\right)^{n_q} \tag{7}$$

where, n_p and n_q are active and reactive power exponents, P_0 and Q_0 are the values of the active and reactive powers at the nominal voltages; V and V_0 are

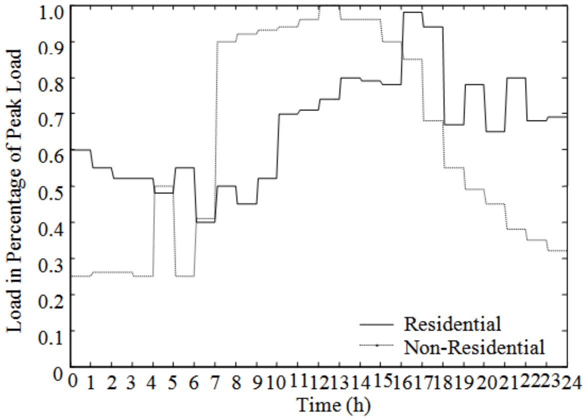

Fig. 1: Daily load curve (Walling and Shattuk, 2007)

Table 1: Values for the exponents of static load models

Load type	n_p	n_q
Residential	0.92	4.04
Industrial	0.18	6.00
Commercial	1.51	3.40

Table 2: Proportion of each load type in the total load on a load node during a day (Qian *et al*., 2011)

Load type	Residential	Industrial	Commercial
Proportion	0.41417	0.30333	0.2825

load bus voltage and load nominal voltage. Common values for exponents of static loads are given in Table 1.

For all kinds of customers the requirement of power varies with time. Here two major classifications have been made for describing the load curve, residential and nonresidential customers. Figure 1 shows the considered daily load curve on a weekday. The load curve shows the variation of load as a percentage of peak loads over a period of 24 h. Load variation per hour has been neglected.

In this study, load demand uncertainties are applied in probabilistic approach. It is assumed that each load node consists of three components of load consumption and the proportion of each load type is constant value in the total load on a weekday as shown in Table 2. Considering exponential load model with daily load curve make system be more realistic.

SOLUTION PROCEDURE

In this section, the probabilistic approach used in optimization procedure, IWO optimization algorithm and proposed algorithm is presented.

Probabilistic load flow using Point Estimate Method (PEM): In probabilistic load flow, the basic load flow model is the same as the one in deterministic load flow except that uncertain inputs are viewed as probabilistic variables. Since the system inputs are probabilistic variable, the system outputs can also be represented by probabilistic variables.

Point Estimate Method (PEM) can be utilized to obtain the numerical characteristics of outputs. This method provides an approximate approach for calculating the moments of a probabilistic output that is formulated as a function of probabilistic inputs. The point estimate method permits us to calculate moments of a random variable that is a function F of random variables, where F relates input and output variables. To obtain these moments, the function F has to be calculated 2m, 2m+1 or 4m+1 times depending on the adopted scheme; where, m is the random input variable number. In this study, the 2 m scheme is used. More details about the application of the point estimate method to probabilistic load flow equations are illustrated by Caramia *et al*. (2010) and Morales and Pérez-Ruiz (2007). The procedure for computing the moments of the output variables using PEM are summarized as follows:

- Number 'm' of input random variables is determined and the numerical characteristics (central moments and standard central moments $\lambda_{t,i}$) of the input random variables are extracted as (8) and (9):

$$M_i(x_t)=\int_{-\infty}^{+\infty}(x_t-\mu_{xt})^i f_{x_t}dx \quad i=3,4 \qquad t=1,2,\dots,m \tag{8}$$

where, μ_{xt} and σ_{xt} are mean and standard deviation of x_t with probability density function f_{x_t}:

$$\lambda_{t,i}=\frac{M_i(x_t)}{\left(\sigma_{x_t}\right)^i} \quad i=3 \tag{9}$$

- For all input variables standard locations ($\xi_{t,1}$, $\xi_{t,2}$), locations ($x_{t,1}$, $x_{t,2}$) and weight factors are determined by (10) to (12):

$$\xi_{t,1}=\frac{\lambda_{t,3}}{2}+\sqrt{m+\frac{1}{4}\lambda_{t,3}^2};\quad \xi_{t,2}=\frac{\lambda_{t,3}}{2}-\sqrt{m+\frac{1}{4}\lambda_{t,3}^2} \tag{10}$$

$$x_{t,i}=\mu_{x_t}+\xi_{t,i}\sigma_{x_t} \quad i=1,2 \tag{11}$$

$$w_{t,1}=\frac{-\xi_{t,2}}{\left(\xi_{t,1}-\xi_{t,2}\right)};\quad w_{t,2}=\frac{\xi_{t,1}}{\left(\xi_{t,1}-\xi_{t,2}\right)} \tag{12}$$

- For all input random variables two input variable vectors as (13) is formed and for each vector the deterministic load flow is run and desired outputs F (X_i) is saved:

$$X_1=\left[\mu_{x_1},\dots,x_{t,1},\dots,\mu_{x_m}\right];\ X_2=\left[\mu_{x_1},\dots,x_{t,2},\dots,\mu_{x_m}\right] \tag{13}$$

- The vector of the j'th moment of the output variable is calculated as (14):

$$E\left(Y^{j}\right)=\sum_{t=1}^{m}\sum_{i=1}^{2}w_{t,i}\left[F\left(X_{i}\right)\right]^{j} \tag{14}$$

Once the first statistical moments are known, it is possible to approximate the Probability Density Functions (PDF) and Cumulative Distribution Function (CDF) of the output variables of interest by Gram-Charlier expansion (or another form of it such as Edgeworth expansion and Cornish fisher expansion) (Zhang and Lee, 2004).

Brief overview of Invasive Weed Optimization (IWO) algorithm: Invasive Weed Optimization (IWO) is a recently developed optimization technique by Mehrabian and Lucas (2006), which is motivated from a common agricultural phenomenon. This algorithm is summarized as follows:

Initialization: An initial population of solutions (weeds) is generated randomly over the D dimensional space.

Reproduction: Each population member is allowed to produce seeds depending on its fitness as well as the colony's lowest and highest fitness. So, the numbers of seeds produced by any weed increases linearly from lowest possible seed generation to its maximum.

Spatial dispersal: The generated seeds are distributed over the search space by normally distributed random numbers with mean equal to zero but varying variance. This step ensures that the produced seeds will be generated around the parent weed, leading to a local search around each plant. However, the standard deviation of the random function is made to decrease over the iterations. The equation for determining the standard deviation for each generation is presented in (15):

$$\sigma_{iter}=\left(\frac{iter_{\max}-iter}{iter_{\max}}\right)^{n}\left(\sigma_{\max}-\sigma_{\min}\right)+\sigma_{\min} \tag{15}$$

where, $iter_{\max}$ is the maximum number of iterations, σ_{iter} is the standard deviation at the current iteration and n is the nonlinear modulation index. This alteration ensures that the probability of dropping a seed in a distant area decreases nonlinearly at each time step which results in grouping fitter plants and elimination of in appropriate plants.

Competitive exclusion: When all seeds have found their position in the search area, plants and offspring are ranked together and the ones with better fitness survive and are allowed to replicate. This mechanism gives a chance to plants with lower fitness to reproduce and if their offspring has a good fitness in the colony then they can survive. The population control mechanism also is applied to their offspring to the end of a given run, realizing competitive exclusion. This process continues until maximum number of plants is reached; now only the plants with higher fitness can survive and produce seeds, others are being eliminated. The process continues until maximum iteration is reached and hopefully the plant with best fitness is the closest to the optimal solution.

Proposed method: As it was said, in order to perform probabilistic load flow by PEM, deterministic load flow is run 2 m times. In this study, the deterministic load flow has been calculated using a direct approach for load flow solutions proposed by Teng (2003). To consider the effects of voltage dependent load models and types of DG, the mathematical models of them have been integrated into load flow program. Also, the constraints are considered in the optimization method. For using PEM, in this study the load values of all load points are assumed as uncertain system inputs. Since the load curve shown in Fig. 1 is like a step-wise curve, the moments and central moments of each load point for each load type are calculated by (16), (17):

$$\alpha_{L\nu}=\sum_{i}p_{i}x_{i}^{\nu} \quad \nu=1,2,\ldots,k \tag{16}$$

$$M_{L\nu}=\sum_{i}p_{i}\left(x_{i}-\alpha_{L1}\right)^{\nu} \tag{17}$$

where, $\alpha_{L\nu}$ and $M_{L\nu}$ are the ν-order moment and ν-order central moment of the load curve and p_i is the probability when the load has the value of x_i that:

$$p_{i}=\frac{t_{i}}{T}$$

where,

t_i : The duration of the load x_i

T : The investigated period

After calculating the moments and central moments of each load point consumption, they could be used in PEM. To consider the proportion of different load consumptions in each load point, the probabilistic load flow is done for each load type and corresponding moments of Objective Function (OF) is obtained. Final moments of OF can be determined considering each load type proportion in total load by (18):

$$E\left(OF^{j}\right)=\sum_{i=1}^{L}E\left(OF_{Li}^{j}\right).\omega_{Li} \tag{18}$$

where,

ω_{Li} : The proportion of load L_i in total load

OF_{Li} : The obtained objective function for load type L_i

In the optimization method, the obtained OFs should be compared with each other. The moments of

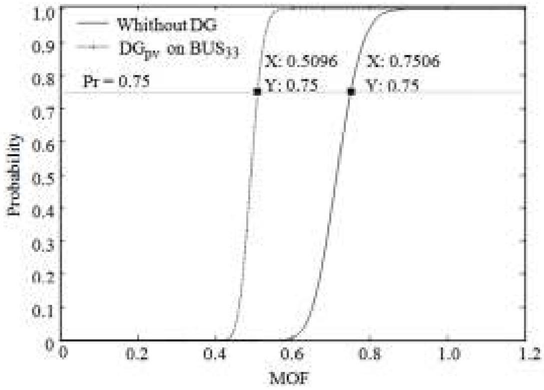

Fig. 2: Different CDFs of objective function

Table 3: Results of proposed algorithm for allocation one DG unit

DG type	PQ (V)	Pf const.	PV
Bus	6	30	6
Pr = 1			
P (MW)	1.05090	0.87156	1.59390
Max_{Prob} $Fitness_{OF}$	0.52505	0.27826	0.27487
Pr = 0.5			
P (MW)	0.98850	0.85132	1.53150
Expected $Fitness_{OF}$	0.48202	0.23760	0.23839
Pr = 0			
P (MW)	0.92240	0.82899	1.43940
Min_{Prob} $Fitness_{OF}$	0.43751	0.19646	0.19751

OF are obtained using probabilistic load flow. Then its CDF can be estimated using Gram Charlier expansion. To compare several CDF, a point with equal probability of them, are compared with each other. So the fitness function is considered as (19):

$$F_{fitness} = \mu + 3\sigma\left(2p_r - 1\right) \tag{19}$$

where, μ and σ are the expected value and standard deviation and p_r is area under PDF curve from 0 to x (20) that can adopt different values of (0, 1):

$$F(x) = P_r\left[X \le x\right] = \int_{-\infty}^{x} f_x(x)dx \tag{20}$$

For example, if p_r is assumed as equal to 0.75, for two CDF of OF shown in Fig. 2, the fitness of them will be 0.5096 and 0.7506. To make a more detailed comparison, optimization procedure is carried out three times considering values of 0, 0.5 and 1 for p_r which gives possible values for minimum, mean and maximum of best fitness. If p_r is 1, the obtained result will be the most probable one.

RESULTS AND DISCUSSION

The problem of the allocation and sizing of distributed generation has been solved for 33-bus distribution test system (Venkatesh *et al.*, 2004). The test system single line diagram and data are presented in the Appendix 1. The limits on control variables are shown in Appendix 2. All the evaluations were carried out with self-developed codes in MATLAB.

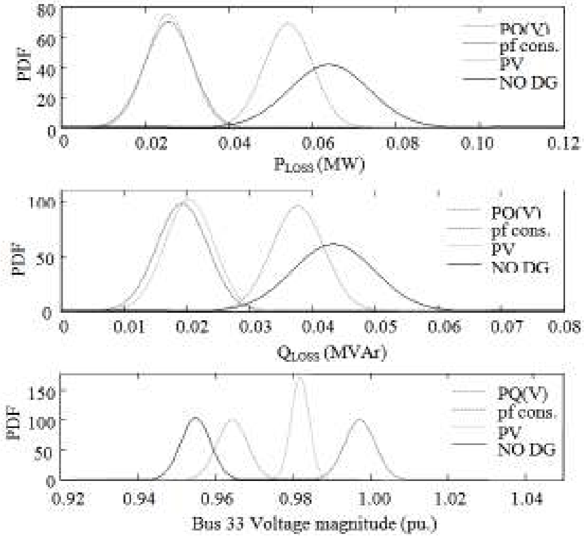

Fig. 3: Results of installation DG with optimal size on optimal location

The proposed approach is used for finding the optimal location and size of different types of DG described in section 2 and results are given in Table 3. The Expected value of $fitness_{OF}$ (p_r = 0.5) and $fitness_{OF}$ in the minimum probability (p_r = 0) and maximum probability (p_r = 1) are listed in this table. Also, active power obtained from different values for pr is shown in this Table. It is obvious that allocating DG of PV type at bus 6 with size 1.5939 (MW) has resulted better fitness for OF. If output active power of this DG is equal to 1.4394, the fitness of OF will be 0.19751 with very low probability. After finding the optimal location and size of each DG unit, the probabilistic load flow is done considering these values. PDFs of total active and reactive loss of network and voltage magnitude of bus 33 are obtained using Gram Charlier expansion represented in Fig. 3. For better comparison PDFs of these variables are shown in these figures when there was no DG in network. Active and reactive power loss reduction and voltage improvement with installing different types of DG is clearly visible. It can be seen that installing a DG of PV type causes the voltage magnitude occurs with less standard deviation around an expected value.

The proposed approach has been performed for determining the optimal location and size of 2 DG units and results are presented in Table 4. It is obvious that by installing two DG units including PV node type and constant power factor type the best fitness is obtained.

For illustrating capability of proposed probabilistic method, the optimal locations and capacities of DG units are also determined with deterministic approach. In this approach, the load power at each bus is considered constant and time invariant. The obtained

Table 4: Results of proposed algorithm for allocation two DG units

		Case 1		Case 2		Case 3	
	DG type	PQ (V)	PV	Pf const.	PV	PQ (V)	Pf const.
	Bus	7	30	30	12	3	30
$P_r = 1$	P (MW)	0.77700	0.562	0.67000	0.623	1.03400	0.838
	$\text{Max}_{\text{Prob}}\ \text{Fitness}_{\text{OF}}$	0.25296		0.13520		0.25058	
$P_r = 0.5$	P (MW)	0.77900	0.590	0.66900	0.595	0.98100	0.826
	Expected $\text{Fitness}_{\text{OF}}$	0.21194		0.10502		0.21647	
$P_r = 0$	P (MW)	0.78400	0.670	0.65500	0.573	0.87400	0.809
	$\text{Min}_{\text{Prob}}\ \text{Fitness}_{\text{OF}}$	0.15682		0.07410		0.18155	

Table 5: Results of deterministic method for allocation one DG unit in cases maximum load level and mean load level

DG type	PQ (V)	Pf const.	PV
Maximum load			
Bus	9	30	6
P (MW)	1.31800	1.42700	2.47600
$\text{Fitness}_{\text{OF}}$	0.62463	0.26161	0.25215
Mean load			
Bus	6	30	6
P (MW)	1.06800	0.91900	1.61700
$\text{Fitness}_{\text{OF}}$	0.59710	0.26309	0.25803

results from deterministic method with considering two level of load power (maximum level and mean level) for one DG unit and Two DG units are presented in Table 5 and 6, respectively. It is obvious that the obtained optimal locations for DG units in some cases are same results of probabilistic method whereas the determined capacities are different. For example, by assuming maximum load level at each bus, the optimum location and size for one DG unit of PQ (V) type will be bus 9 and 1.318 MW. Although the results of deterministic method are optimal in maximum or mean load power level separately, they aren't optimum for general case.

Assuming time invariant and constant loads causes the simulation of system to be less realistic. In addition, by installing and exploiting of DG units with values obtained using deterministic method, the calculated fitness may not be reachable. Hence, the probabilistic method results of case Pr = 1 is proposed for optimal locations and capacities of DG units. The obtained results considering time variant loads are more reliable than deterministic method results. Using these values for DG units exploiting causes the fitness of OF to be less than calculated fitness certainly.

CONCLUSION

In this study, to determine the optimal size and location of DG units in distribution systems, a probabilistic method was proposed. In this approach, time varying demands for voltage dependent loads were considered as uncertain variables of distribution system. It is assumed in the studied system that each load node consists of three components of load consumption that makes results be more realistic. Then, a probabilistic load flow was applied based on the Point Estimate Method (PEM). Active loss, reactive loss and voltage profile indices were considered as components of objective function and IWO algorithm was used to solve the optimization problem for different types of DG units. By comparing obtained fitness function for different types of DG, their effects could be evaluated in the network.

Table 6: Results of deterministic method for allocation two DG units in cases maximum level and mean level for constant loads

		Case 1		Case 2		Case 3	
	DG type	PQ (V)	PV	Pf const.	PV	PQ (V)	Pf const.
Maximum load	Bus	9	30	30	13	10	30
	P (MW)	0.81400	1.123	1.10800	0.845	0.73000	1.330
	Fitness of OF	0.20518		0.09765		0.21013	
Mean load	Bus	7	30	13	30	3	30
	P (MW)	0.76600	0.641	0.42600	0.788	1.01200	0.888
	Fitness of OF	0.22732		0.11684		0.23884	

Appendix 1:

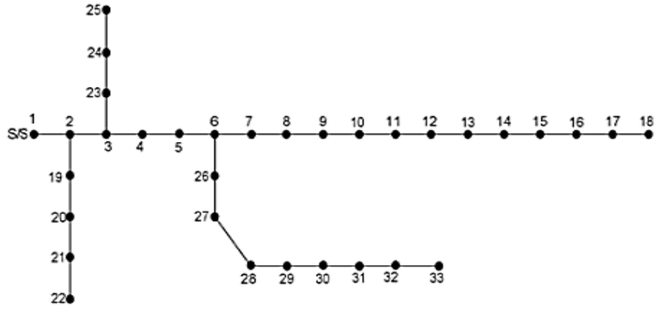

Fig. A1: Single line diagram of 33 bus test system

Table A1: Line data

From	To	R	X	From	To	R	X
1	2	0.0922	0.0477	17	18	0.7320	0.5740
2	3	0.4930	0.2511	2	19	0.1640	0.1565
3	4	0.3660	0.1864	19	20	1.5042	1.3554
4	5	0.3811	0.1941	20	21	0.4095	0.4784
5	6	0.8190	0.7070	21	22	0.7089	0.9373
6	7	0.1872	0.6188	3	23	0.4512	0.3083
7	8	1.7114	1.2351	24	25	0.8960	0.7011
8	9	1.0300	0.7400	6	26	0.2030	0.1034
9	10	1.0400	0.7400	26	27	0.2842	0.1447
10	11	0.1966	0.0650	27	28	1.0590	0.9337
11	12	0.3744	0.1238	28	29	0.8042	0.7006
12	13	1.4680	1.1550	29	30	0.5075	0.2585
13	14	0.5416	0.7129	30	31	0.9744	0.9630
14	15	0.5910	0.5260	31	32	0.3105	0.3619
15	16	0.7463	0.5450	32	33	0.3410	0.5302
16	17	1.2890	1.7210				

Table A2: Peak load value

Bus	P (kW)	Q (kVAr)	Bus	P (kW)	Q (kVAr)
2	100	60	19	90	40
3	90	40	20	90	40
4	120	80	21	90	40
5	60	30	22	90	40
6	60	20	23	90	50
7	200	100	24	420	200
8	200	100	25	420	200
9	60	20	27	60	25
10	60	20	28	60	20
11	45	30	29	120	70
12	60	35	30	200	600
13	60	35	31	150	70
14	120	80	32	210	100
15	60	10	33	60	40
16	60	20			

Appendix 2:

Voltage magnitude limit: 0.95-1.05 pu
Size of DG: 0.04-4MW
Power factor for constant pf type of DG: 0.95

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