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Research Article

The Electricity Portfolio Decision-making Model Based on the CVaR under Risk Conditions

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Abstract: With the gradual opening up of China's power sector, electricity investment is growing. Risk analysis should be applied to the investment optimization decisions. This study describes a CVaR-based investment optimization model, which established electricity portfolio decision-making model to optimize the ratio of investment decision-making and achieve the maximum yield of the total investment target between the various modes of generation. An example was given to verify the validity of the model based on the actual data. Based on simulation results of the example, the ratio of investment in a certain confidence level has been well optimized. The model can play purposes for overall investment risk reduction.

Keywords: Risk analysis, the electricity investment, the optimization model

INTRODUCTION

With Chinese rapid economic development, the society's demand for electricity is increasing. In order to meet the needs of economic and social development, we must increase the installed capacity of the various power generation (Alexander, 2006). With the rapid growth of China's power installed capacity, electricity market restructuring has taken new steps. At the present, the installed capacity of hydropower growth increased from 117 million kilowatts to 197 million kilowatts in the "Eleventh Five-Year" period. The wind power installed capacity reached 24.12 million kilowatts, ranking third in the world. The construction of nuclear power is significantly faster. The country in the construction of nuclear powerbase has reached 23, a total of 25.4 million kilowatts (Kemal and Ilhan, 2007). China has become the world's largest country in the construction of nuclear power scale. In view of the development of the electricity market, electricity investment becomes a hot investment area in recent years.

"How to choose the field of power generation investment in order to obtain the highest income" has become the most concern to investors when making investment decisions (Wenjie *et al*., 2010). When analyzing the power investment, investors will generally use the traditional methods of technical and economic feasibility analysis to calculate the investment yield. However, in the analysis process, investors tend to ignore the existence of the risk (Pun and Shiu, 2001). In Fire, water, wind, nuclear, solar and other energy carriers there are various risks, such as financial risks, climate risks, natural risks, policy risks. These risks have a direct impact on investment income after the last power plant is built.

Risk measurement refers to the estimated and measured against the scope and extent of the likelihood of specific risks or losses (Chen, 2011; Claro and Sousa, 2012). Only be accurately measured risk, it helps to choose an effective tool for the purpose of disposal risks and achieve the best risk management effectiveness with minimum expenses. As can be seen from the definition of risk, it is more true, accurate measurement to describe risk with the loss extent of the transaction than other indicators, such as income uncertainty, loss of uncertainties and other. The theory of VaR and CvaR of risk measurement indicators are based yields lower partial moment (Goh *et al*., 2012; Lim *et al*., 2011). The risk lower partial moment measurement theory has obvious advantages than variance theory. This theory making the "loss" as only one risk measurement factor reflects the real psychological feelings of investors (market members) to risk, in line with the behavioral science principles. In study (Schaumburg, 2012) a framework is introduced allowing us to apply nonparametric quantile regression to Value at Risk (VaR) prediction at any probability level of interest. A monotonized double kernel local linear estimator is used to estimate moderate (1%) conditional quantiles of index return distributions. For extreme (0.1%) quantiles, nonparametric quantile regression is combined with extreme value theory. In study (Yau *et al*., 2011) a two-stage Stochastic Integer Programming (SIP) model with a Conditional Value-at-Risk (CVaR) constraint to incorporate risk aversion is developed. Computational results are presented that demonstrates the CVaR

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approach and the results are compared with a corresponding expected cost minimization approach. The SIP model with CVaR will allow acceptance of contracts at lower prices compared to an approach based on a corresponding risk-neutral model as a hedge against uncertainty and mis-specified arbitrage. Foregoing considerations, based on the risk research results of the financial sector securities markets, the introduction of VaR and CVaR Risk Measurement in the electricity investment market, the pape established a electricity portfolio decision-making model. In the context of specific economic indicators, this model is able to find the best portfolio of power generation field with the maximum benefit rate target.

COMPREHENSIVE INTEGRATED MODELING APPROACH

Basic Model: VaR (Value at Risk) is a risk measure, refers to the maximum possible losses of a financial asset or portfolio of securities in a specific period of time in the future under normal market conditions and given confidence, called "risk value "or" VaR". Prob ($\Delta P>VaR$) = $1-\beta$, ΔP is the portfolio loss in " Δt" holding period, VaR is the value at risk under the confidence level β. VaR model, the use of financial theory and mathematical statistics theory, measures the market risk of an asset or portfolio with a single indicator (VaR value).

In order to overcome the deficiencies of VaR, the researchers invented CVaR Risk Measurement theory and applied to portfolio optimization. The CVaR theory derived from VaR, also known as the average excess of loss (Mean Execs Loss), refers to the conditional mean losses exceed VaR, reflects the suffer size of average potential losses exceed VaR values (Fig. 1), reflecting potential value-at-risk better than VaR.
Its main advantages are as follows:

- As VaR, CVaR also belong to the lower partial moment class risk measurement indicators, it does not require the same market factor must be for a normal distribution and makes only reducing the below loss as the target. Thus, in theory, it is considered superior to the variance
- CVaR meets transform volatility, orthogonal time and monotonous, which is a coherent risk measure
- CVaR is calculated by constructing functions into a convex optimization problem, easy handling in math
- By calculating CVaR, the corresponding the VaR value also can be obtained simultaneously at the same time, thereby against the risk of "double limit" supervision, more insurance than simple VaR. Figure 1 illustrates the CVaR and VaR position and relationship in the loss distribution

Portfolio optimization model based on CVaR: Portfolio risk refers to the risks associated with a number of asset investment. In general, the use of a number of assets to invest to reduce the investment risk, the magnitude of the risk reduction depends on the degree of correlation between the assets. Modern portfolio theory is a theory based on mathematics and statistics about the investment. In this theory, the risk of an individual asset investment yield variance is described. About the risk of the portfolio assets, you need to use the covariance of the yield to describe the degree of correlation between the assets.

CVaR is a consistency risk measurement indicator. Thus, the study shows that: CVaR can be applied to any distribution form of portfolio optimization. Assuming $\phi(x)$ is the risk function, $R(x)$is the revenue function,x is the decision vector, $\mu_1$is the risk factor parameters, ρ is the minimum return requirements, ω is the lowest risk limits, 0 <β<1 given. Suppose that the function $\varphi(x)$ and $R(x)$ is a function of the decision-making vector x.The following optimization problem can be shown:

$$min_x\, \phi(x) - \mu_1 R(x), x \in X, \mu_1 \geq 0 \quad (1)$$

$$min_x\, \phi(x), R(x) \geq \rho, x \in X \quad (2)$$

$$min_x\, -R(x), \phi(x) \leq \omega, x \in X \quad (3)$$

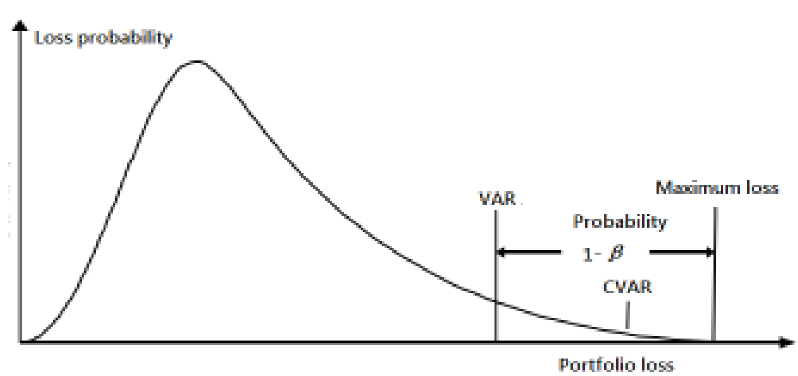

Fig. 1: The CVaR schematic diagram

The assumed constraints $R(x)\geq\rho$, $\varphi(x)\leq\omega$ with internal points, the transformation parameters μ_1, ρ and ω, formula (1), formula (2), formula (3) having the same efficient frontier. If $\varphi(x)$ is a convex function, $R(x)$ is a concave function, the feasible set X is a convex set, then the problem formula (1) and formula (2), formula (3) produce the same efficient frontier and have the same optimal solution X *when it is appropriate to select the parameters μ_1, ρ, ω.

In general optimization problems, the risk function $\varphi(x)$ can be replaced with CVaR risk function$\phi_{\beta(x)}$. The optimization problem (formula (2)) described the optimization model, which goal is the smallest CVaR. In the model, the change in the minimum expected return ρ can obtained mean-CVaR efficient frontier, which is equivalent to the objective function for maximize revenue while meeting certain risk constraints. Therefore, in the optimization problem (formula (2)) this model can swap the CVaR function with the expected return. This is the form of optimization problem (formula (3)): minimize negative revenue function, at the same time meeting the CVaR constraints.

It is proved that$F_\beta(x,\alpha)$ can replace the optimization problem hazard function $\emptyset_\beta(X)$.The study (Glasserman *et al*., 2002) gives two theorems to prove that optimization problems formula (1) and formula (3) have the same solution with the following optimization problem formula (4) and formula (5):

$$min_x -R(x), F_\beta(x,\alpha)\leq\omega, x\in X, \tag{4}$$

$$min_x F_\beta(x,\alpha)-\mu_1 R(x), x\in X, \mu_1\geq 0 \tag{5}$$

The study (Glasserman *et al*., 2002) proved that assuming the investment return rate for a normal distribution, the mean-CVaR model and mean - variance model have the same resulting efficient frontier:

$$\alpha+\frac{1}{m(1-\beta)}\sum_{k=0}^{m} Z_k \tag{6}$$

Assume that $f(x, y)$ is the loss function, then $R(x)$ is the revenue function. In the $\widetilde{F_\beta}$(x,α), the introduction of dummy variables (Z_k=1, 2, m), then the function $\widetilde{F_\beta}$(x, α) is replaced by the linear function and the linear constraint ($Z_k\geq 0$, $Z_k\geq f(x, y^k)-\alpha$). Based on optimization problem (P2 and P3) form the mean-CVaRmodel as follows:

- In order to minimize the risk of CVaR objective and meet the expected revenue for the constraints, the mean-CVaR optimization model as follows:

$$min_{(x,\ \alpha)\in(X,\ R)}\ \tilde{F}_\beta\ (x,\alpha,z)\ =min\Big[\alpha+ 1m1-\beta k=0mZk \tag{7}$$

$$s.t. x_i\geq 0, (i=1,2,\dots N), \textstyle\sum_{k=0}^{N} x_i=1 \tag{8}$$

$$R(x\geq\rho) \tag{9}$$

$$Z_k\geq 0 \tag{10}$$

$$Z_k\geq f(x,y^k)-\alpha \tag{11}$$

- In order to maximize the expected revenue (minimum loss) and meet CVaR constraints, the mean-CVaR optimization model as follows:

$$min_{x,\alpha} -R(x) \tag{12}$$

$$s.t. \alpha+\frac{1}{m(1-\beta)}\sum_{k=0}^{m} Z_k \tag{13}$$

$$x_i\geq 0, (i=1,2,\dots N), \textstyle\sum_{i=1}^{N} x_i=1 \tag{14}$$

$$Z_k\geq f(x,y^k)-\alpha \tag{15}$$

$$Z_k\geq 0 \tag{16}$$

CVaR-based electricity portfolio optimization allocation model: High-yield power project development is always accompanied by high risk. The more high-risks what power development projects have, the more the successful development what the project can get a monopoly on the market. Low risk can only bring low-income. To more than for an income, you must assume more risks. The investment in the field of electricity production tends to have high capital, long time, long payback period characteristics. Due to the high risk of a variety of new energy power generation network, so when investors choose to invest in the field of power generation there are more concerns. In order to diversify risk and reduce the risk of return, investors tend to select the portfolio. The goal of the investor is in the investment and the premise of low risk, to maximize the total yield of the portfolio. Problems about optimal allocation of the total investment in the construction of a number of power generation projects, that need to be studied include: Considering the conditions of the power generation cost and market fuel prices factors such, how to optimize the total investment allocation in fire, water, wind, nuclear, solar and other power projects, with high total expected income and low level risk after the project completion; Or in a certain level of risk (indicators), to improve the total revenue with consider raising revenue and reducing risks.

In the study, the behavior that investors determine investment ratio in the field of multiple power generation, is called portfolio strategy. The study establishes a mean-CVaR optimization allocation model of the total portfolio investment, by the risk measurement indicators of CVaR (Condition value at risk), considering the risks and the expected rate of return. The application of the model can reasonably prorated the total amount of investment in several power projects, guarantee expect the premise of the yield with the minimum CVaR risk, with the premise of the minimum CVaR risk and certain expected rate. With the background of four to fire, water, wind, nuclear power generation mode, the example calculates the efficient frontier and the distribution ratio of the total investment and provides a new idea for the portfolio strategy of power suppliers.

Assume that $X^T = (x_1, x_2, \ldots, x_N) \in X$ is a portfolio of investors, including component $x_i$represents the proportion of the total amount of investment in power generation projects i .Satisfy the condition:

$$x_i \geq 0, (i = 1,2, \ldots N), \sum_{i=1}^{N} x_i = 1 \quad (17)$$

Assume that y^i is the i-th power project yield, then multivariate random variable$y^T = (y_1, y_2, \ldots, y_N)$is investors of the portfolio yield vector. The mean vector μ of y and covariance matrix Σ are as follows:

$$\mu^T = (\mu_1, \mu_2, \ldots, \mu_N), \Sigma = (\sigma_{ij})_{N\times N} \quad (18)$$

Defined *R (x, y)* for the portfolio revenue function, the combining gain Mean *E [R (x, y)]* and variance σ^2[R (x, y)] are as follows:

$$E[R(x, y)] = E(r_x) = x^T, \quad \sigma^2[R(x, y)] = \sigma^2(r_x) = x^T \Sigma x \quad (19)$$

Portfolio investment loss function f (x, y) = -R (x, y), which can be given by the following formula:

$$f(x, y) = -(x_1 y_1 + x_2 y_2 + \cdots + x_n y_n) = -x^T y \quad (20)$$

CVaR formula as follows:

$$F_\beta(x, \alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in R^m} [f(x, y) - \alpha]^+ p(y)\, dy \quad (21)$$

$[f(x, y) - \alpha]^+$ represents max (0, *f* (*x*, *y*)-*α*). Formula substitutions can obtain *F_β* (*x*, *α*) in the formas follows:

$$F_\beta(x, \alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in R^m} [-x^T y - \alpha]^+ p(y)\, dy \quad (22)$$

Taken market yields y sample values $y_1, y_2, \ldots, y_q$, the estimation of the above equation is as follows:

$$\tilde{F}_\beta(x, \alpha) = \alpha + \frac{1}{q(1-\beta)} [-x^T y^k - \alpha]^+ \quad (23)$$

A dummy variable $z_k (k = 1,2, \ldots, q)$, so $z_k = [-x^T y^k - \alpha]^+$, $k = 1,2, \ldots, q$, and $z_k \geq 0, z_k \geq -x^T y^k - \alpha$.So minimize CVaR risk investors portfolio optimization model:

Table 1: The investment income distribution of four kinds power generation

Proportion of revenue is	Thermal power	Hydropower	Wind power	Nuclear power
Average value (μ_{yi})	0.231	0.178	0.072	0.105
Standard deviation (σ_{yi})	0.376	0.196	0.053	0.018

$$\min_{(x, \alpha) \in (X, R)} \tilde{F}_\beta(x, \alpha, z) = \min \Big[\alpha +$$
1q1−βk=1qZk
(24)

$$s.t.\, x_i \geq 0, (i = 1,2, \ldots N), \sum_{i=1}^{N} x_i = 1 \quad (25)$$

$$x^T \mu \geq E \quad (26)$$

$$Z_k \geq 0 \quad (27)$$

$$z_k \geq -x^T y^k - \alpha \quad (28)$$

The meaning of the constraint formula (26) is that a combination of expected return must be greater than the income lower bound *E*(*0≤E≤*1). The above model is to seek risk minimization under fixed expected revenue levels.

In summary, based on CVaR investors portfolio investment model can be described as follows: to seek the optimal portfolio, so that in the future a certain period of time (usually a year) in the probability of a given level of confidence, guarantee annual expectations power generation operations revenue constraints, so that investors may suffer losses CVaR minimum.

EMPIRICAL ANALYSIS

The examples selected four investment field of thermal power, hydropower, wind power, nuclear power. Examples collected history data for the average yield and the standard deviation yield data of these areas in China from 2001 to 2010 year, which are shown in Table 1.

Normally distributed random produced a yield of 100 groups of samples: $y^k = (y_1^k, y_2^k, y_3^k, y_4^k)$, where k = 1,2, ..., 100, confidence level β = 0.95 and β = 0.99, expected rate of return lower limit of *E=0.20* and *E* =0.30. Set X_1, X_2, X_3, X_4 be the investment allocation ratio for fire, water, wind, nuclear construction project.

Let M_n^kbe the random investment ratio for the sample randomized. So the total investment yield of sample randomized"E"can be rewritten as:

$$E = M_1^k y_1^k + M_2^k y_2^k + M_3^k y_3^k + M_4^k y_4^k,$$

The total investment optimization yields "F" can be rewritten as:

$$F = y_1^k X_1^k + y_2^k X_2^k + y_3^k X_3^k + y_4^k X_4^k,$$

Table 2: The results of the investment optimize proportion

Expected revenue limit is	Confidence level is	X_1	X_2	X_3	X_4	VaR	CVaR
$E = 0.20$	$\beta = 0.95$	0.151	0.287	0.186	0.45	0.075	0.145
	$\beta = 0.99$	0.208	0.143	0.236	0.412	0.172	0.153
$E = 0.30$	$\beta = 0.95$	0.242	0.246	0.382	0.002	0.177	0.248
	$\beta = 0.99$	0.399	0.324	0.196	0.136	0.303	0.307

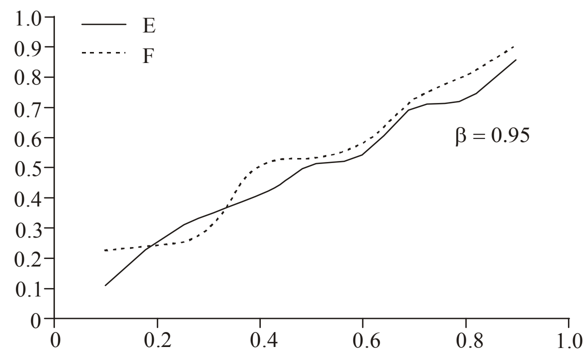

Fig. 2: The comparative results of overall investment yield ($\beta = 0.95$)

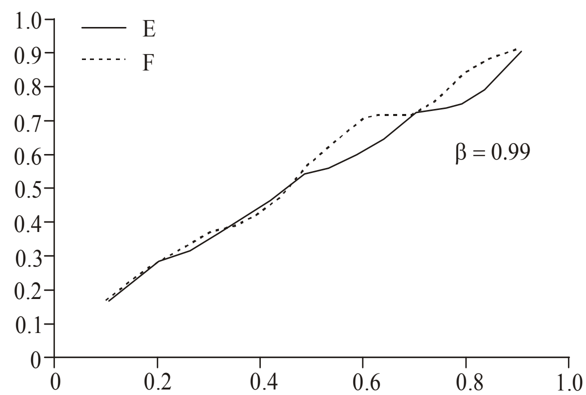

Fig. 3: The comparative results of overall investment yield ($\beta = 0.99$)

where,

$$X_1^k + X_2^k + X_2^k + X_2^k = 1, M_1^k + M_2^k + M_2^k + M_2^k = 1$$

Respectively using matlab software, the optimum results are computed as shown in Table 2, including: x_1, x_2, x_3 and x_4 for fire, water, wind, nuclear construction project investment allocation ratio. The simulation results of all samples are shown in Fig. 2 and 3 in the figure. It is clear that the overall yield indicators of the optimization model is better than the random sample yields. The results also proved the superiority of the model.

CONCLUSION

Portfolio optimization is necessary for the power construction projects based on the characteristics of investment management status of power projects. WithCVaR as risk measurement indicators, the established tender combination of mean-CVaR model CVaR risk minimization. Because the CVaR's target is to reduce below the loss and thus this model is suitable for the electricity portfolio loss protection. It is necessary for the power industry funds with high-risk characteristics that risk indicators are introduced to the investment evaluation in the model. Based on simulation results of the example, the ratio of investment in a certain confidence level has been well optimized. The model can play purposes for overall investment risk reduction. CVaR Risk Measurement-based power investor portfolio investment model can truly reflect the essential characteristics of the market risk faced by the electricity investors, which provides new tools and ideas for the investor's investment decision-making and risk assessment.

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