

Research Article

Mathematical Models for in-Plane Moduli of Honeycomb Structures-A Review

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Abstract: Honeycomb structures are light weight cellular structures having high strength to weight ratio with enormous applications in aerospace industry, high speed automobiles, computers and other electronics equipment bodies and recently as flexible structures and mechanisms. In this paper a review of mathematical models for stress strain behaviour of two dimensional honeycomb structures is presented. As proposed by different authors, expressions for in-plane Elastic Moduli and shear Modulus are presented and compared on same scale dimensions. In addition to that, effects of number of unit cells on effective in plane and out of plane Moduli of the testing specimen for regular honeycombs and open and closed cell foams, are also reviewed.

Keywords: Auxetic honeycombs, flexure model, honeycomb structures, hinging model, stretching model

INTRODUCTION

Honeycomb cellular materials are now widely used in light weight construction especially in aircrafts, missiles, space vehicles, compliant mechanisms and flexible structures and high speed automobiles. Usually these materials are used as sandwich panels. The typical structural sandwich panel or shell consists of three layers. Two relatively thin, high-density and high strength face sheets are adhesively bonded to a soft, light weight, and relatively thicker core as shown in Fig. 1. The core carries the transverse shear load and keeps relative distance between the face sheets, while face sheets carry in plane loads and bending moment. Honeycomb sandwich structure possesses high specific strength and specific rigidity and it can resist high wind pressure, reduce vibrations, isolate sound, maintain temperature, retard fire and possesses less density etc. In addition, there is no need of large area riveting which alleviates stress concentration and thus greatly enhances the fatigue strength.

Tom (1997) as well as Hoffman (1958) report some earlier examples, the Mosquito aircraft is considered as the first industrial scale application. Initially honeycombs were used as sandwich structures for high out-of-plane stiffness and as low density impact energy absorption materials (Zhao and Grrard, 1998; Vinson, 1999; Khan, 2006). The in-plane properties are two to three orders less in magnitude than that of highly stiff and strong out-of-plane properties. For this reason in-plane properties were considered to be limited for design applications. But in recent years researchers are motivated to use the lower in-plane

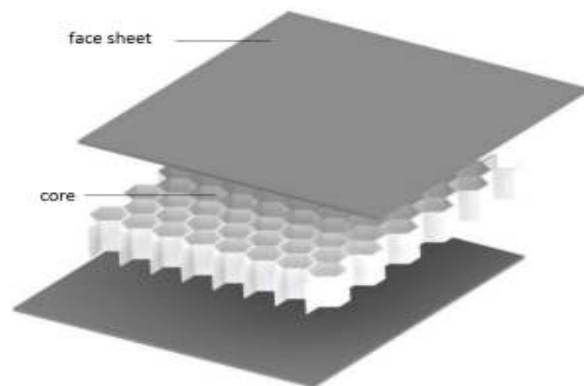


Fig. 1: A typical sandwich panel with regular hexagonal honeycomb core

stiffness for designing flexible meso-structures for applications that need high deformation for given loads. Such as morphing air-craft skins e.g. (Olympio and Gandhi, 2007; Bubert *et al.*, 2008; Jaehyung *et al.*, 2010) proposed high shear strength and strain honeycomb structures, (Kwangwon Kim *et al.*, 2012) proposed FEA models of modified Auxetic honeycombs as high strain flexures.

The computational time for finite element honeycomb sandwich models increase rapidly as the number of cells in the core increase, that also require a more sophisticated computer. Therefore, to attain efficiency in numerical analysis, the honeycomb core is usually replaced with an equivalent continuum model and analysed in terms of their effective properties rather than by consideration of their real cellular structure, by

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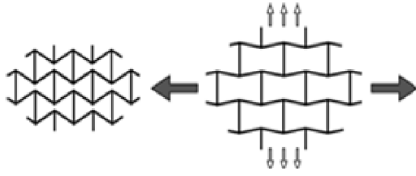


Fig. 2a: 2D Re-entrant honeycomb, (a) Undeformed, (b) Deformed (Yanping and Hong, 2010)

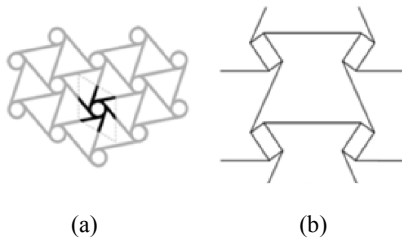


Fig. 2b: Chiral honeycombs, (a) Formed with the same chiral units, (b) Formed with symmetrical chiral units (Grima *et al.*, 2008b)

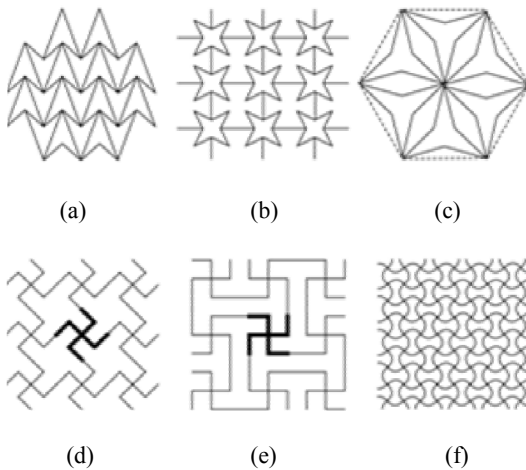


Fig. 2c: Re-entrant structures, (a) Double arrowhead structure, (b) Star honeycomb structure, (c) Structurally hexagonal re-entrant honeycomb, (d) Structure formed from lozenge grids, (e) Structure formed from square grids, (f) Structure formed from sinusoidal ligaments (Yanping and Hong, 2010)

application of an appropriate composite or sandwich shell theory as given by Habip (1964), Lui and Habip (1965) and Frostig (1998). The early analytical studies on effective properties have been reported in 1950's such as the studies of Garrard (1949), Charles and Ralph (1951) and Horvay (1952) on the overall elastic properties of sandwich plates in the assembled situation as well as the studies of Akasaka and Takagishi (1959), Hoffman (1958) and Kelsey *et al.* (1958) on the effective transverse shear moduli and the in-plane Poisson's ratio of hexagonal honeycomb cores appeared. All of these studies used a homogenization procedure, which is based on a redistribution of the stresses on the external surfaces of a representative

volume element. In most cases, only the transverse shear properties of the core are considered while the in-plane properties are neglected.

Most of the early researchers focused on only regular hexagonal honeycombs for sandwich panels as a core material. But since last two decades due to the availability of faster and more sophisticated techniques new cell geometries are developed to meet the in-plane mechanical properties requirements. Yanping and Hong (2010), made a comprehensive review of Auxetic cellular structures, those which have negative Poisson's Ratio, as show in Fig. 2a to c some proposed geometries such as Theocaris *et al.* (1997) proposed star-shaped microstructure, Chiral honeycombs by Grima *et al.* (2008b), others like Larsen *et al.* (1997), Smith *et al.* (2000) and Gaspar *et al.* (2005). Triangular kagome and diamond cell honeycombs cell are known to be extension dominated cell structures which is good for high modulus structural design. On the other hand square and hexagonal cell honeycombs are known to be bending dominated structures which are good for flexible flexure design. Kwangwon Kim *et al.* (2012) and Jaehyung *et al.* (2010) also proposed modified cell shape. But most of these proposed cell geometries have limitations of manufacturing and load bearing capacities and not as much use full practically.

This article presents a review on mathematical models proposed by different authors for in-plane and out-of plane mechanical properties of honeycomb structures. Emphases are focused on in-plane Effective Elastic Moduli E_1 and E_2 and Effective shear modulus G_{12} . Using cell wall bending model which is called Cellular Material Theory (CMT), Gibson and Michael (1988) determined the in plane Moduli by considering cell walls as flexures that is fixed at one end and guided at other end. Masters and Evans (1996) modified the model by introducing three mechanisms of flexing, hinging and stretching. Hohe and Becker (2002) presented a comprehensive review of the homogenization methods as well as material models but did not report studies of the effect of the wall curvature at the intersection points. Balawi and Abot (2007b) and Balawi and Abot (2008) proposed a refined model for commercial honeycombs, with double wall thickness in vertical direction and some curvature in the vicinity of the intersection points of the hexagons.

GIBSON AND ASHBY'S MODEL

As proposed by Abd El-Sayed *et al.* (1979) and Gibson and Michael (1988) determined the in plane Moduli by considering cell walls as flexures that is fixed at one end and guided at other end. The in plane Tensile and compressive deformation mechanisms for three different types of materials were studied by progressive loading and photographing the structures. Three types of materials, an elasto-metric honeycomb (rubber), elastic plastic honeycomb (a metal) and an elastic-brittle (ceramics) were studied. For compression

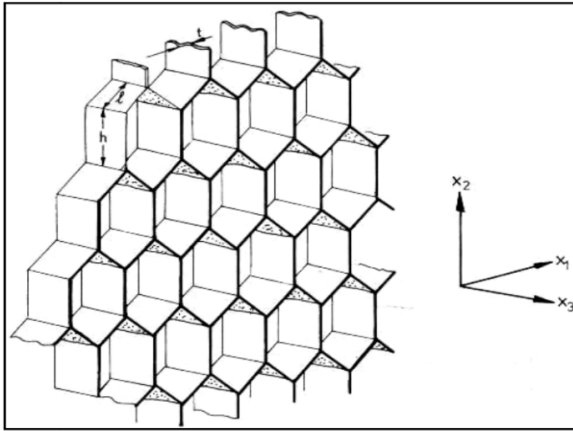


Fig. 3: A honeycomb with hexagonal cells. The in-plane properties are in X_1 - X_2 plane while responses to the loads applied in X_3 direction are referred as out-of-plane properties (Gibson and Michael, 1988)

all showed linear elastic behaviour, followed by a plateau stress value, and finally a steep increase in stress. In tension elasto-metric honeycombs don't show any plateau and fail after elastic range, plastic honeycombs show a similar behaviour as in compression while brittle fails abruptly at a tension which is lower than the true crushing strength.

In-plane moduli: If honeycombs are regular hexagon, having all sides equal with angles 120° internal angles, and the cell walls are of same thickness, these are called regular honeycombs and their in-plane properties are isotropic. Such a structure requires only two independent elastic Moduli, a Young's Modulus E and a Shear Modulus G and a Poisson's ratio ν . But when the hexagon is irregular and having different wall thickness, then it requires four in plane Moduli, ($E_1, E_2, G_{12}, \nu_{12}$ where ν is Poisson's ratio). All the other parameters considered are shown in Fig. 3.

The honeycombs considered for analysis have low density so that (t/l) is lower. The relative density is given by:

$$\frac{\rho^*}{\rho} = \frac{t/l(\frac{h}{l}+2)}{2\cos\theta(\frac{h}{l}+\sin\theta)} \quad (1)$$

The in-plane loading mechanisms for Young's Moduli E_1, E_2 is shown in Fig. 4a and for Shear Modulus G_{12} is shown in Fig. 4b.

The E_1, E_2, G_{12} and Poisson's ratios ν_{12} & ν_{21} are calculated as following expressions:

$$\frac{E_1}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{\sin^2\theta(\frac{h}{l}+\sin\theta)} \quad (2)$$

$$\frac{E_2}{E_s} = \left(\frac{t}{l}\right)^3 \frac{(\frac{h}{l}+\sin\theta)}{\cos^3\theta} \quad (3)$$

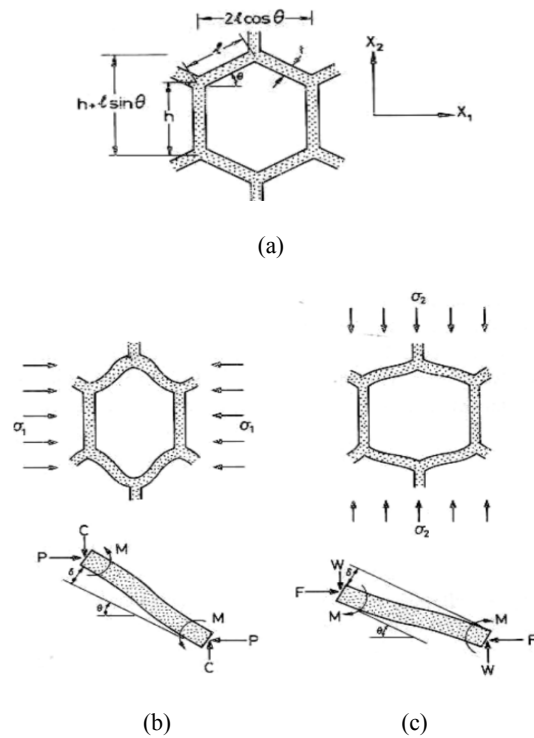


Fig. 4a: Deformation by cell wall bending phenomenon, giving linear-elastic extension or compression of the honeycomb, (a) the un-deformed honeycomb, (b) and (c) the bending caused by loads in X_1 and X_2 direction (Gibson and Michael, 1988)

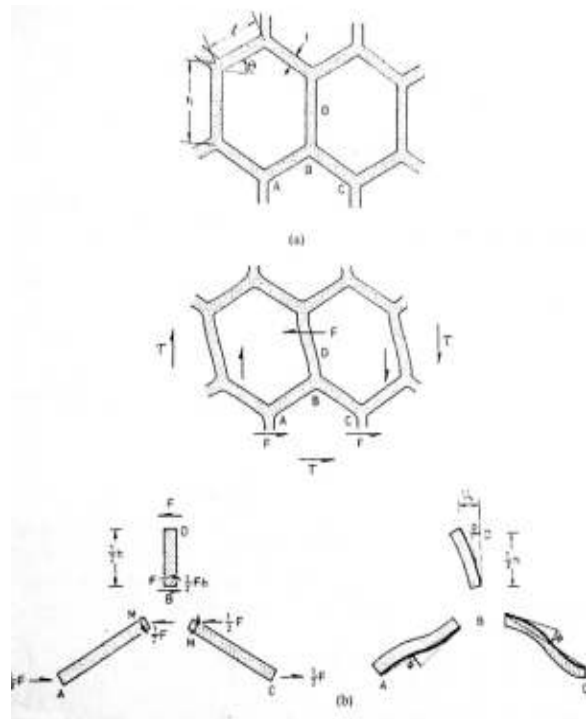


Fig. 4b: Un-deformed honeycomb, (b) Linear elastic shear of honeycombs under shear stress (Gibson and Michael, 1988)

$$v_{12} = \frac{\cos^2\theta}{\left(\frac{h}{t} + \sin\theta\right) \sin\theta} \quad (4)$$

$$\frac{G_{12}}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\frac{h}{t} + \sin\theta}{\left(\frac{h}{t}\right)^2 (1 + 2\frac{h}{t}) \cos\theta} \quad (5)$$

And $v_{12} = 1/v_{21}$. Where, E_s is the modulus of the cell wall material. Gibson and Michael (1988), in addition to bending proposed that there exists axial and shearing stresses that cause axial and shear deformations, but were considered as negligible for small values of t/l . The non-linear beam-column effect was also neglected calculating the elastic Moduli in limit of small deflection. In case of shear the deformation is considered entirely due to bending of vertical cell walls BD and their rotation about joints B, no stretching of cell walls and change in relative distance between the joints was considered.

Gibson and Michael (1988) also introduced the hexagonal core properties for double wall thickness in the vertical direction. As their analysis considers no deformation in vertical walls so there is no effect of double wall on in plane properties and hexagonal honeycombs will still be transversely isotropic.

Out of plane Moduli: For out of plane loading the walls compress or extend, and the moduli are much larger than the in-plane moduli. The Young's Modulus in X_3 direction is calculated as:

$$\frac{E_3}{E_s} = \frac{t}{l} \left\{ \frac{t/l(\frac{h}{t}+2)}{2\cos\theta(\frac{h}{t}+\sin\theta)} \right\} = \frac{\rho^*}{\rho} \cong \frac{t}{l} \quad (6)$$

The Poisson's ratios v_{31} & v_{32} are simply equal solid itself v_s .

The exact value of shear moduli G_{13} and G_{23} can only be found by numerical solutions, because of non-uniform deformation of each cell due to constraints applied by the neighbouring cells also the initially plane cells don't remain plane. However upper and lower bounds were found using "Theorem of minimum Potential Energy" and "Theorem of minimum complementary energy" respectively and their values are found to be same for upper and lower bounds:

$$\frac{G_{13}}{G_s} \leq (t/l) \frac{\cos\theta}{\frac{h}{t} + \sin\theta} \quad (7)$$

$$\frac{G_{23}}{G_s} \leq \frac{1}{2} (t/l) \frac{\frac{h}{t} + 2\sin^2\theta}{\left(\frac{h}{t} + \sin\theta\right) \cos\theta} \quad (8)$$

The out of plane moduli depend linearly with density (t/l) , while in-plane moduli scale as $(t/l)^3$. In general out of plane moduli are greater than those in-plane by a factor of $(t/l)^2$.

Nkansah *et al.* (1994) also used the flexure model for molecular structures. In these structures stretching

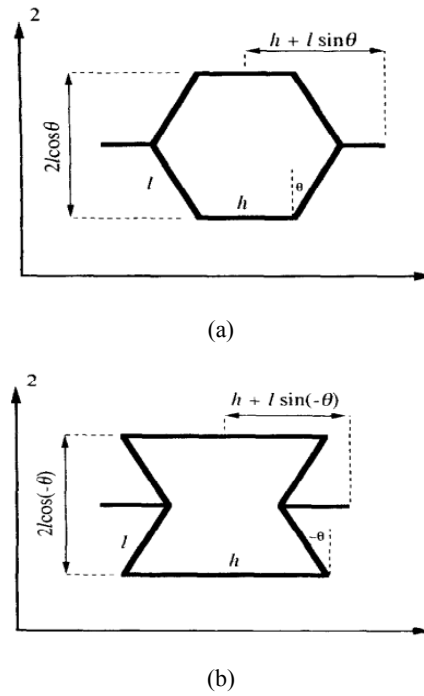


Fig. 5: The cell geometry and co-ordinate system for, (a) regular hexagonal honeycombs and (b) re-entrant cells (Masters and Evans, 1996)

of molecular chains tends to increase the longitudinal deformation at the expense of transverse thus reducing the Poisons ratio, which indicate that flexing is dominant mechanism in these structures but there is also some contribution by hinging and stretching.

I. G. MASTERS AND K. E. EVANS MODEL

Masters and Evans (1996) modified the model by introducing three mechanisms of flexing, hinging and stretching. The force constants ($F = K_i * \delta$) relate the displacement of the cell walls of a honeycomb to the applied force which causes it. For all three mechanisms the force constants were defined and then using these constants expressions for E_1 , E_2 , G_{12} , v_{21} and v_{12} for all mechanisms were calculated separately and also combined to give a general model. Thickness of all the cell walls was considered same, also no bonding was present between the cell walls. The direction 1 is parallel to h , and direction 2 is perpendicular to h . All the other parameters are shown in Fig. 5.

Flexure model: As proposed by Gibson and Michael (1988) the cell walls act like flexures and modelled as cantilever beams fixed at one end and guided at other end. The moduli are dependent on the ratio of cell wall thickness to length $(t/l)^3$ as described in equations below. Flexure force constant K_f :

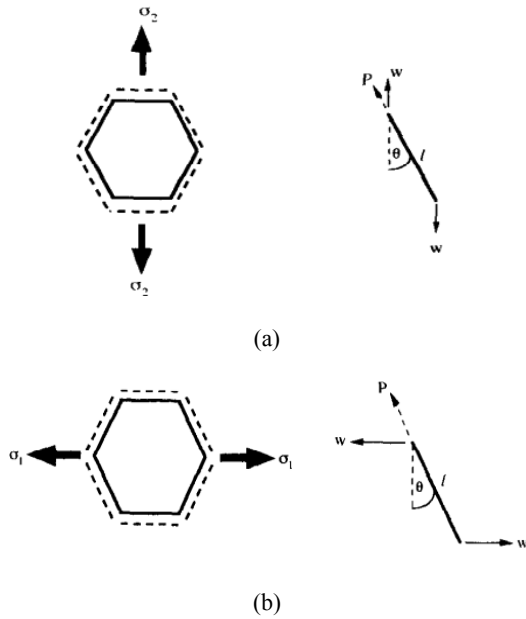


Fig. 6: Hexagonal cell deforming by stretching of the cell walls due to tensile load applied in, (a) direction 2 and (b) direction 1. Forces acting on the walls length 1 are shown on the right (Masters and Evans, 1996)

$$K_f = \frac{E_s b t^3}{l^3} \tag{9}$$

$$E_1 = \frac{K_f (\frac{h}{l} + \sin\theta)}{b \cos^3 \theta} \tag{10}$$

$$E_2 = \frac{K_f \cos \theta}{b \sin^2 \theta (\frac{h}{l} + \sin\theta)} \tag{11}$$

$$G_{12} = \frac{K_f (\frac{h}{l} + \sin\theta)}{b (\frac{h}{l})^2 (1 + \frac{2h}{l}) \cos \theta} \tag{12}$$

$$v_{21} = \frac{\cos^2 \theta}{(\frac{h}{l} + \sin\theta) \sin \theta} \tag{13}$$

And $v_{12} = 1/v_{21}$ where, E_s is Young's Modulus of the cell wall material, b is width of structure, t is thickness, and l is length of wall. The Young's moduli and Poisson's ratio expressions for the flexure model comply with the reciprocal relation $E_1 \cdot v_{21} = E_2 \cdot v_{12}$.

Stretching model: In this model it was assumed that cell walls are like shock absorbers that only stretch along their length and no change in angle or rotation as shown in Fig. 6. Stretching force constant K_s is given as:

$$K_s = \frac{E_s b t}{h} \tag{14}$$

The elastic constants derived are given by following equations:

$$E_2 = \frac{K_s}{b \cos \theta (\frac{h}{l} + \sin\theta)} \tag{15}$$

$$v_{21} = - \left(\frac{\sin\theta}{\frac{h}{l} + \sin\theta} \right) \tag{16}$$

$$E_1 = \frac{K_s (\frac{h}{l} + \sin\theta)}{b \cos \theta (2\frac{h}{l} + \sin^2\theta)} \tag{17}$$

$$v_{12} = - \left(\frac{\sin\theta (\frac{h}{l} + \sin\theta)}{2\frac{h}{l} + \sin^2\theta} \right) \tag{18}$$

$$G_{12} = \frac{K_s}{b} \left[\frac{l \cos\theta (h + l \sin\theta)}{(1 \cos^2 \theta (h + l \sin \theta) \sin \theta)^2} \right] \tag{19}$$

where, E_1 and E_2 are the Young's moduli in direction 1 and 2, v_{12} and v_{21} are poisson's ratios and G_{12} is shear modulus.

Hinging model: This model considers the cells as six bar compliant mechanisms (the lumped compliance mechanisms where links are theoretically rigid along the lengths and rotates due to living hinge mechanism). Like a rotational joint, the walls are stiff both in transverse and axial directions and Elastic hinges at the joints enable the cell to deform when a load is applied. Here no change in length or bending of whole cell wall, the cell deforms only due to change in angle between walls. The actual mechanism by which hinging occurs can be envisaged as one of two processes; global shear (which was considered important only for small-celled foams and molecular networks but is unrealistic for the macro-networks, like honeycombs) or local bending, Fig. 7a and b. Hinging force constant K_h is given as;

$$K_h = \frac{E_s b t^3}{6l^2 q} \tag{14}$$

where, q is length of short imaginary hinge usually 1/10th of the cell wall length. The other constants were derived as followed:

$$E_2 = \frac{K_h \cos\theta}{b \sin^2 \theta (\frac{h}{l} + \sin\theta)} \tag{15}$$

$$E_1 = \frac{K_h (\frac{h}{l} + \sin\theta)}{b \sin^3 \theta} \tag{16}$$

$$G_{12} = \frac{K_h}{b \cos \theta} \left[\frac{Cl(h+l\sin\theta)}{Ch(h+l \sin \theta) + l(2l - Ch \sin \theta)} \right] \tag{17}$$

$$v_{21} = \frac{\cos^2 \theta}{(\frac{h}{l} + \sin\theta) \sin \theta} \tag{18}$$

$$v_{12} = \frac{(\frac{h}{l} + \sin\theta) \sin \theta}{\cos^2 \theta} \tag{19}$$

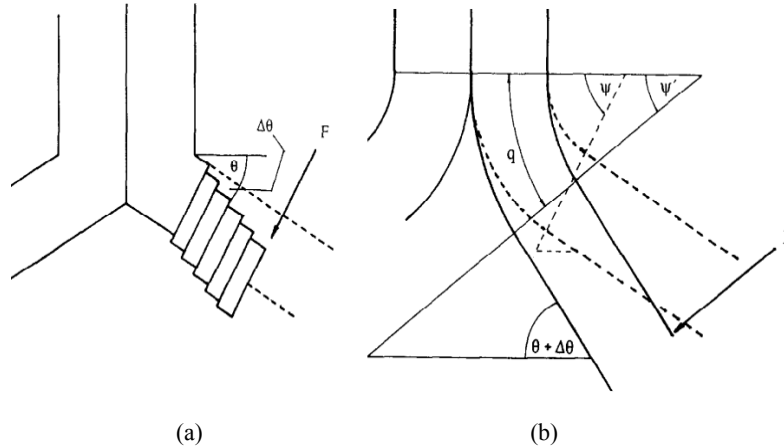


Fig. 7a: Hexagonal cell deforming by hinging, (a) global shear, (b) local bending (Masters and Evans, 1996)

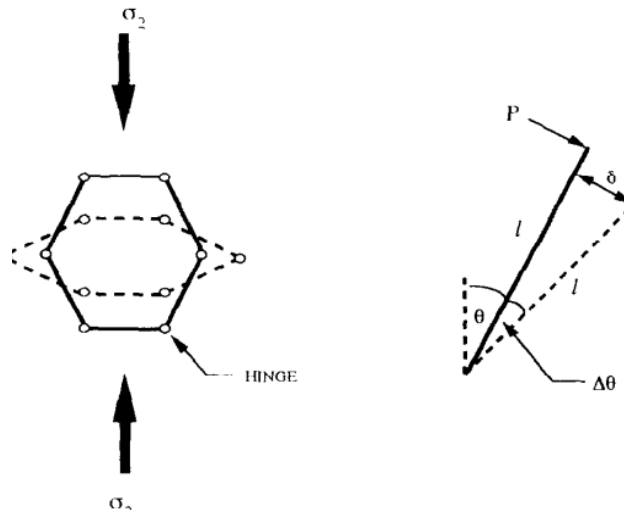


Fig. 7b: Hexagonal cell deforming by hinging due to a compressive stress applied in direction 2 also showing the forces acting on a wall length l (Masters and Evans, 1996)

General model: By summing the strains calculated by all three models, and dividing by total stress component in that direction, elastic moduli can be calculated as:

$$E_1 = \frac{1}{b \cos \theta \left[\frac{\cos^2 \theta}{K_f} + \frac{\cos^2 \theta}{K_h} + \frac{\left(\frac{2h}{l} + \sin \theta\right)}{K_s} \right]} \quad (20)$$

$$\frac{h}{l} + \sin \theta$$

$$E_2 = \frac{1}{b \left(\frac{h}{l} + \sin \theta\right) \left[\frac{\sin^2 \theta}{K_f \cos \theta} + \frac{\sin^2 \theta}{K_h \cos \theta} + \frac{\cos \theta}{K_s} \right]} \quad (21)$$

$$v_{21} = \frac{-\sin \theta \cos \theta \left[\frac{1}{K_f} + \frac{1}{K_h} - \frac{1}{K_s} \right]}{\left(\frac{h}{l} + \sin \theta\right) \left[\frac{\sin^2 \theta}{K_f \cos \theta} + \frac{\sin^2 \theta}{K_h \cos \theta} + \frac{\cos \theta}{K_s} \right]} \quad (22)$$

$$v_{12} = \frac{-\sin \theta \left(\frac{h}{l} + \sin \theta\right) \left[-\frac{1}{K_f} - \frac{1}{K_h} + \frac{1}{K_s} \right]}{\left[\frac{\cos^2 \theta}{K_f} + \frac{\cos^2 \theta}{K_h} + \frac{\left(\frac{2h}{l} + \sin \theta\right)}{K_s} \right]} \quad (23)$$

The expression for shear modulus is given as:

$$G_{12} = 1 / \left[\frac{\left\{ \frac{bh^2(l+2h)\cos\theta}{k_f l^2(h+l\sin\theta)} \right\} + \frac{1}{k_s} \left\{ \frac{Ch^2+2l^2}{Cl(h+l\sin\theta)} \right\}}{b \cos \theta} + \frac{b \left\{ l \cos^2 \theta + (h+l\sin\theta)\sin\theta \right\}}{K_s} \left\{ \frac{\cos\theta}{(h+l\sin\theta)} + \frac{\sin\theta}{\cos\theta} \right\} \right] \quad (24)$$

In the above equation if we assume that there is no hinging and stretching phenomenon this equation reduced to that of flexure model as proposed by Gibson and Michael (1988).

It is observed in these models that higher value of the force constant, lesser will be the contribution of that model in the overall cell deformation. With increase in thickness the force constant value increases, showing less deformation that validates the correlation of Gibson and Michael (1988) at lower (t/l) . Experimental tests

were performed by Masters and Evans (1993) and Masters and Evans (1996), and the value of K_s found to be higher than the K_h and K_f so stretching can be ignored usually. The stretching force constant value is lower only in range (t/l) 0.01 ~0.02. In this region K_f has the lowest value that's why flexure model explains successfully. Due to local damage at hinge the value of K_h is usually lower than K_f so the hinging mechanism dominates as in card honeycombs.

For same cell angle the modulus is higher for re-entrant structures than corresponding honeycombs, but this increase is at the expense of increase in weight due to higher density. By using the transformation equation, derived for an orthotropic material, the behaviour of honeycomb materials for different orientations of load was also investigated in 1-2 plane. By polar plots of elastic properties it was observed that for flexure, hinging and stretching models, the regular hexagon honeycombs ($h = 1, \theta = 30^\circ$) generates a material which is truly isotropic in the plane.

While the re-entrant structures ($h = 2l, \theta = -30^\circ$) showed different behaviour. Deforming by flexing and hinging the honeycomb is clearly square symmetric, not isotropic while for the stretching case the honeycomb is clearly not isotropic or square symmetric. By equating the Poisson's Ratio ν_{12}, ν_{21}, E and G were plotted against the "load orientation angle ϕ " to investigate the possibility of finding the isotropic re-entrant cell. The plots showed that apparent near isotropy can only be achieved in a re-entrant cell if we consider that deformation is by stretching mechanism. The value of shear modulus G was maximum when E minimum at $\phi = 45^\circ$ for hinging and flexure models and at $\phi = 45^\circ, 90^\circ$ for stretching models.

Masters and Evans (1996) and Olympio and Gandhi (2007) considered the curvature at the intersection points but only for the prediction of the hinging shear deformation stiffness. This curvature was not considered to determine the bending or axial stiffness of the honeycomb unit cell.

S. BALAWI, AND J.L. ABOT MODEL

Balawi and Abot (2008) proposed a refined model for commercial honeycombs, with double wall thickness in vertical direction and some curvature in the vicinity of the intersection points of the hexagons resulted in corrugation or expansion process during manufacturing as shown in Fig. 8. All the parameters are shown in Fig. 9 i.e., R is radius of curvature in the vicinity of intersections, t is cell wall thickness, L is cell wall length, and θ is the angle with horizontal line. This model was also validated by Finite Element Analysis and experimental data for lower as well as higher

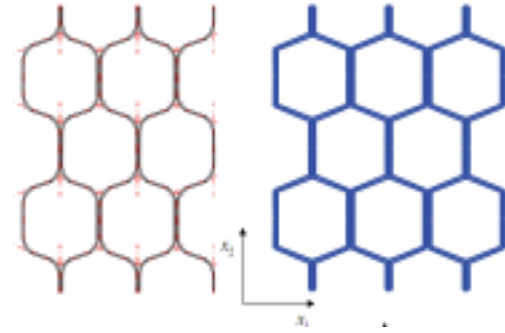


Fig. 8: Regular hexagonal honeycomb structure, (a) commercial honeycomb with curved walls at the intersection points, (b) theoretical honeycomb with straight walls (Balawi and Abot, 2008)

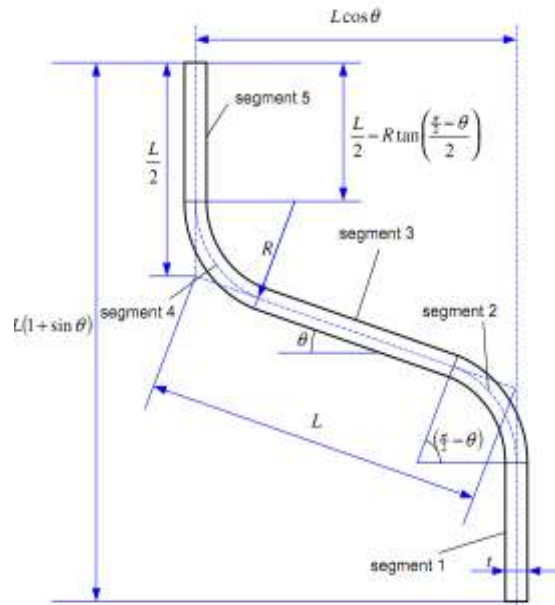


Fig. 9: Modified unit cell of a general hexagonal honeycombs with curved walls at the intersection points and single wall thickness in vertical direction (Balawi and Abot, 2008)

density commercial honeycombs. Due to double thickness in vertical direction the relative density of commercial honeycombs is higher than theoretical ones, so it is calculated by dividing the total hexagonal cell area with cell wall area, and is given by where ρ_s is density of cell walls material (Balawi and Abot, 2008):

$$\frac{\rho}{\rho_s} = \frac{8t}{3\sqrt{3}L} - \frac{4}{9} \left(\frac{t}{L}\right)^2 \quad (25)$$

A unit load method (Cook and Young, 1999) was used to calculate the deformation due to bending, extension and shear deformation in both x_1 and x_2 directions. For x_1 direction loading it was considered that only curved and inclined sections experience

deformations, while straight walls experience only rigid body motion. For x_2 direction, vertical walls experience only extension. By adding all three deformations, strain was calculated and effective Moduli were simply the ratio of applied stress to strain in the respective direction. The E_1 and E_2 are given by Eq. (27 and 28) while the shear modulus is calculated by:

$$G_s = \frac{E_s (1+\nu_s)}{2} \tag{26}$$

For validation purpose, these values were calculated for straight wall regular honeycombs $R = 0$ and $\theta = 30^\circ$ and the result was found similar to that predicted by Masters and Evans (1996). It was observed that effective moduli in both x_1 and x_2 directions for curved walls are very sensitive to radius of curvature R , decreasing with increase in R . This reduction is qualitatively similar but less in magnitude x_2 direction as compared to x_1 . Bending was the most dominant mechanism for honeycombs of lower relative densities. At $R = L$, the bending deformation reaches values larger than two times in x_2 and 18 times in x_1 direction, than those of calculated for straight-walled honeycombs. Shearing deformation increases with R in x_1 direction while decreases slightly in x_2 direction while the reverse effect is observed for extension deformation. For lower values of cell angle θ than 30° , the curved wall honeycombs are stiffer and more compliant for higher values of θ in x_1 direction than those of straight regular honeycombs. This effect increases as R increases. For direction 2 stiffness increases for large cell angles and shows no prominent effect of R . When θ is small inclined walls are oriented more along x_1 direction, when the θ is larger than 30° , there is more material oriented along the x_2 direction, thus causing the unit cell to deform more in the x_1 direction.

For experimental validation of this model the effective elastic moduli were calculated in both the x_1 and x_2 directions for two commercial honeycombs Plascore (8.1-3/16-003) and Hexcel (12.0-1/16-0015) having relative densities 0.029 and 0.0318, respectively. By considering an average curvature of 0.4 the ratio between elastic moduli E_1 and E_2 was observed 0.0481 theoretically. It was quite similar to the experimental one 0.0439 that confirmed the validity of refined model. As Gibson and Michael (1988) predict a ratio between the effective in-plane elastic moduli of 1, while Masters and Evans (1996) predicted a value of 0.99. This also predicts that the effect of curvature generates anisotropic behaviour and can't be neglected. By considering all three mechanisms of deformation predicted by Masters and Evans (1996), the experimental results of Balawi and Abot (2008) showed good correlation for lower values of t (lower relative density) while for higher values of t the experimental values are higher than that of predicted by Balawi and Abot (2008).

$$\frac{E_1}{E_s} = \frac{(2t^3 \cos\theta)}{\left[(1+\sin\theta) \left(\frac{L^3 - 4R^3(28+3\pi-60) - 12LR^2(3+2\pi-4\theta) + 3LR^2(\pi-4-2\theta) + 12R\cos\theta(L^2+4R^2+2LR(\pi-2\theta)) - L\cos2\theta(L^2+12R^2+3LR(\pi-2\theta))}{64R^3\sin(\frac{\theta}{2}) + \frac{8R(3L^2+2R^2+6LR(\pi-2\theta)\sin(\frac{\theta}{2}))}{(\sin(\frac{\theta}{2})+\cos(\frac{\theta}{2}))^3} + \frac{16R^2(3L+R(-2-3\pi-6\theta))}{\sin(\frac{\theta}{2})+\cos(\frac{\theta}{2})} + \frac{1}{1+\sin\theta}} \right) + \frac{2R\sin2\theta(3L^2+2R^2) + t^2(L+\pi R-2R\theta-4R\cos\theta+L\cos2\theta+R\sin2\theta) + 2t^2(1+\nu_s)\left(R(\pi-2\theta+\sin2\theta)+2\sin^2\theta\left(L-2R\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right)\right)}{2} \right]} \tag{27}$$

$$\frac{E_2^{eff}}{E_s} = (2t^3 \sec\theta(1+\sin\theta)) / \left[\begin{aligned} & 2t^2(1+\nu_s)(L+\pi R-2R\theta-4R\cos\theta+L\cos2\theta+R\sin2\theta) + \\ & t^2(2L+\pi R-2R\theta+2L\sin^2\theta+R\sin2\theta-4R(1+\sin^2\theta)\tan(\frac{\pi}{4}-\frac{\theta}{2})) + \\ & 2 \left(L(L^2+24R^2+3LR(\pi-2\theta))\cos^2\theta - 6R\cos\theta(L^2+8R^2+2LR(\pi-2\theta)-L^2\sin\theta) + \right. \\ & \left. 2R^2(6L+9R(\pi-2\theta)-4R\sec\theta+R(17+\cos2\theta)\tan\theta+6\sin\theta(L(-2+\sin\theta)-2R\tan\theta)) \right) \end{aligned} \right] \tag{28}$$

where, G_s is shear Modulus and E_1^{eff} and E_2^{eff} are effective moduli in 1 and 2 directions, E_s is modulus of cell wall material.

EFFECT OF SPECIMEN SIZE ON MECHANICAL PROPERTIES

An important issue in the experimental determination of mechanical properties of honeycombs is the “specimen size with respect to cell size”. Size effect becomes also important in design when the object dimensions are order of meso-scale (2-6 mm scale). Due to presence of some stress-free cut cell edges at the surface of a specimen and

constraints applied by neighbouring cells or boundary, specimen properties can give different results than single or infinite size specimens. Brezny and Green (1990) used three point bend specimens which subject the more compliant top and bottom surfaces of the beam to the highest normal stresses, magnifying the size effect in smaller beams and suggested the critical value of ratio specimen to cell size to 15 to achieve same results as that of single or infinite size specimens. Their tests were performed on a brittle reticulated vitreous carbon foam which exhibits a Weibull size effect, confounding the specimen size/cell size effect. Bastawros (Mellquist and Waas, 2002) performed uni-axial compression tests on prismatic aluminium foam specimens of constant length and width and varying depth. They found that stiffness and strength became essentially constant when the depth was greater than about 4 times the cell size.

Onck *et al.* (2001a, b) analysed the effect of the ratio of specimen size to cell size, for the uni-axial compression, shear and indentation response of regular hexagonal honeycombs. They provided mathematical models for regular hexagonal honeycombs and results were extended to foams and experimentally verified their results for open-cell aluminium (6101-T6) foam (trade name Duocel; ERG) and closed-cell aluminium foam (trade name Alporas ; Shinko Wire). The Young's modulus and uni-axial strength were found to increase with increasing specimen size and reaches up to a plateau level at "specimen size/cell size = 6" for both foams. This effect was due to increased constraints at inner cell walls and less "area fraction" of stress free cell walls. The shear moduli and strength decrease with increasing specimen size and becomes independent of the specimen thickness at specimen size/cell size = 2.67. Shearing phenomenon becomes easy as cell size increased due to "more constrained applied boundary conditions" on specimen boundary. The Indentation Peak stress was found to decrease as the indenter diameter increases with respect to cell size and for large indenter diameters approaches a value slightly higher than the uni-axial compressive strength of the material.

Mellquist and Waas (2002, 2004) studied the effect of out of plane crushing on circular cell polycarbonate honeycombs and also validated experimentally and numerically. Experiments were conducted on hexagonal cell honeycombs ranging in size from 1 to 14 cells and it was observed that number of cells does not have an effect on the compressive strength (per unit area) of the honeycomb, but the way that the cells are arranged does have an effect on the response. FEA models containing 1, 3, 4, 5, and 9 cells were created and analysed The FEA results were also in good agreement with the experimental results. Choon *et al.* (2007) studied the effect of specimen size on in-plane and out of plane Moduli for of Nomex Paper made of

aromatic polyamide fibers. Theoretically E_{33} is independent of number of cells while both experimental and FEM results indicated that E_{33} decreases as number of cells increases and converges at large number of cells. In plane Moduli also showed same contradiction with theoretical results. This discrepancy was due to anisotropic behaviour of Nomax paper as compared to polycarbonate honeycombs used by Mellquist and Waas (2004).

RESULTS AND DISCUSSION

In early years two dimensional cellular structures had been widely as core material for light weight sandwich panels because of their higher strength to weight ratio, so main focus of research was their transverse normal and shear properties. Since last two decades in-plane properties are widely investigated due to higher global strain abilities useful for compliant mechanisms applications. More over the increasing computational power enabled the use of improved sandwich plate and shell theories, which require a refined knowledge of the mechanical behaviour of the core. The in-plane properties had been neglected compared to the transverse properties in the early sandwich plate and shell theories but advanced sandwich plate and shell theories in many cases consider the in-plane material response of the core as well. Many refined core cell geometries were proposed and investigated to meet the requirements such as positive Poisson's ratio cellular structures.

Using cell wall bending model which is called Cellular Material Theory (CMT), Gibson and Michael (1988) also determined the in plane Moduli by considering cell walls as flexures that are fixed at one end and guided at other end as considered by Abd El-Sayed *et al.* (1979). The effect of double wall thickness cells such as in commercial honeycombs was neglected because stretching and hinging phenomenon was considered negligible. Masters and Evans (1996) modified the model by introducing three mechanisms of flexing, hinging and stretching. The curvature at the intersection points was considered but only for the prediction of the hinging shear deformation stiffness. This curvature was not considered to determine the bending or axial stiffness of the honeycomb unit cell. But out-of plane constants were not determined also no double wall thickness was considered. Balawi and Abot (2007b) proposed a refined model for commercial honeycombs, with double wall thickness in vertical direction and some curvature in the vicinity of the intersection points of the hexagons. The model was validated experimentally and anisotropic behaviour of commercial honeycombs was attributed to curvature and double wall thickness effect. In this model FEA model was presented for one fourth unit cell, showing

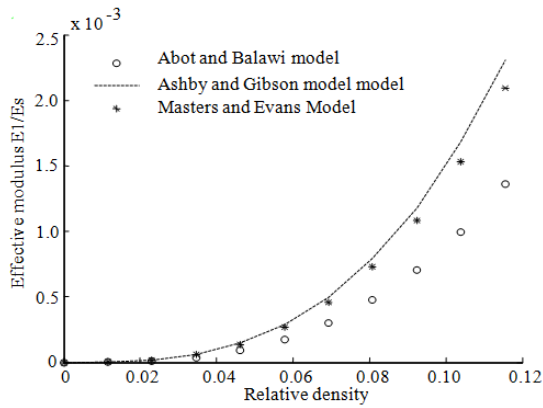


Fig. 10a: Effective modulus E_1 versus relative density

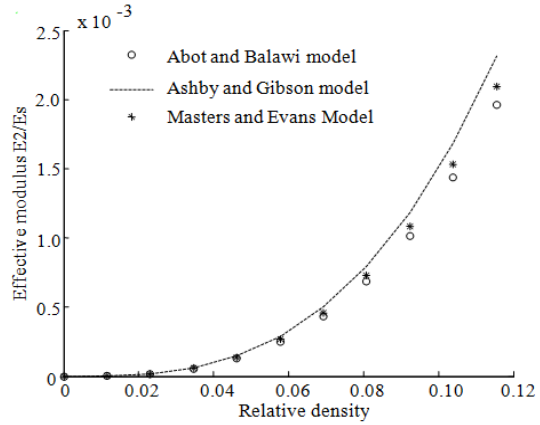


Fig. 10b: Effective modulus E_2 versus relative density

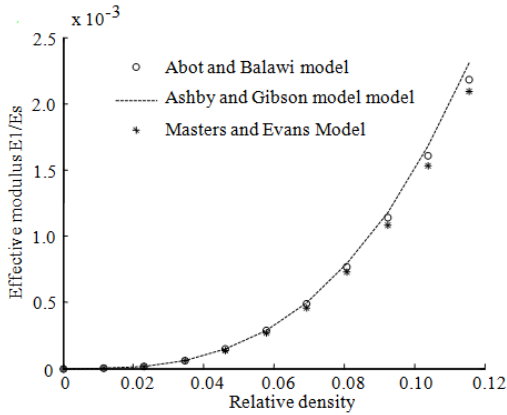


Fig. 11a: Effective modulus E_1 versus relative density for $r = 0$

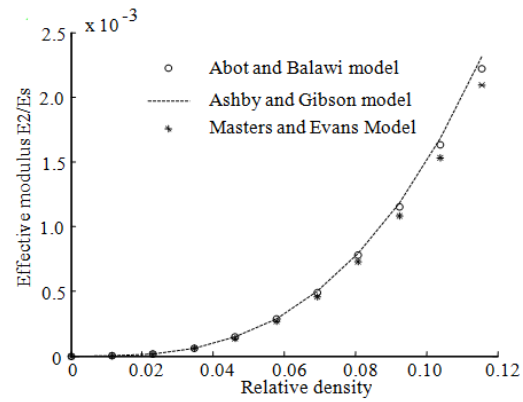


Fig. 11b: Effective modulus E_2 versus relative density for $r = 0$

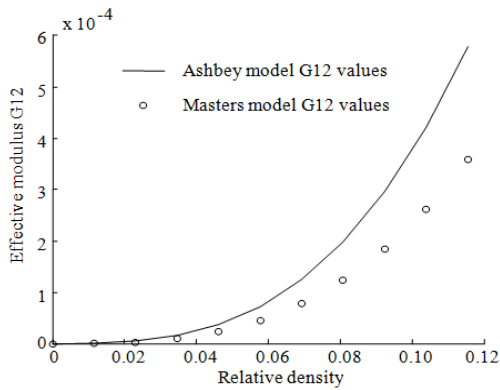


Fig. 12: Effective modulus G_{12} versus relative density

that the vertical walls remains vertical, but that is not in actual case, so experimental results showed discrepancy from theoretical for higher density honeycombs.

Masters and Evans (1996) calculated the off-axis elastic constants to determine isotropic or anisotropic as compared to Gibson and Michael (1988). Also Balawi and Abot (2007b) determined the E_1 and E_2 constants to

investigate anisotropic behaviour but no off-axis constants were determined. For comparison purpose graphs are plotted for effective Moduli (E_1/E_s) and (E_2/E_s) against relative density (ρ/ρ_s) for three models on same scale of dimensions, for regular 30° angle hexagonal honeycombs Fig. 10a and b. Cell wall length $l = 1$, $h = 1$ and thickness t is varied from 0 to 0.1 because most of commercial honeycombs are available in this range. A radius of curvature $r = 0.01$ is considered for Balawi's model.

Graph for $r = 0$ is also in Fig. 11a and b, which shows same values for Gibson and Michael (1988) and Balawi and Abot (2008) models while a very little discrepancy at higher density for Masters and Evans (1996). The model showed by Masters and Evans (1996) correlates well with the experimental results of Balawi and Abot (2008) for lower relative density but unable to estimates for higher relative densities. The shear modulus G_{12} for regular hexagonal honeycombs $h = l = 1$ and 30° angle is also compared in Fig. 12 for Gibson and Michael (1988) and Masters and Evans (1996) models as described by Eq. (5) and (24),

respectively while Balawi and Abot (2008) don't describe shear modulus values in their model although they used materials shear modulus G_s and shearing phenomenon to calculate Elastic Moduli.

The specimen to be used for testing is also important when the size of object is comparable with cell size, due to stress free cut cell walls at the boundary and constraints applied by neighbouring cells. Different authors investigated the effect using different materials and general trend is decrease in elastic Moduli for lower number of cells and increase in shear modulus for lower number of cells. The specimen properties gain a plateau value and become independent to number of cells in range 4~15 for Elastic moduli and 2~3 for shear modulus.

CONCLUSION

In early years honeycomb structures were used as sandwich panels, using their property of "high out of plane strength to weight ratio" as compared to constitute materials. Since last two decades their high in-plane strain ability have presented them better candidates as flexible materials. As proposed by different authors, mathematical models for in plane properties are compared using same scale dimensions. For lower relative densities all three models show same results but for higher values of density, (Gibson and Michael, 1988) and Masters and Evans (1996) models shows close results (a little bit discrepancy at higher densities) while for Balawi and Abot (2007b). Balawi and Abot (2008) model the values of elastic moduli are lower at higher density and this decreasing effect is more for E_1 as compared to E_2 . The anisotropic behaviour of honeycomb materials can't be described without using all three mechanisms of deformation, flexing, hinging and stretching. The shear modulus values are same for lower relative densities while for higher relative density the shear modulus described by Gibson and Michael (1988) is higher than that of Masters and Evans (1996) because of neglecting stretching and hinging mechanisms for deformation.

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