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Research Article

Ultimate Flexural Capacity of Unbounded Prestressed Concrete Pier

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Abstract: Based on the plastic-hinge method, the influence of different factors on the ultimate flexural capacity is analyzed, including relationship between the ultimate bearing capacity and prestressing tendon area, the regular reinforcement area, the strength of concrete and other influence factors. The results show that, the increase in the area of regular reinforcement and the strength of concrete can enhance the ultimate flexural capacity. Moreover, the ultimate bending moment increases first and then decreases, with the increasing of the prestressing tendon. Thus, the most effective method to increase the ultimate flexural capacity of prestressed concrete bridge pier is to increase regular reinforcement ratio, and to control the number of prestressing tendon simultaneously.

Keywords: Pier, prestressed concrete, ultimate flexural capacity, unbounded

INTRODUCTION

The advantages of using prestressed concrete bridge pier include accelerating bridge construction, reducing construction period and maintaining construction quality (Kwan and Billing, 2003). This type of bridge column has been widely used, such as Donghai Bridge (China), Humin Viaduct in Shanghai (China), Kok Island Bridge (Japan) and Northumberland Strait Bridge (Canada) (Ou *et al*., 2010; Yang, 2011). The prestressed concrete bridge pier can be categorized as overall cast-in-situ pier and precast installation pier which is unbonded prestressed concrete bridge pier. Nowadays the study on the unbonded prestressed concrete beams and other members subjected to flexure is much more than the study on unbonded prestressed concrete bridge pier (Du *et al*., 2008; Zhang *et al*., 2011). This study mainly focus on ultimate flexural capacity of the unbonded prestressed concrete bridge pier.

MATERIALS AND MATHODS

Mechanical behavior of the unbounded prestressed concrete bridge pier: The mechanical behavior of the unbounded prestressed concrete bridge is different from that of bonded prestressed concrete flexural members. Because the feature of bonded prestressed concrete flexural members is deformation compatibility. Namely, the deformation of prestressing tendon in any section and the surrounding concrete is harmonious when the member is subjected to load. For the unbonded prestressed concrete flexural members, the stress of unbounded prestressing tendon nearly distributes uniformly along the full length and moreover, the longitudinal relative sliding between unbounded prestressing tendon and concrete occurs.

The key to calculate the ultimate capacity of the unbounded prestressed concrete flexural member is to determine the ultimate stress of the unbounded prestressing tendon when the member reaches the ultimate bearing capacity. After unbounded prestressed concrete flexural members bear some loads, the strain along the prestressing tendon is almost the same because of the relative sliding between unbounded prestressing tendon and concrete. The variation value of the strain is equal to the average strain of the surrounding concrete along the full length of the prestressing tendon. Thus, the stress increment of unbounded prestressing tendon is small before concrete cracks but fast after that (Kim and Lee, 2012).

The ultimate stress of unbounded prestressing tendon f_{ps} can be calculated by the following Eq. (1) in ACI code:

$$f_{ps} = f_{pe} + 105 \tag{1}$$

where, f_{pe} effective stress of prestressing tendon, MPa. This equation is quite simple and conservative.

Assumptions: The following assumptions are made based on plastic-hinge method (Kim and Lee, 2012; Du and Liu, 2003):

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- The section of the reinforced concrete remain plane after bending (plane section assumption)
- Most regions are in elastic stage except plastic hinge regions of the pier bottom (lumped plastic hinge theory)
- The compressive strain of margin concrete under ultimate stress state is ε_{cu}, without consideration of the ultimate tensile strength of concrete
- Regular compressive reinforcement has reached its yield value
- Compared with that in plastic region, the elongation (nominal elongation) and shortening of the concrete which is at the same position with unbonded prestressing tendon can be neglected, without considering the friction effect of unbonded prestressing tendon.

Mechanical analysis for the section of unbonded prestressed concrete bridge pier in ultimate state:

- **Equivalent length of plastic hinge and relative height of neutral axis:** The parameter ψ, equivalent length of plastic hinge and k_u, height of relative neutral axis is introduced for convenient derivation (Wang, 2007; Tam and Pannell, 1976).
  - Equivalent length of plastic hinge ψ:

$$\psi = L_P / L \tag{2}$$

where,

L_p = Equivalent length of plastic hinge

L = The height of the pier

$$L_P = 0.15L + 0.134\left[1 - 5.564[\omega_s - \omega_s' + 0.25(\omega_p - \omega_p')] + n\right]h_0 \tag{3}$$

ω_s, ω_s', ω_p, ω_p' is reinforcement index:

$$\omega_s = \rho_s f_y / f_c,\ \omega_s' = \rho_s' f_y' / f_c,\ \omega_p = \rho_p f_{py} / f_c,\ \omega_p' = \rho_p' f_{py}' / f_c$$

where,

f_c = Compressive strength of concrete,

f_y, f_y' = The tensile and compressive yield strength of regular reinforcement

f_{py}, f_{py}' = The tensile and compressive yield strength of prestressing tendon

ρ_s, ρ_s' = The steel ratio of regular reinforcement in tensile and compressive area

ρ_p, ρ_p' = The steel ratio of prestressing tendon in tensile and compressive region

h_0 = The effective height of section.

$n = \dfrac{N + N_{pe}}{f_c bh}$ = Longitudinal axial compression ratio

N_{pe} = Prestressing axial pressure

N = External axial force

- The height of relative neutral axis k_u:

$$k_u = x_u / h \tag{4}$$

where,

x_u = The height of neutral axis in ultimate state

h = The height of section

The following equation is the relation between the height of relative neutral axis k_u and the height of relative compressive area ξ_n:

$$\xi_u = \frac{\beta x_u}{h_0} = \frac{\beta k_u h}{h_0} \tag{5}$$

where,

β = The height coefficient of rectangular graphics of concrete

h_0 = The effective height of section

The mechanical analysis for the section of unbonded prestressed concrete bridge pier in ultimate state: The concrete strain which is at the same position of unbonded tendon at tension and compression area is, ε_c ε_c', the ultimate compressive strain of concrete is ε_{cu} as Fig. 1 shows .According to basic assumption (2) and (5), the total elongation or shortening of unbonded tendon along its full length is equal to the elongation or shortening of concrete at the same position. So the strain increment of unbonded tendon under ultimate state is:

$$\begin{cases} \triangle\varepsilon_p = \dfrac{\varepsilon_c L_p}{L} \\ \triangle\varepsilon_p' = \dfrac{\varepsilon_c' L_p}{L} \end{cases} \tag{6}$$

According to equilibrium equation:

$$\begin{cases} N = \alpha\beta k_u f_c bh + f_y' A_s' - \sigma_s A_s - [(\sigma_{pe} + \psi\varepsilon_c E_p)A_P + (\sigma_{pe}' - \psi\varepsilon_c' E_p)A_P'] \\ M = \alpha\beta k_u f_c bh(\dfrac{h}{2} - \dfrac{\beta k_u h}{2}) + \sigma_s A_s(\dfrac{h}{2} - a_s) + f_y' A_s'(\dfrac{h}{2} - a_s') \\ \quad -[(\sigma_{pe}' - \psi\varepsilon_c' E_p)A_P'(\dfrac{h}{2} - a_p') - (\sigma_{pe} + \psi\varepsilon_c E_p)A_P(\dfrac{h}{2} - a_p)] \end{cases} \tag{7}$$

where,

A_s, A_s' = The total area of regular longitudinal reinforcement in tension and compression area

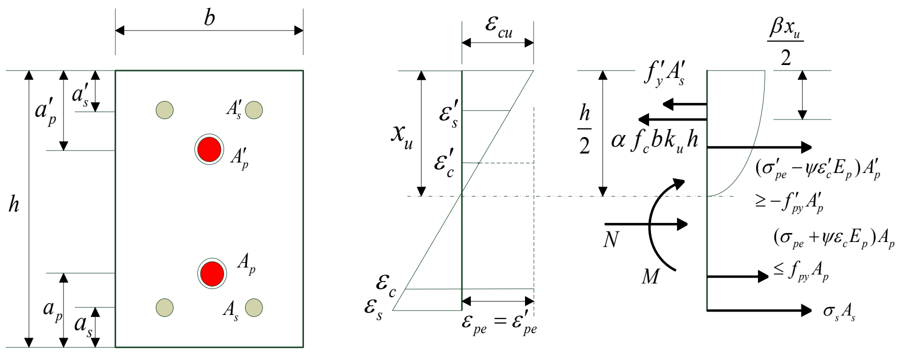

Fig. 1: Force analysis of unbounded PRC pier under limit state

A_p, A'_P = The total area of prestressing tendon in tension and compression area

a_s, a'_s = The thickness of cover of regular longitudinal reinforcement in tension and compression area

a_p, a'_p = Thickness of cover of prestressing tendon in tension and compression area

E_p = Elastic modulus of prestressing tendon

α = Stress coefficient of concrete rectangular graphics

RESULTS AND DISCUSSION

Ultimate bearing capacity of unbounded prestressed concrete bridge pier: The following equations can be calculated based on assumptions (1), (3), (4):

$$\begin{cases} \varepsilon'_c = \dfrac{k_u h - a'_p}{k_u h}\varepsilon_{cu} \\ \varepsilon_c = \dfrac{(1-k_u)h - a_p}{k_u h}\varepsilon_{cu} \\ \varepsilon_c = \dfrac{(1-k_u)h - a_s}{k_u h} \end{cases} \quad (8)$$

And generally it's reinforced symmetrically for most unbonded prestressed concrete bridge pier, namely, $A_S = A'_S$, $A_P = A'_P$, $a_s = a'_s$, $a_p = a'_p$ and generally $\sigma_{pe} = \sigma'_{pe}$

- **Large eccentric compression:** The regular longitudinal reinforcement in tension area has yielded under large eccentric compression, namely $\sigma_s = f_y = f'_y$, substituted Eq. (7) and combined with Eq. (8), M_u can be calculated by the following equation:

$$M_u = [\frac{1}{2}\alpha\beta k_u(1-\beta k_u) + 2\rho\frac{f_y}{f_c}(\frac{1}{2}-\frac{a_s}{h}) + \frac{h-2a_p}{k_u h}\frac{A}{f_c}(\frac{1}{2}-\frac{a_p}{h})]f_c bh^2 \quad (9)$$

where,

$$k_u = \frac{-(2A - f_c n) + \sqrt{(2A - f_c n)^2 + 4\alpha\beta f_c A}}{2\alpha\beta f_c}, \quad A = \psi E_p \varepsilon_{cu}\rho_p$$

- **Small eccentric compression:** The regular longitudinal reinforcement in tension area has not yielded under small eccentric compression, namely $\sigma_s = E_s\varepsilon_s < f_y$, substituted Eq. (7) and combined with Eq.(8):

$$M_u = [\frac{1}{2}\alpha\beta k_u(1-\beta k_u) + (\rho_s\frac{f_y}{f_c} + \frac{(1-k_u)h - a_p}{k_u h}\frac{B}{f_c})(\frac{1}{2}-\frac{a_s}{h}) + \frac{h-2a_p}{k_u h}\frac{A}{f_c}(\frac{1}{2}-\frac{a_p}{h})]f_c bh^2 \quad (10)$$

where,

$$k_u = \frac{\sqrt{(2A - f_c n + B + f_y\rho_s)^2 + 4\alpha\beta f_c[A + (1-\frac{a_p}{h})B]}}{2\alpha\beta f_c} - \frac{2A - f_c n + B + f_y\rho_s}{2\alpha\beta f_c}$$

$$A = \psi E_p \varepsilon_{cu}\rho_p, \; B = E_s\varepsilon_{cu}\rho_s$$

Discussion about the ultimate flexural capacity of the unbounded prestressed concrete bridge pier: According to Eq. (9) and (10), the ultimate flexural capacity of the unbounded prestressed concrete bridge pier is related to the area of prestressing tendons, the area of regular reinforcement and concrete strength. A bridge pier model of 5 m height is developed to further discuss about each factor's effect on ultimate flexural capacity.

The bridge pier with symmetric reinforcement is 5 m in height and with a 250 cm×150 cm cross section. Regular reinforced concrete cover and prestressed reinforced concrete cover is 10 cm, 30 cm respectively. The ultimate compressive strain of concrete ε_{cu} = 0.0033, the effective stress of prestressing tendon f_{pe} = 1000 Mpa, external axial force is 100 kN.

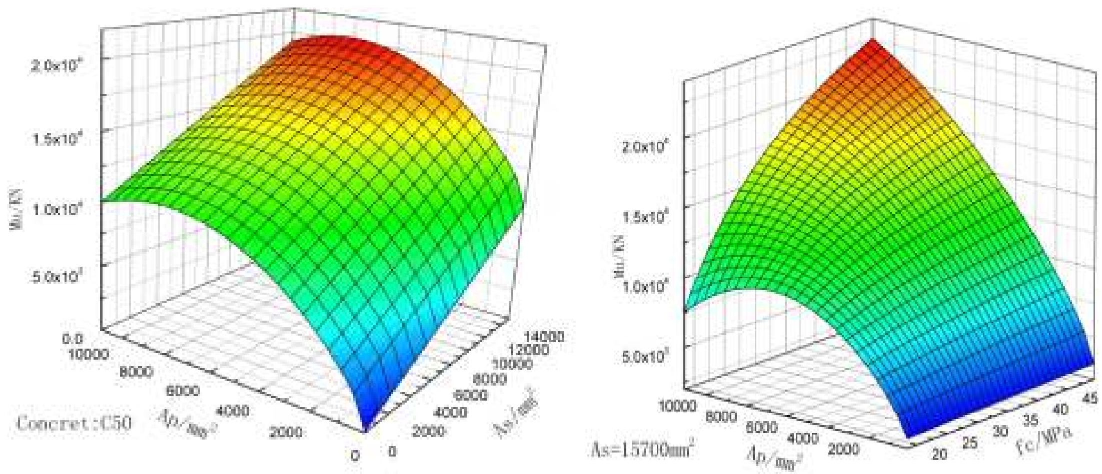

Fig. 2: Relation between M_u, A_p, A_s and f_c or unbounded PRC pier under large eccentric compression

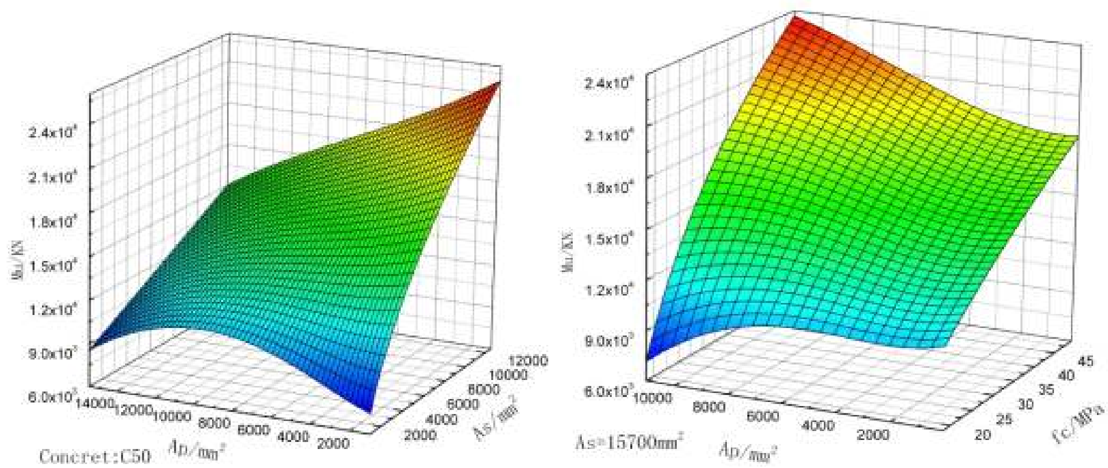

Fig. 3: Relation between M_u, A_p, A_s and f_c for unbounded PRC pier under small eccentric compression

For bridge pier under large eccentric compression, changed curved surface of M_u and A_p, A_S and f_c based on Eq. (9) are shown in Fig. 2.

For bridge pier under small eccentric compression, changed curved surface of M_u based on Eq. (10) are shown in Fig. 3.

Figure 2 and 3 show that, for prestressed reinforced concrete bridge pier, the ultimate bending moment almost increases linearly with the amount of regular reinforcement; the ultimate bending moment increases fast initially and gently later, with concrete strength; the ultimate bending moment increases first and then decreases, with the increasing of the prestressing tendon. Because increasing the amount of tendon reinforcement can increase not only steel ratio but also axial compression ratio. When axial compression ratio reaches a certain degree, the concrete of cross section has been crushed by prestressing tendon. So the most effective method to increase the ultimate flexural capacity of prestressed concrete bridge pier is to increase regular reinforcement ratio, and to control the number of prestressing tendon simultaneously.

CONCLUSION

The calculation formula for the ultimate flexural capacity of the unbounded prestressed concrete bridge pier has been derived, and the relationship between the ultimate bearing capacity and prestressed reinforcement area, the regular reinforcement area, the strength of concrete and other influence factors has been analyzed based on plastic hinge method in this paper. The results show that the ultimate bending moment almost increases linearly with the amount of regular reinforcement; the ultimate bending moment increases fast initially and gently later with concrete strength; the ultimate bending moment increases first and then decreases, with the increasing of the prestressed reinforcement. Therefore the most effective method to increase the ultimate flexural capacity of prestressed concrete bridge pier is to increase regular reinforcement ratio, and to control the number of prestressed reinforcement simultaneously.

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