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Research Article

Stabilization Analysis and Synthesis of Discrete-Time Descriptor Markov Jump Systems with Partially Unknown Transition Probabilities

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Abstract: A sufficient condition for the open loop system to be regular, causal and stochastically stable is proposed for a class of discrete-time descriptor Markov jump systems with partly unknown transition probabilities. The proposed criteria are in the form of a set of strict linear matrix inequalities and convenient for numerical realization. The presented condition used the information of unknown transition probabilities in an effective way and is less conservative. Furthermore, the stabilization control of the researching systems is realized by designing the state feedback controller to make the close-looped systems be regular, causal and stochastically stable. At last, a numerical example is given to demonstrate the validity of the proposed results.

Keywords: Descriptor Markov jump systems, Linear Matrix Inequality (LMI), partially unknown, stability analysis, stabilization control, Transition Probabilities (TPs)

INTRODUCTION

Descriptor systems is a kind of more general model that described many real dynamic systems such as electronic systems, economic systems, robot systems and aerospace systems (Dai, 1989). The research of descriptor systems has much meanings in theory and application. Since 1980s, the theory has been fully developed and the research tendency becomes more and more complicated recently (Chaibi and Tissir, 2012; Huang and Mao, 2011; Ma *et al*., 2011; Ma and Boukas, 2011).

Meanwhile, there are many dynamic systems in engineering application which has abrupt structure changing caused by random reason such as failure of system components, changing of sub-systems connection, abrupt change of environment condition and so on. This kind of random jumps always followed the rule of Markov process (Costa *et al*., 2005) and this kind of systems is called Markov Jump Systems (MJSs) (Liberzon, 2003; Boukas, 2005).

When the structure parameters of descriptor systems abruptly changed by the rule of Markov process, they became Descriptor Markov Jump Systems (DMJSs) (Xu and Lam, 2006; Boukas, 2008). The research of DMJSs becomes one hotspot of control domain in recent years (Feng and Lam, 2012; Chang *et al*., 2012; Boukas and Xia, 2008; Xia *et al*., 2009).

Present results of DMJSs are based on the condition of known about all Transition Probabilities (TPs) of system mode switching, however, in fact, because of experiment complexity, feasibility, expensive cost and so on, it is impossible to get all TPs. These circumstances are more suitable to engineering reality, so it has more general meaning in practice and theory to study DMJSs with partly unknown TPs. Zhang and Boukas (2009a, b) discussed the stability and stabilization of MJSs respectively. Based on unknown TPs, Che and Wang (2010) studied a new method of H∞ control of discrete-time MJSs. Because of un-casualness, descriptor system is different from normal ones. As a result, the research of DMJSs with partly unknown TPs is much more complicated. As far as we know, there are no references focusing on this problem.

In this study, some new results of DMJSs are extended from MJSs by a technique of expressing unknown TPs with known ones through equivalent transformation. Firstly, a sufficient condition in strict linear matrix inequalities form for guaranteeing a DMJS with partly unknown TPs to be regular, causal and stable is discussed. Moreover, the state feedback controller can be designed by solving the strict linear matrix inequalities, hence a new stabilization control method is obtained to make sure the close-looped system is regular, causal and stable. At last, an example is used to show the effectiveness of the proposed results.

PROBLEM PRELIMINARY

Consider an discrete-time DMJS on probability space (Ω, F, P):

$$\boldsymbol{E}(r_k)\boldsymbol{x}(k+1) = \boldsymbol{A}(r_k)\boldsymbol{x}(k) + \boldsymbol{B}(r_k)\boldsymbol{u}(k) \qquad (1)$$

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where, $x(k) \in \Re^n$ is the state, $u(k) \in \Re^m$ is the control input, $E(r_k) \in \Re^{n\times n}$ is singular and $rank(E(r_k)) = r \le n$, $\{r_k, k\ge 0\}$ is a discrete-time homogeneous Markov chain, which takes values in a finite set $\Gamma = \{1,2,\ldots, N\}$ and $\forall i, j \in \Gamma$, the transition probability from mode i to mode j is $\Pr(r_{k+1} = j | r_k = i) = \pi_{ij}$, which satisfies $\forall i, j \in \Gamma$, $\pi_{ij} \ge 0$ and $\sum_{j\in\Gamma}\pi_{ij} = 1$, $\forall i \in \Gamma$. The transition probability matrix is defined:

$$\pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N1} & \pi_{N1} & \cdots & \pi_{NN} \end{bmatrix} \tag{2}$$

for $r_k = i \in \Gamma$, $A(r_k) \in \Re^{n\times n}$, $B(r_k) \in \Re^{n\times m}$ are known constant matrices with proper dimensions, respectively, when $r_k = i \in \Gamma$, $A(r_k)$ and $B(r_k)$ of mode i becomes A_i and B_i for short, also for $E(r_k)$ to E_i and so on.

In this study, we consider DMJSs with partly unknown TPs condition. For this, given the following definition:

$$\Gamma = \Gamma^i_K \cup \Gamma^i_{UK}, \forall i \in \Gamma \tag{3}$$

where, $\Gamma^i_K = \{j: \pi_{ij} \text{ is known}\}$, $\Gamma^i_{UK} = \{j: \pi_{ij} \text{ is unknown}\}$. At the same time, we define:

$$\Gamma^i_K = (K^i_1,\ldots, K^i_m), \forall 1 \le m \le N \tag{4}$$

$$\Gamma^i_{UK} = (UK^i_1,\ldots, UK^i_w), \forall 1 \le w \le N \tag{5}$$

where, $K^i_m \in N^+$ means that the i^{th} element in the m^{th} column of condition (2) is known and values m, $UK^i_m \in N^+$ means that the i^{th} element in the w^{th} column of condition (2) is unknown and values w.

Meanwhile, we define $\pi^i_K = \sum_{j\in\Gamma^i_K}\pi_{ij}$ and $R_i \in \Re^{n\times n}$ with $R_i \times E_i = 0$ in the following.

When $u(k) = 0$, system (1) is simplified to the following autonomous system:

$$E(r_k)\, x(k+1) = A(r_k)x(k) \tag{6}$$

First of all, we introduce the following definitions.

Definition 1: Xu and Lam (2006) the system (6) is said to be:

- Regular if, for each mode $i \in\Gamma$, $\det(zE_i - A_i)$ is not identically zero.
- Causal if, for each mode $i \in\Gamma$, $\deg(\det(zE_i - A_i)) = rank(E_i)$.
- Stochastically stable if, for any initial state $x_0 \in \Re^n$ and $r_0 \in\Gamma$, there exist a scalar $M(x_0, r_0) > 0$ which satisfies:

$$\mathrm{E}[\textstyle\sum_{k=0}^{\infty}\|x(k)\|^2 \;|x_0, r_0] \le M(x_0, r_0)$$

where, $\mathbf{E}$ stands for the mathematics expectation.
- Stochastically admissible if, the system (6) is regular, causal and stochastically stable.

Lemma 1: (Boukas and Xia, 2008) system (6) is stochastically admissible, if and only if for each $i \in \Gamma$, there exist symmetric positive definite matrices Q_i and symmetric nonsingular matrices Φ_i satisfying the following:

$$A^T_i\left[\textstyle\sum_{j\in\Gamma}\pi_{ij}Q_j - R_i^T \Phi_i R_i\right]A_i - E^T_i Q_i E_i < 0 \tag{7}$$

STABILITY ANALYSIS

Firstly, we study the open looped stability of autonomous system (6) with partly unknown TPs.

Theorem 1: The autonomous system (6) with condition (3) is stochastically admissible, if for each $i \in \Gamma$, there exist symmetric positive definite matrices Q_i and symmetric nonsingular matrices Φ_i satisfying the following LMIs:

$$A^T_i\left[Q^i_K + (1-\pi^i_K)\textstyle\sum_{j\in\Gamma^i_{UK}}Q_j - R_i^T \Phi_i R_i\right]A_i - E^T_i Q_i E_i < 0 \tag{8}$$

where, $Q^i_K = \sum_{j\in\Gamma^i_K}\pi_{ij}Q_j$.

Proof: Consider autonomous system (6) with partly unknown TPs (3), condition (7) is equivalent to:

$$\boldsymbol{A}_i^{\mathrm{T}}(\boldsymbol{Q}^i_K + \boldsymbol{Q}^i_{UK} - \boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{\Phi}_i\boldsymbol{R}_i)\boldsymbol{A}_i - \boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{Q}_i\boldsymbol{E}_i < \mathbf{0} \tag{9}$$

where,

$$Q^i_K = \textstyle\sum_{j\in\Gamma^i_K}\pi_{ij}Q_j\,,\; Q^i_{UK} = \sum_{j\in\Gamma^i_{UK}}\pi_{ij}Q_j\,.$$

Since:

$$\pi_{ij} \ge 0,\; Q^T_i = Q_i > 0,\; \textstyle\sum_{j\in\Gamma}\pi_{ij} = 1$$

We have:

$$Q^i_{UK} \le (\textstyle\sum_{j\in\Gamma^i_{UK}}\pi_{ij})(\sum_{j\in\Gamma^i_{UK}}Q_j) = (1 - \pi^i_K)(\sum_{j\in\Gamma^i_{UK}}Q_j)$$

So if (8) holds, then (9) holds and then we obtain (7). According to lemma 1, the autonomous system (6) with partly unknown TPs (3) is stochastically admissible. The proof is completed.

Remark 1: By using equation $\sum_{j\in\Gamma^i_{UK}}\pi_{ij} = (1 - \sum_{j\in\Gamma^i_K}\pi_{ij})$, the unknown TPs are reformed with the known TPs. This technique uses the information of unknown parts effectively and makes the sufficient condition less conservative.

Remark 2: Theorem 1 extends the stability criteria of DMJS to the condition with partly unknown TPs. The sufficient condition in Theorem 1 covers all known and all unknown TPs as two special cases and expresses in strict linear matrix inequalities form to show its more powerful practical significance and theory value. When the TPs are all known, theorem 1 has the same formation as lemma 1, so lemma 1 can be considered as one special case of theorem 1.

Corollary 1: We have (8), if for each $i \in \Gamma$, there exist nonsingular matrices W_i satisfying the following LMIs:

$$\begin{bmatrix} \Theta_i & A_i^T W_i - W_i^T \\ * & Q_K^i + Q_{UK}^i + W_i - W_i^T \end{bmatrix} < 0 \tag{10}$$

where, $Q_K^i = \sum_{j \in \Gamma_K^i} \pi_{ij} Q_j$, $Q_{UK}^i = (1-\pi_K^i) \sum_{j \in \Gamma_{UK}^i} Q_j$, $\boldsymbol{\Theta}_i = \boldsymbol{A}_i^T \boldsymbol{W}_i - \boldsymbol{W}_i^T \boldsymbol{A}_i - \boldsymbol{A}_i^T \boldsymbol{R}_i^T \boldsymbol{\Phi}_i \boldsymbol{R}_i \boldsymbol{A}_i - \boldsymbol{E}_i^T \boldsymbol{Q}_i \boldsymbol{E}_i$ and "*" denotes the term that is induced by symmetry (same as follow).

Proof: $[I \ A^T_i]$ is full row rank, so there exist nonsingular matrices W_i to reform (8) as followed:

$$A_i^T \left[Q_K^i + (1-\pi_K^i) \sum_{j \in \Gamma_{UK}^i} Q_j - R_i^T \Phi_i R_i \right] A_i - E_i^T Q_i E_i$$

$$= A_i^T \left(Q_K^i + Q_{UK}^i - R_i^T \Phi_i R_i \right) A_i - E_i^T Q_i E_i$$
$$= [I \ A^T_i] \begin{bmatrix} \Theta_i & A_i^T W_i - W_i^T \\ * & Q_K^i + Q_{UK}^i + W_i - W_i^T \end{bmatrix} \begin{bmatrix} I \\ A_i \end{bmatrix} < 0$$

So if (10) holds, then (8) holds. The proof is completed.

Theorem 2: Given any scalar α_i and β_i, the autonomous system (6) with condition (3) is stochastically admissible, if for each $i \in \Gamma$, there exist symmetric positive definite matrices P_i, symmetric nonsingular matrices $\boldsymbol{\psi_i}$ and nonsingular matrices H_i satisfying the following LMIs:

$$\begin{bmatrix} \Sigma_i & H_i^T A_i^T - H_i & 0 & 0 \\ * & -H_i^T - H_i & (Z_K^i)^T & (Z_{UK}^i)^T \\ 0 & * & -\chi_K^i & 0 \\ 0 & * & 0 & -\chi_{UK}^i \end{bmatrix} < 0 \tag{11}$$

where,

$$\Sigma_i = H_i^T A_i^T + A_i H_i - \alpha_i R_i A_i H_i - \alpha_i H^T_i A^T_i R^T_i + \alpha^2_i \boldsymbol{\psi_i} - \beta_i H^T_i E^T_i - \beta_i E_i H_i + \beta^2_i P_i,$$

$$Z_K^i = \left[\sqrt{\pi_{iK_1^i}} H_i^T \ \sqrt{\pi_{iK_2^i}} H_i^T \cdots \sqrt{\pi_{iK_m^i}} H_i^T \right]^T$$

$$Z_{UK}^i = \left[\sqrt{1-\pi_K^i}\, H_i^T \ \sqrt{1-\pi_K^i}\, H_i^T \cdots \sqrt{1-\pi_K^i}\, H_i^T \right]^T$$

$$\chi_K^i = diag\ \{P_{K_1^i}, P_{K_2^i}, \dots, P_{K_m^i}\}$$

$$\chi_{UK}^i = diag\ \{P_{UK_1^i}, P_{UK_2^i}, \dots, P_{UK_m^i}\}$$

Proof: Let $H_i = W_i^{-1}$, pre and post-multiplied (10) by $diag\{H^T_i, H^T_i\}$ and its transpose, we obtain:

$$\begin{bmatrix} \overline{\Theta}_i & H_i^T A_i^T - H_i \\ * & H_i^T Q_K^i H_i + H_i^T Q_{UK}^i H_i + H_i^T - H_i \end{bmatrix} < 0 \tag{12}$$

where,

$$\overline{\Theta}_i = H_i^T A_i^T - A_i H_i - H_i^T A_i^T R_i^T \Phi_i R_i A_i H_i - H_i^T E_i^T Q_i E_i H_i$$

Let $P_i = Q^{-1}_i$ and $\boldsymbol{\psi_i} = \Phi_i^{-1}$, according to Schur complement, (12) can be equivalent transformed to:

$$\begin{bmatrix} \Xi_i & H_i^T A_i^T - H_i & 0 & 0 \\ * & -H_i^T - H_i & (Z_K^i)^T & (Z_{UK}^i)^T \\ 0 & * & -\chi_K^i & 0 \\ 0 & * & 0 & -\chi_{UK}^i \end{bmatrix} < 0$$

where,

$$\Xi_i = H_i^T A_i^T - A_i H_i - H_i^T A_i^T R_i^T \boldsymbol{\psi_i}^{-1} R_i A_i H_i - H_i^T E_i^T P_i^{-1} E_i H_i$$

Given any scalar α_i and β_i, the following inequalities hold:

$$0 \le (H_i^T A_i^T R_i^T - \alpha_i \boldsymbol{\psi_i}) \boldsymbol{\psi_i}^{-1} (R_i A_i H_i - \alpha_i \boldsymbol{\psi_i}) = H_i^T A_i^T R_i^T \boldsymbol{\psi_i}^{-1} R_i A_i H_i - \alpha_i R_i A_i H_i - \alpha_i H_i^T A_i^T R_i^T + \alpha^2_i \boldsymbol{\psi_i}$$

$$0 \le (H_i^T E_i^T - \beta_i P_i) P_i^{-1} (E_i H_i - \beta_i P_i) = H_i^T E_i^T P_i^{-1} E_i H_i - \beta_i E_i H_i - \beta_i H_i^T E_i^T + \beta^2_i P_i$$

Then we have:

$$- H_i^T A_i^T R_i^T \boldsymbol{\psi_i}^{-1} R_i A_i H_i \le - \alpha_i R_i A_i H_i - \alpha_i H_i^T A_i^T R_i^T + \alpha^2_i \boldsymbol{\psi_i}$$
$$- H_i^T E_i^T P_i^{-1} E_i H_i \le - \beta_i E_i H_i - \beta_i H_i^T E_i^T + \beta^2_i P_i$$

So we get $\Sigma_i \ge \Xi_i$.

According to the above, it is obtained that if (11) holds, then (12) holds and then (10) holds. According to corollary 1, (8) holds and so the autonomous system (6) is stochastically admissible. The proof is completed.

STABILIZATION CONTROL

In this section, consider system (1), we design a state feedback controller to guarantee the close looped system to be regular, causal and stochastically stable. Under the observability of all system states, we can get the following state feedback controller:

$$\boldsymbol{u}(k) = \boldsymbol{K}(r_k)\boldsymbol{x}(k) \tag{13}$$

Substituting controller (13) in system (1), the close looped system is given:

$$\boldsymbol{E}(r_k)\boldsymbol{x}(k+1) = \boldsymbol{A}_{cl}(r_k)\boldsymbol{x}(k) \tag{14}$$

where $\boldsymbol{A}_{cl}(r_k) = \boldsymbol{A}(r_k) + \boldsymbol{B}(r_k)\boldsymbol{K}(r_k)$ and $\boldsymbol{A}_{cli} = \boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i$ for short.

Theorem 3: Given any scalar α_i and β_i, the close looped system (14) with condition (3) is stochastically admissible, if for each $i \in \Gamma$, there exist symmetric positive definite matrices P_i, symmetric nonsingular matrices $\boldsymbol{\psi_i}$, nonsingular matrices H_i and suitable dimension matrices Y_i satisfying the following LMIs:

$$\begin{bmatrix} \Omega_i & \Delta_i & 0 & 0 \\ * & -H_i^T - H_i & (Z_K^i)^T & (Z_{UK}^i)^T \\ 0 & * & -\chi_K^i & 0 \\ 0 & * & 0 & -\chi_{UK}^i \end{bmatrix} < 0 \tag{15}$$

where,

$$\Omega_i = Y_i^T B_i^T + H_i^T A_i^T + B_i Y_i + A_i H_i - \alpha_i R_i B_i Y_i - \alpha_i R_i A_i H_i - \alpha_i H_i^T A_i^T R_i^T + \alpha^2{}_i \boldsymbol{\psi_i} - \alpha_i Y_i^T B_i^T R_i^T - \beta_i H_i^T E_i^T - \beta_i E_i H_i + \beta^2{}_i P_i,$$

and $\Delta_i = Y_i^T B_i^T + H_i^T A_i^T - H_i$, Z_K^i, Z_{UK}^i, χ_K^i, χ_{UK}^i are the same as theorem 2. If (15) can be solved, then the gain matrices of state feedback controller (13) are $K_i = Y_i H^{-1}{}_i$.

Proof: Consider close looped system (14), substituting $\boldsymbol{A_i}$ of (11) by $A_{cli} = A_i + B_i K_i$, we get:

$$\begin{bmatrix} \bar{\Omega}_i & \bar{\Delta}_i & 0 & 0 \\ * & -H_i^T - H_i & (Z_K^i)^T & (Z_{UK}^i)^T \\ 0 & * & -\chi_K^i & 0 \\ 0 & * & 0 & -\chi_{UK}^i \end{bmatrix} < 0 \tag{16}$$

where,

$$\bar{\Omega}_i = H_i^T(A_i^T + K_i^T B_i^T) + (A_i + B_i K_i)H_i - \alpha_i R_i (A_i + B_i K_i)H_i - \alpha_i H_i^T(A_i^T + K_i^T B_i^T) R_i^T + \alpha^2{}_i \boldsymbol{\psi_i} - \beta_i H_i^T E_i^T - \beta_i E_i H_i + \beta^2{}_i P_i$$

$$\bar{\Delta}_i = H_i^T(A_i^T + K_i^T B_i^T) - H_i$$

According to theorem 2, if (16) holds, then the close looped system (14) is stochastically stable. Let $Y_i = K_i H_i$, it is obtained that (15) is equivalent to (16). So if (15) holds, then the close looped system (14) is stochastically stable. The proof is completed.

Remark 3: The nonlinear items of (16) are eliminated in (15) and the formalization is strict linear matrix inequalities in theorem 3, so it is convenient for real engineering application.

Remark 4: The singular matrix E of system (1) can abruptly changes with system modes in theorem 3. This model covers the situation that the fast and slow sub-systems of system (1) abruptly change with system modes, so it has more general significance in practical applications.

EXAMPLES

Consider system (1) with 4 modes and state vector of 3 dimensions:

Mode 1:

$$E_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.8 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 1.2 & 0.3 & 0.5 \\ -0.3 & 0.6 & 0 \\ 0.2 & -0.5 & 0.7 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$$

Mode 2:

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.5 & 0.7 & 1.1 \\ 0.3 & 0.8 & 0.7 \\ 0.4 & 0.7 & 0.2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0.6 \\ 0.2 & 0.5 \\ 0 & 0.3 \end{bmatrix}$$

Mode 3:

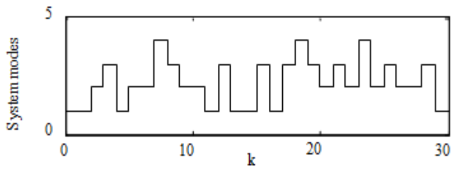
$$E_3 = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.2 & 0.4 & 0.6 \\ 0.5 & 0.2 & -1.5 \\ 0.4 & 0.2 & -0.8 \end{bmatrix}, B_3 = \begin{bmatrix} 1.6 & 1.2 \\ -0.2 & 0.3 \\ 0.2 & 0 \end{bmatrix}$$

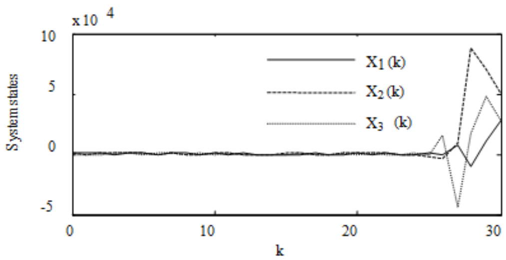
Mode 4:

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -0.4 & 0.2 & 0.3 \\ 0.1 & 0.5 & 0.7 \\ 0.6 & 0.1 & -0.2 \end{bmatrix}, B_4 = \begin{bmatrix} 0.8 & 0.4 \\ 0.7 & 0.6 \\ 0 & 0.1 \end{bmatrix}$$



(a) Switching process



(b) Open-loop

Fig. 1: Switching process and open-looped state trajectories

Transition probability matrix is:

$$\pi = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.5 & 0.1 \\ 0.5 & 0.1 & 0.2 & 0.2 \\ 0.6 & 0.1 & 0.1 & 0.2 \end{bmatrix}$$

Two cases of partly unknown TPs denoted as "Case 1" and "Case 2" are as followed:

$$\pi_{C1} = \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ ? & 0.3 & ? & 0.1 \\ 0.5 & ? & ? & 0.2 \\ 0.6 & 0.1 & 0.1 & 0.2 \end{bmatrix},$$

$$\pi_{C2} = \begin{bmatrix} 0.4 & ? & ? & 0.1 \\ ? & 0.3 & ? & 0.1 \\ 0.5 & ? & ? & 0.2 \\ 0.6 & ? & ? & ? \end{bmatrix}$$

where ,"?" means the unknown item.

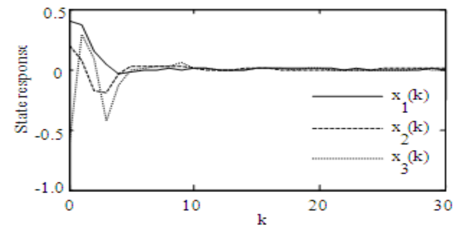
The switching process of system modes and state trajectories of open looped system are shown as Fig. 1 (a) and (b) respectively. From Fig. 1 (b), it can be clearly seen that the autonomous system is unstable before no feedback controller is introduced.

Discuss Case1 and Case 2, respectively. Given $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\alpha_3 = -0.1$, $\alpha_4 = 0.5$, $\beta_1 = 1.5$, $\beta_2 = -1.5$, $\beta_3 = 1$, $\beta_4 = -1$, according to theorem 3, we can obtain the gain matrices of state feedback controller by solving LMIs of (15) as followed:

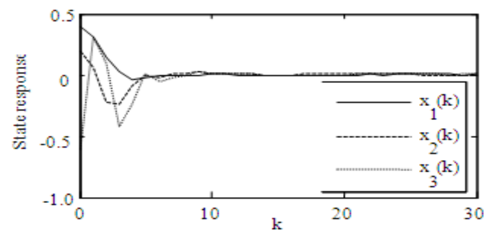
Case 1:

$$K_1 = \begin{bmatrix} -0.6717 & -0.2041 & -2.7298 \\ 0.2148 & 0.9913 & 1.6373 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -1.6266 & 4.3814 & -2.8966 \\ 0.3841 & -1.3978 & -0.9019 \end{bmatrix}$$



(a) Case 1



(b) Case 2

Fig. 2: Close-looped state trajectories for the two cases

$$K_3 = \begin{bmatrix} 0.7686 & -1.2280 & -0.8318 \\ -0.3694 & 1.2138 & 0.4392 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 3.6843 & -0.3658 & -3.0017 \\ -4.4202 & 0.9981 & 3.2011 \end{bmatrix}$$

Case 2:

$$K_1 = \begin{bmatrix} -0.5675 & -0.2868 & -2.6602 \\ 0.1767 & 1.0215 & 1.6118 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -1.7202 & 4.4131 & -2.7138 \\ 0.3858 & -1.3984 & -0.9052 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.7681 & -1.1625 & -0.6786 \\ -0.3685 & 1.0975 & 0.1852 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 4.0438 & -0.6193 & -2.5253 \\ -4.8753 & 1.3191 & 2.5979 \end{bmatrix}$$

Given initial condition $x(0) = [0.4\ 0.2\ -0.6]^T$, $r(0) = 1$, the state trajectories of close looped system for the two cases are shown as Fig. 2 (a) and (b) show Case 1 and Case 2 respectively. From Fig. 2, it can be clearly seen that in two cases, the close looped systems are both stabilized after introducing the state feedback controller designed by theorem 3 when the system structures abruptly changes between the 4 modes. So the stabilization control of system (1) is realized by the controller design method of theorem 3.

CONCLUSION

In this study, by effectively using information of unknown transition probabilities, a new general stability criteria of DMJSs with partly unknown TPs in a set of strict linear matrix inequalities is proposed.

Furthermore, a method of designing state feedback controller is discussed to realize the stabilization control of considered systems. More suitable for the real engineering background, the proposed results have more practical significance and theory value.

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