Research Journal of Applied Sciences, Engineering and Technology 7(5): 930-936, 2014
DOI:10.19026/rjaset.7.337
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: January 29, 2013 Accepted: February 25, 2013 Published: February 05, 2014

Research Article

Study on VMI Inventory Control Mode based on the Third-Party Logistics

[1, 2]Haoxiong Yang and [1]Jindan Li
[1]School of Business, Beijing Technology and Business University, Beijing 100048, China
[2]Research Base of Capital Retail, Beijing 100048, China

Abstract: Adding the third party logistics enterprises between the suppliers and the retailers is a kind of the development of VMI mode, in this mode; inventory pressure is transferred to the third party logistics enterprise. In view of this situation, the VMI inventory control model which treats total inventory control costs as the objective function is built based on from four dimensions: the inventory holding costs, the fixed delivery costs, replenishment costs and customer waiting costs. After solving the model and sensitivity analyzing related parameters, it can be inferred that related parameters in the VMI model of the introduction of the third party logistics have effects on inventory control costs.

Keywords: Inventory control, sensitivity analysis, third-party logistics, vendor managed inventory

INTRODUCTION

Inventory management is an important part in the production and operation of various enterprises and is an important link to realize value-added value chain. But enterprises in the supply chain manage inventory in their own way and strategy of inventory control is not the same, so "bullwhip effect" inevitably appears in the supply chain, which leads to high inventory and unbalanced production. Vendor Managed Inventory (VMI), which is an inventory mode in a supply chain environment, links the inventory management of upstream enterprises to downstream. Demand and inventory information sharing and collaboration in upstream and downstream enterprises can effectively reduce total inventory in the entire supply chain, so as to promote the synchronization of the supply chain and efficient operation (Burke, 1996; Cottrill, 1997).

About the inventory control model under the VMI environment, many scholars at home and abroad have done a lot of research and applications examples. The retailer can set the transshipment prices to coordinate the two-echelon supply chain consisting of suppliers and retailers (Rudi *et al*., 2001). In addition, two-echelon supply chain model when a supplier faced multiple retailers was studied, in which all retailers was owned by l economic entity targeting at maximizing the overall profits of all retailers. The study analyzes two cases: one case assumes that the wholesale price is fixed; in these case retailers entities' optimal select for various retailers inventory was obtained (Dong and Rudi, 2004). In Stackelberg competition model described by some scholars, suppliers by determining the wholesale price is the leader and retailers economic entity needs to determine the optimal inventory for various retailers. The model analyzed and compared the equilibrium solutions in different parameter values (Larivier and Porteus, 2001). Analysis on three representative integrated inventory shipment model under the VMI environment was proposed, which is respectively: time-based integrated delivery model, integrated delivery model based on quantity, integrated delivery model based on the time and quantity and calculated the optimal solution, by setting the parameter values and comparing the pros and cons, determined applicable environment of each model (Liu and Yuan, 2003); Some scholars studied time-based integrated delivery strategies and integrated delivery strategies based on quantity under the VMI environment when the level of service required by the customer is different in the different demand region (Ji *et al*., 2006); The VMI inventory strategy was adopted between chain supermarkets and suppliers, taking full account of the mutual interests of the suppliers and demanders and proposing a inventory replenishment model targeting at minimizing average cost and the global optimal solution of the model calculation was given (Zhang and Wang, 2008); Inventory costs control in VMI mode was studied, in order to circumvent the shortcomings of the traditional supply chain management, established the inventory cost control model of delivery strategies based on quantity. It got a complete total cost formula by decomposing inventory costs and got optimized number of delivery, delivery times and optimal operating costs by optimizing the method to solve the model (Pan *et al*., 2009).

Corresponding Author: Haoxiong Yang, School of Business, Beijing Technology and Business University, Beijing 100048, China, Tel.: 13426053556

To sum up, the current researches about VMI consider the case of retailers managing inventory, the realization of this model requires strong information exchange between retailers, or simply assume that retailers belong to an entity. For inventory control research under the VMI environment, regardless of what to do to improve the model, it is mostly based on the assumptions of the single-vendor and multi-client. In the process of implementation of VMI, a growing number of manufacturing companies (suppliers) put the sales outsourcing to specialized third-party logistics enterprises, namely increased third-party logistics enterprises between suppliers and vendors and pressure on the stock transfer to the third-party logistics enterprises. Inventory control study of increasing third-party logistics enterprises in VMI mode, is still rare. The study is based on VMI inventory control model and studies VMI inventory control problems after the introduction of third-party logistics enterprises from four cost dimensions: inventory holding costs, fixed costs of shipments, replenishment costs and customer waiting cost.

INVENTORY CONTROL MODEL

- **Problem description:** There are mainly two services modes of third party logistics: one is the type of strategic cooperation and that only to build a strategic partnership with a single manufacturing enterprise, only providing logistics services for a manufacturing company; another is the provision of logistics services for a number of manufacturing companies and it is responsible for distribution to multi-area stores. Since the first case is relatively rare, the model established in this study points at the latter, namely a logistics enterprise faces multiple suppliers and multiple customers of different regions (R_1, R_2….), as shown in the Fig. 1.

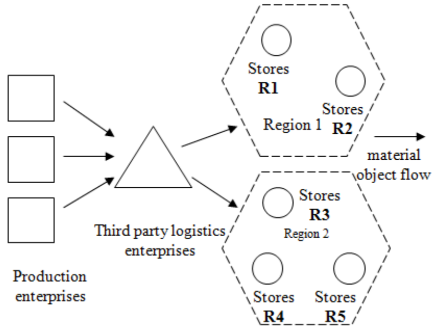

Fig. 1: VMI model after the introduction of third-party logistics enterprises

The model assumes that the customer demand is random and obeys the Poisson distribution of the different intensity. In an integrated delivery cycle, distribution enterprises come together customer needs and form an optimal economic order quantity and then shippe. After the delivery of third-party logistics enterprises through the current inventory minus the accumulated demand on an integrated delivery cycle. For the (s, S) inventory management mode of most third-party logistics companies, namely checking inventory status at any time, they start ordering when the inventory reorder level reduces to s, the largest inventory maintains the same constant S after ordering. If inventory is s when orders, the order quantity will be S-s. Once you make a delivery decision, all needs can be met. If the current inventory is insufficient to meet all the needs at shipment time point, third-party logistics enterprises will immediately replenish the inventory to the supplier in order to meet the requirements. Order point s considered here is determined to the safety stock and the forecast demand during replenishment, so there is no time delay in the replenishment model.

Based on the above assumptions, the study will analyze optimal replenishment quantity, shipments quantity and delivery frequency when third party logistics companies minimize the total cost per unit time in the case of the certain demand.

The total cost includes four parts: the replenishment cost, inventory holding costs, customer waiting cost and delivery cost. Replenishment cost and shipping cost refer to the fixed costs, regardless of the replenishment quantity but only the replenishment frequency.

- **Description of the relevant parameters and variables:** Parameters related with the model are as follows:

$\lambda_{m,\,i}$ = Demand intensity of point R_i (Poisson stream), i = 1, 2, 3, … (their goods are in the charge of the m-th supplier, m = 1, 2, …)
C_R = The fixed cost of replenishment
C_D = The fixed cost of shipment
h = Holding costs of unit time every unit inventory
w = Waiting cost of unit time every unit demand of customer
$L(t)$ = Demand of not meeting at t point
$I(t)$ = Inventory levels at t point
Q_m = The target inventory levels that the m-th supplier is responsible for, namely inventory levels the m-th supplier achieves after replenishmen, m = 1, 2, …; Defining $Q = \sum_{m=1}^{\infty} Q_m$, that Q is the goal inventory levels of the entire warehouse.
N_j = Shipment quantity during the j-th delivery cycle, j = 1, 2, …
T_j = The length of time of the j-th delivery cycle, j = 1, 2, …

The definition of $S_{i,\ n}$ is arrival time for the n-th demand of the point R_i, get $E[S_i] = \sum_{i=0}^{\infty} \frac{n_i}{\lambda_i}$

Defining N(t) = sup{n: $S_{i,\ n} \leq t$, found that N(t) is a demand within t units time, get $E[N(t)] = \sum_{i=0}^{\infty} \lambda_i t$.

When the cumulative demand of all shipments during each replenishment cycle is more than Q, you need replenishment. Shipment quantity K within a replenishment cycle is:

$$K = \inf\{k : \sum_{j=1}^{k} N_j > Q\}$$

Define two adjacent replenishment intervals for a replenishment cycle. In case demand obeys Poisson distribution, inventory levels I(t) can be considered as independent and identically distributed random variable according to replenishment cycle.

C represents the expected long-term average cost of the inventory replenishment and integrated delivery model:

$$C = \frac{E[C_{total\cos ts}]}{E[\overline{T}]}$$

- **VMI inventory control model:** Inventory control model selection based on the number of shipments strategy to get closer to the actual situation of the enterprise. When the total demand of goods of every region accumulates to Q_e units, third-party logistics companies deliver goods to meet the demand. The length of each integrated delivery cycle is a random variable, therefore, expected long-term average cost of inventory replenishment and integrated delivery model is a function about Q_e and K, so the essence to solve the problem in this strategy is:

$$\begin{aligned} &min\, C(Q_e, K) \\ &s/to\ \ Q_e \geq 1,\quad K \geq 1 \end{aligned}$$

Integrated delivery model is updated once each time the demand accumulated to Q_e. Shipment quantity is K during each replenishment cycle, so:

$$E[T] = K \frac{Q_e}{\sum \lambda_{m,i} \times Q_m / Q} \tag{1}$$

- **Inventory holding costs:**

$$E[I(t)] = \begin{cases} (K-1)Q_e, when \\ 0 \leq t \leq \dfrac{Q_e}{\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \\ (K-2)Q_e, when \\ \dfrac{Q_e}{\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \leq t \leq 2\dfrac{Q_e}{\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \\ \cdots \\ 0, when \\ (K-1)\dfrac{Q_{\mathrm{e}}}{\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \leq t \leq K\dfrac{Q_{\mathrm{e}}}{\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \end{cases}$$

$$\begin{aligned} \overline{E[I(t)]} &= (K-1)Q_e + (K-2)Q_e + \ldots\ldots \\ &+ [K-(K-1)]Q_e + 0 = \frac{K(K-1)Q_e}{2} \end{aligned}$$

Then:

$$E[\overline{I(t) \times T \times h}] = E[\overline{I(t)}] \times E[T] \times h = \frac{(K-1)KQ_e^2 h}{2\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \tag{2}$$

- **The fixed cost of the shipment:** The delivery times within each replenishment cycle is K, so:

$$E[\overline{KC_D}] = KC_D \tag{3}$$

- **Replenishment cost:** Due to replenishment one time during integrated shipment cycle, so the shipping cost is C_R:

- **Customer waiting costs:**

$$\begin{aligned} E[\overline{T_j - S_i}] &= E[\ (T_j - S_1) + (T_j - S_2) + \ldots + (T_j - S_{Q_e})] \\ &= wE(Q_e T_j - S_1 - S_2 - \ldots\ldots - S_{Q_e}) \\ &= wQ_e[E(T) - E(S_i)] \quad = wQ_{\mathrm{e}} \frac{Q_e - 1}{2\sum \lambda_{\mathrm{m,i}} \times Q_m / Q} \end{aligned}$$

Then,

$$E[\overline{(T_j - S_i) \times K}] = wK \frac{Q_e^2 - Q_e}{2\sum \lambda_{m,i} \times Q_m / Q} \tag{4}$$

So, to sum up, by (1), (2), (3), (4) get the units inventory control costs are as follows:

$$\begin{aligned} C(Q_e, K) = [C_R + CA_D + \frac{(K-1)KQ_e^2 h}{2\sum \lambda_{m,i} \times Q_m / Q} + \\ wK \frac{Q_e^2 - Q_e}{2\sum \lambda_{m,i} \times Q_m / Q}] \div E[T] \end{aligned}$$

Finishing, to obtain:

$$C(Q_e, K) = \frac{C_R \sum \lambda_{m,i} \times Q_m / Q}{KQ_e} + \frac{C_D \sum \lambda_{m,i} \times Q_m / Q}{Q_e} + \frac{(K-1)Q_e h}{2} + w\frac{Q_e - 1}{2}$$

MODEL ANALYSIS

Model solution: Making C(Q_e, K) be the smallest solution, cross terms contained in Q_e and K are found by observing. In order to calculate the minimum value, Q_e and K are seen as continuous variables. Obtaining the Hessian matrix H of C(Q_e, K) is:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial Q_e^2} & \frac{\partial^2 f}{\partial Q_e \partial K} \\ \frac{\partial^2 f}{\partial K \partial Q_e} & \frac{\partial^2 f}{\partial K^2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2\sum \lambda_{m,i} \times Q_m / Q}{Q_e^3}(\frac{C_R}{K} + C_D) & \frac{C_R \sum \lambda_{m,i} \times Q_m / Q}{Q_e^2 K^2} \\ \frac{C_R \sum \lambda_{m,i} \times Q_m / Q}{Q_e^2 K^2} & \frac{2C_R \sum \lambda_{m,i} \times Q_m / Q}{Q_e K^3} \end{bmatrix}$$

To obtain,

$$det[H] = \frac{3(\sum \lambda_{m,i} \times Q_m / Q)^2 C_R^2}{Q_e^4 K^4} + \frac{4(\sum \lambda_{m,i} \times Q_m / Q)^2 C_R C_D}{Q_e^4 K^3} > 0$$

So, *[H]* is Positive definite matrix, C(Q$_e$, K) is a strict convex function about Q$_e$ and K. Setting C(Q*$_e$, K*) is optimal solution of the problem and Q$_e$≥1, K≥1, making $\frac{\partial f}{\partial Q_e} = 0, \frac{\partial f}{\partial K} = 0$.

Getting the solution to target problem is:

When $w \geq \left(\frac{C_D}{C_R} + 1\right) h$

$$K^* = [\sqrt{\frac{(w-h)C_R}{hC_D}}],$$

$$Q_e^* = \max\{[\sqrt{\frac{2\sum \lambda_{m,i} \times Q_m / Q}{(K^*-1)h + w}(\frac{C_R}{K^*} + C_D)}],1\}$$

When $w < (\frac{C_D}{C_R} + 1)h$,

$$K^* = 1,$$

$$Q_e^* = \max\{[\sqrt{\frac{2\sum \lambda_{m,i} \times Q_m / Q}{w}(C_R + C_D)}],1\}$$

Its practical significance: when only making shipment one time during each integrated delivery cycle, factors affecting economic shipping bulk are demand intensity for various retail outlets, target inventory levels of each supplier, the goal inventory of the entire warehouse, waiting cost in a unit time each unit demand of customers; demand intensity for various retail outlets and each supplier's target inventory levels are positively related to economic delivery batch. The entire warehouse target inventory and customer waiting cost are negatively related to economic delivery batch. If more than one time shippment during each entire shipment cycle, delivery frequency is positively correlated to the fixed costs of order, difference between customers waiting costs and inventory holding costs and is negatively related to inventory holding costs, fixed costs of shipment; Factors which are positively correlated with economic shipping bulk are demand indensity for various retail outlets, each supplier's target inventory levels, fixed costs of orders and fixed costs of shipments. The negative factors are

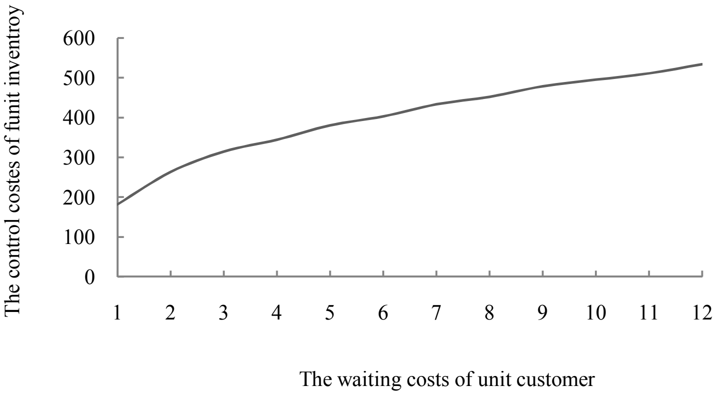

Fig. 2: Sensitivity analysis diagram of w to the control costs of unit inventory

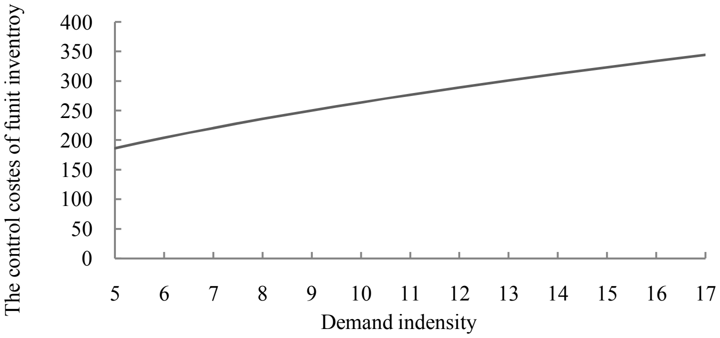

Fig. 3: Sensitivity analysis diagram of λ to the control costs of unit inventory

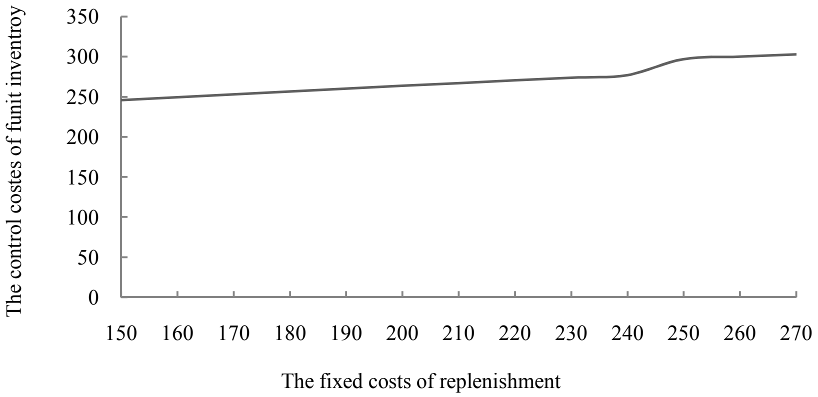

Fig. 4: Sensitivity analysis diagram of C_R to the control costs of unit inventory

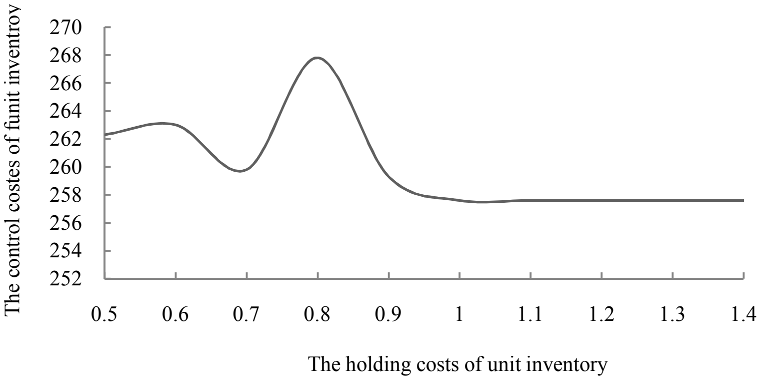

Fig. 5: Sensitivity analysis diagram of h to the control costs of unit inventory

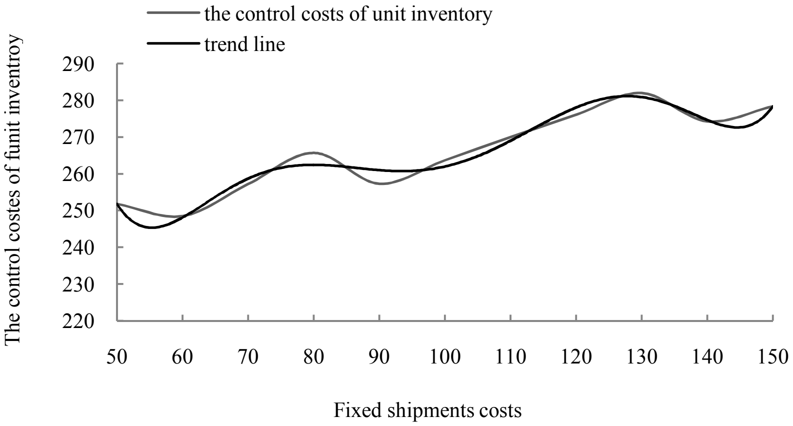

Fig. 6: Sensitivity analysis diagram of C_D to the control costs of unit inventory

inventory holding costs and shipments frequency during integrated cycle.

Sensitivity analysis of the model parameters: In order to facilitate evaluation of the parameters for the impact on the costs of unit inventory control, analysis of the sensitivity of each variable in the optimal solution is made. Set its original parameter value: w = 2, h = 1, CD = 100, CR = 200,

$$\sum \lambda_{m,i} \times Q_m / Q = \lambda = 10$$

Sensitivity analysis diagram on the control costs of unit inventory is as follow in the Fig. 2 to 6.

Conclusion one: The incensement of w (the waiting costs of unit customer), λ (demand intensity) and C_R (the fixed costs of replenishment) will induce to the incensement of C (inventory control costs) and w (the waiting costs of unit customer) above three is the greatest influence on C (inventory control costs).

Evidently obtaining from Fig. 2-4, the waiting costs of unit customer, demand indensity and the fixed costs of replenishment are positively related to the control costs of unit inventory. The greater these three variables are the higher the inventory control costs are. Three variables have effects on inventory control costs in different degree, from high to low are: the waiting costs of unit customer, demand indensity, the fixed cost of replenishment. the waiting costs of unit customer are higher, in order to maintain the level of customer service, delivery times will inevitably increase to meet demand and the holding costs of inventory falls, leading to a surge in shipping costs, causing the rising of inventory control costs.

Conclusion two: When $w \geq \left(\frac{C_D}{C_R}+1\right)h$, the effect which h (the holding costs of unit inventory) have on C (inventory control costs) is undulated; When $w < \left(\frac{C_D}{C_R}+1\right)$, h (the holding costs of unit inventory) have no effect on C(inventory control costs):

According to Fig. 5, it is obvious that the inventory holding costs have fluctuated impact on inventory control costs when $w \geq \left(\frac{C_D}{C_R}+1\right)h$. Inventory control costs are composed of shipment costs, replenishment costs, customer waiting costs and inventory holding costs together. The level of inventory holding costs determines the frequency of delivery, that affects the number of shipments and replenishment, this one variable substantially affects the three elements constituting inventory control costs, resulting in the undulator of the total inventory control costs. When the inventory holding cost increases to a certain value, due to $w < \left(\frac{C_D}{C_R}+1\right)$, inventory holding costs have no effect on K*, Q_e, so is C (control costs of unit inventory), inventory control costs are fixed values after it . Therefore, do not blindly reduce inventory holding costs to achieve the purpose of reducing the total inventory control costs, which is meaningless. Figure 5 shows that, when the inventory holding cost is reduced to a boundary value, it has no effect on the inventory control costs; inventory control costs will not be reduced because of the continued reduction of inventory holding costs.

Conclusion three: The effects which C_D (fixed shipments costs) has on C (the control costs of unit inventory) are undulated up: Fixed shipments costs for inventory control costs undulator rise (solid line as shown in Fig. 6, the general trend is still rising, that the fixed shipments costs are approximate positive correlation to inventory control costs. This is mainly caused by the change of delivery times due to the changes in the fixed shipment costs. By observing that, in the case of the same delivery times, fixed shipments costs are lower, the total inventory control costs are lower. The sensitivity analysis of entire fixed shipments cost to the control costs of unit inventory tends to a 6 times polynomial (dashed line as shown in Fig. 6. It can be more convenient to estimate total inventory control costs by fitting a trend line.

CONCLUSION

In this study, in the context of the third-party logistics introduced in implementation process of the VMI model, under VMI environment integrated delivery model based on the numbers is improved to make it more in line with the mode, that real third-party logistics enterprises in the face of multiple suppliers, multi-client of different demand intensity in the same region. It build a VMI inventory control model which treats inventory control total costs as the objective function, from four cost dimensions: inventory holding costs, fixed shipment costs, replenishment costs and customers waiting costs. It has strong practical significance.

ACKNOWLEDGMENT

The author thanks the anonymous reviewers for their valuable remarks and comments. This study is supported by National Social Science Fund of China (Grant No. 11CGL105) and Beijing Philosophy Social Science Planning Project (Grant No. 12JGC100).

REFERENCES

Burke, M., 1996. It's time for vendor managed inventory [J]. Ind. Distribut., 85(2): 90-95.

Cottrill, K., 1997. Reforging the supply chain [J]. J. Bus. Strat., 18(6): 35-39.

Dong, L.X. and N. Rudi, 2004. Who benefits from transshipment? Exogenous vs. endogenous wholesale prices [J]. Manag. Sci., 50(5): 645-657.

Ji, S.F., C.J. Fu and X.Y. Huang, 2006. Choosing stock replenishment/delivery model in view of different service levels as customers required [J]. J. Northeastern Univ. Nat. Sci., 27(10): 1169-1172.

Larivier, M.A. and E.L. Porteus, 2001. Selling to the newsvendor: An analysis of price-only contracts [J]. Manuf. Serv. Oper. Manag., 3(4): 293-305.

Liu, L.W. and J.R. Yuan, 2003. Inventory and dispatch models in VMI systems [J]. Chinese J. Manag. Sci., 11(5): 31-36.

Pan, W.Q., W.S. Li, C.M. Gao and M.A. Yong-Hong, 2009. Optimization of quantity-based dispatching policy in a Vendor Managed Inventory (VMI) environment [J]. J. Beijing Univ. Chem. Ind. Nat. Sci., 26(1): 102-104.

Rudi, N., K. Sandeep and F.P. David, 2001. A two-location inventory model with transshipment and local decision making [J]. Manag. Sci., 47(12): 1668-1680.

Zhang, W.N. and X.L. Wang, 2008. Inventory optimized model and simulation of Chain supermarkets based on VMI [J]. Logist. Technol., 27(12): 49-54.