Research Journal of Applied Sciences, Engineering and Technology 7(6): 1125-1132, 2014
DOI:10.19026/rjaset.7.369
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: February 06, 2013 Accepted: March 08, 2013 Published: February 15, 2014

Research Article

Nonlinear Vibration Study on the Gear System based on the Non-dimensional Differential Equation of Dynamic Model

[1]Fan-Ping Qing, [2]Song-Shao Fang, [1]Xing-Yan Feng and [1]Zhang-Song Yun
[1]Shanghai University of Engineering and Science, Shanghai
[2]Elomatic Consulting and Engineering Company, Shanghai

Abstract: The study derives the kinetics equation of the system and analyzes vibration performance considering the nonlinear factors like time-varying mesh stiffness, gear back lash, mesh error. With the time-varying mesh stiffness of the gear increasing, the vibration of the gear system is more intense. The gear backlash has little effect on the aperiodic behaviour. However, with the clearance increasing, the system response quickly converts to chaos response from single frequency response with the increase of gap and the mesh impact is more serious. The excitation frequency is close to the resonance frequency of system, the system appears chaotic response and the vibration amplitude increases. When the frequency is far from the natural frequency, the response of the system tends to be steady. So it is easy to predict the dynamic performance in the design and make the system avoid the resonance frequency. In the following profile modification of gear, it also provides the data sources.

Keywords: Gear system, nonlinear vibration, time-varying mesh stiffness

INTRODUCTION

The analysis of tooth mesh stiffness is based on the classic material mechanics early, which includes Equivalent toothed law, Cantilever method (Li and Jun, 1997). And then the elastic mechanics method is used to study the deformation of gear meshing, but this kind of method are very different compared with actual working condition in shape, load, boundary conditions, etc. With the development of computer technology, people begin using the finite element method to calculate the elastic deformation of the gear teeth and tooth root stress (Wang and Liu, 2003). However, the contact force (engaging force) is a distributed force rather than concentrated force. To solve this problem, the introduction of a numerical contact method is chosen.

There are two kinds of nonlinear dynamics model of gear system which are rigid impact model (Shaw, 1985) and elastic impact model. The first model hypothesizes that impact object is rigid and uses a compensation coefficient to describe the energy loss (Smith and Liu, 1992). Although this method can not be directly used for the analysis of the gear system, but some of the viewpoints and conclusions for the gear system is of great value. The elastic impact model (Dubowsky and Freudenstein, 1971; Veluswami and Crossley, 1975) can reveal some important characteristics of nonlinear clearance vibration. Two kinds of damping are considered in the model: viscous damping and impact damping (Dubowsky and Freudenstein, 1971). The analysis is based on the elastic impact model in study.

THE KINETICS MODEL OF GEAR

Nonlinear dynamic model: Assuming that the system is composed with the only having elastic spring and the only having inertial quality block, this study uses the lumped mass method (Li and Jun, 1997; Tamminana *et al*., 2006) to build the nonlinear dynamic model, ignoring the elastic deformation of the transmission shaft and support system and considering about the nonlinear factors of the time-varying stiffness k(t), mesh error e(t), gear back lash 2b. The model is showed as Fig. 1.

According to the Newton Law and the nonlinear dynamic model of the gear, the motion differential Eq. (1) is established as follow:

$$\begin{cases} I_1\ddot{\theta}_1 + cr_{b1}[r_{b1}\dot{\theta}_1 - r_{b2}\dot{\theta}_2 - \dot{e}(t)] + r_{b1} \\ k(t)f(r_{b1}\theta_1 - r_{b2}\theta_2 - e(t)) = T_1 \\ I_2\ddot{\theta}_2 + cr_{b2}[r_{b1}\dot{\theta}_1 - r_{b2}\dot{\theta}_2 - \dot{e}(t)] - r_{b2} \\ k(t)f(r_{b1}\theta_1 - r_{b2}\theta_2 - e(t)) = -T_2 \end{cases} \tag{1}$$

where,
t = Time

Corresponding Author: Fan-Ping Qing, Automotive Engineering College, Shanghai University of Engineering Science, Shanghai, Tel.: 13524976219

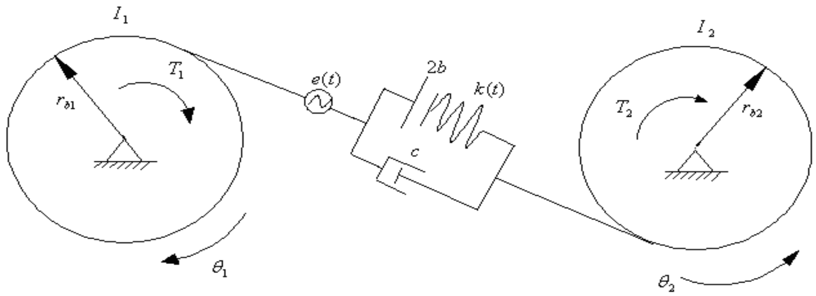


Fig. 1: The nonlinear dynamic model of the gear

c = The meshing damping
r_{bi} = The radius of base circle of the gear i (i = 1, 2)
θ_i = The torsional displacement of the gear i (i = 1, 2)
I_i = The moment of inertia of gear i (i = 1, 2)
T_i = The torque of the gear i (i = 1, 2)
$k(t)$ = The time-varying stiffness
$e(t)$ = The general error of mesh gears measured along the base circle tangent direction

The dynamic transmission error of gear:

$$x = r_{b1}\theta_1 - r_{b2}\theta_2 \tag{2}$$

Deduce by the Eq. (1) and the Eq. (2):

$$m_e\ddot{x} + c[\dot{x} - \dot{e}(t)] + k(t)f(x - e(t)) = F_m \tag{3}$$

$$m_e = \frac{I_1 I_2}{(I_1 r_{b2}^2 + I_2 r_{b1}^2)} \tag{4}$$

$$F_m = \frac{T_1}{r_{b1}} = \frac{T_2}{r_{b2}} \tag{5}$$

where,
m_e = The equivalent mass
F_m = The total normal force, without considering the fluctuation of the prime mover and the load
$f(x)$ = The nonlinear function of the backlash

The defined transmission error is the difference between dynamic transmission error and the static transmission error: $q(t) = x(t) - e(t)$.
So the Eq. (3) can be transformed into as follows:

$$m_e\ddot{q}(t) + c\dot{q}(t) + k(t)f(q(t)) = F_m - m_e\ddot{e}(t) \tag{6}$$

where,
F_m = The external excitation
$-m_e\ddot{e}(t)$ = The gear's internal excitation (the error stimulation)

Equation parameters:

- **Gear mesh stiffness $k(t)$:** Using the finite element contact method, the mesh stiffness curve is obtained as shown in Fig. 2. Time-varying stiffness changes periodically, so the gear mesh stiffness can be developed into Eq. (7) in form of the Fourier series (Wang and Howard, 2005):

$$k(t) = k_m + \sum_{j=1}^{\infty}[A_j\cos(j\omega_e t) + B_j\sin(j\omega_e t)] = k_m + \sum_{j=1}^{\infty}k_j\cos\left(j\omega_e t + \phi_j\right) \tag{7}$$

where,
k_m = The average meshing stiffness
n_i, z_i = Gear rotational speed and tooth number (i = 1, 2)
ω_e = Meshing frequency
ϕ_j = Phase angle
k_j = Amplitude of the j-th harmonic

- **General teeth errors $e(t)$:** In the gear transmission process, the mesh error takes the gear meshing frequency ω_e (Munro, 1992; Raclot and Velex, 1999) as the basic frequency. Assuming all the engaging position occurring on the theoretical meshing line, the mesh error is converted into the form of the Fourier series based on the gear mesh frequency, as displayed in Eq. (8):

$$e(t) = e_m + \sum_{j=1}^{\infty}e_{Aj}\left[\cos(j\omega_e t) + e_{Bj}\sin(j\omega_e t)\right] = e_m + \sum_{j=1}^{\infty}e_j con(j\omega_e t + \theta_j) \tag{8}$$

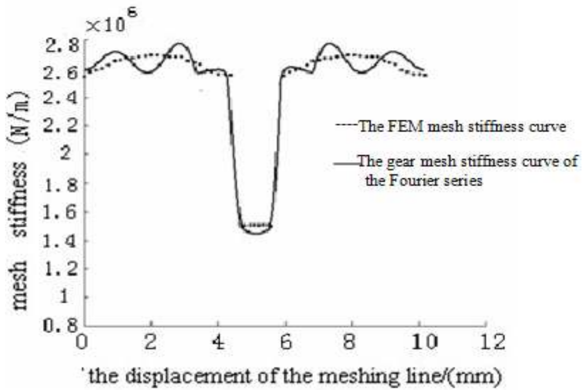


Fig. 2: Gear mesh stiffness curve

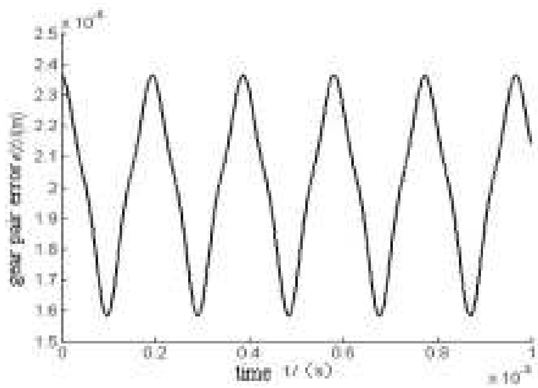


Fig. 3: The gear mesh error curve

Table 1: The values of mesh error

	Static transfer error/(m)	
Order j	Amplitude e_j (10^{-6})	Phase angle θ_j (rad)
0	20	
1	3.45	0
2	0.26	π
3	0.45	0

where,

e_m = The average amplitude of gear error

e_{Aj} , e_{Bj} , e_j = Each harmonic component amplitude of error

θ_j = The harmonic phase angle of error

Every harmonic component amplitude of error is defined as the Munro experimental values (Munro, 1992) in analysis the affection meshing error on gear vibration, which are shown in Table 1 and Fig. 3.

- **Meshing damping:** The equation (Li and Jun, 1997) of meshing damping is as follows:

$$c = 2\varsigma\sqrt{\frac{k_m r_{b1}^2 r_{b2}^2 I_1 I_2}{r_{b1}^2 I_1 + r_{b2}^2 I_2}} \tag{9}$$

where, ς is the damping ratio

- **Nonlinear function of gear back lash:** The backlash (Wang and Liu, 2003; Theodossiades and Natsiavas, 2000) is circumferential wobbles of one gear calculated on the pitch circle when another tooth is fixed for the assembled gear pair. Backlash referred in this study is the measured value in the engagement line. Defining the gear backlash as 2b, the nonlinear function of the gear backlash is obtained Eq. (10) assuming the backlash is symmetric:

$$f(x) = \begin{cases} x-b & x \ge b \\ 0 & -b \le x \le b \\ x+b & x \le -b \end{cases} \tag{10}$$

DIMENSIONLESS ON THE DIFFERENTIAL EQUATION OF GEAR SYSTEM

In the study, the differential equation of gear is converted into non-dimensional form (Tamminana *et al*., 2006; Raclot and Velex, 1999; Theodossiades and Natsiavas, 2000). This dimensionless equation doesn't depend on the physical quantity any longer, so that it can avoid excessive difference of magnitude between parameters in the numerical analysis and it can offer facilities for controlling error and defining step.

From the Eq. (6), the natural frequency ω_0 of SDOF gear system:

$$\omega_0 = \sqrt{k_m / m_e}$$

where,

k_m = The mean of mesh stiffness
m_e = Equivalent mass

Defining dimensionless time $\tau = \omega_0 t$, $q(t) = lu(t)$, (l is the characteristic length, taking unit length as 10e-6) $\dot{q}(t)$ and $\ddot{q}(t)$ can be deduced to Eq. (11) and Eq. (12):

$$\begin{aligned} \dot{q}(t) &= \frac{dq}{dt} = \frac{dq}{d\tau}\frac{d\tau}{dt} \\ &= \frac{d(l \cdot u)}{d\tau} \cdot \frac{d(t\omega_0)}{dt} = l\omega_0 \dot{u}(\tau) \end{aligned} \tag{11}$$

$$\ddot{q}(t) = \frac{d\dot{q}}{dt} = \frac{d(l\omega_0 \dot{u})}{d\tau} \cdot \frac{d\tau}{dt} = l\omega_0^2 \ddot{u}(\tau) \tag{12}$$

Taking the meshing stiffness Eq. (7) and transmission error Eq. (8) into the Eq. (6), the analysis model of dimensionless is obtained as follows:

$$\ddot{u}(\tau)+2\varsigma\dot{u}(\tau)+(1+\sum_{n=1}^{\infty}k_{bm}\cos\left(n\Omega\tau\right)f\left(u\left(\tau\right)\right)$$
$$=F_0+\Omega^2\sum_{j=1}^{\infty}j^2e_{jb}\cos(j\Omega\tau+\theta_j)) \tag{13}$$

$$f(u(\tau))=\begin{cases}u(\tau)-b/l & u(\tau)>b/l\\ 0 & -b/l<u(\tau)<b/l\\ u(\tau)+b/l & u(\tau)<-b/l\end{cases} \tag{14}$$

where,

$$\varsigma=\frac{c}{2m_e\omega_0},\ \Omega=\frac{\omega_e}{\omega_0},\ k_{bm}=\frac{k_j}{k_m},\ F_0=\frac{F_m}{m_e l\omega_0^2},\ e_{bj}=\frac{e_j}{l},$$

$f(u(\tau))$ is the dimensionless nonlinear function of clearance.

NUMERICAL CALCULATION AND RESULTS ANALYSIS

Numerical calculation: This study uses the variable step size fourth order Runge-Kutta to solve the nonlinear differential equation. First, the second order differential Eq. (13) is turned into two first-order differential equations (He *et al*., 2008) and then solve the equation directly by the fourth order Runge-Kutta. In order to obtain two first-order differential equations from Eq. (13), it is necessary to define a new variable u:

$$v=\{v_1,v_2\}'=\{u,\dot{u}\}' \tag{15}$$

So the Eq. (13) can be deduced into the form of state space (Eq. 16):

$$\begin{cases}v_2=\dot{v}_1\\ \dot{v}_2(\tau)+2\varsigma v_1(\tau)+(1+\sum_{n=1}^{\infty}k_n\cos(n\Omega\tau)f(u(\tau)))\\ =F_0+\Omega^2\sum_{j=1}^{\infty}j^2e_{jb}\cos(j\Omega\tau+\theta_j)\end{cases} \tag{16}$$

The static deformation solved by tooth stiffness is considered as initial values. Successive iterating won't stop until results are close to the desired solution. Numerical solution of Eq. (15) is finished by matlab software. In addition, considering the impact of the initial value, it is necessary to delete the response of hundreds of cycle at the beginning, so that the ideal phase diagram is gained.

Analysis of results: In analysis, the parameters of the gear is shown in Table 2 (Al-Shyyab and Kahraman, 2005; Ambarisha and Parker, 2007; Wang and Howard, 2005).

- **The effect of time varying mesh stiffness for gear vibration:** The above convert the mesh stiffness into the form of the Fourier series Eq. (7), the high-order harmonic has the greater impact at low frequencies because the n-th harmonic produces resonance only in $\Omega=1/n$ near. The mesh stiffness is converted into the form of harmonic function, as shown in Eq. (17) and Eq. (18), so as to study the effect of time-varying stiffness on system vibration:

$$k(\tau)=1+\sigma\cos(\Omega t+\phi) \tag{17}$$

Table 2: The parameters of the gear system

Gear	Tooth number z	Modulus m (mm)	Addendum coefficient	Tooth width	Moment of inertia
1	62	2.5	1	85	37257
2	42	2.5	1	85	82103

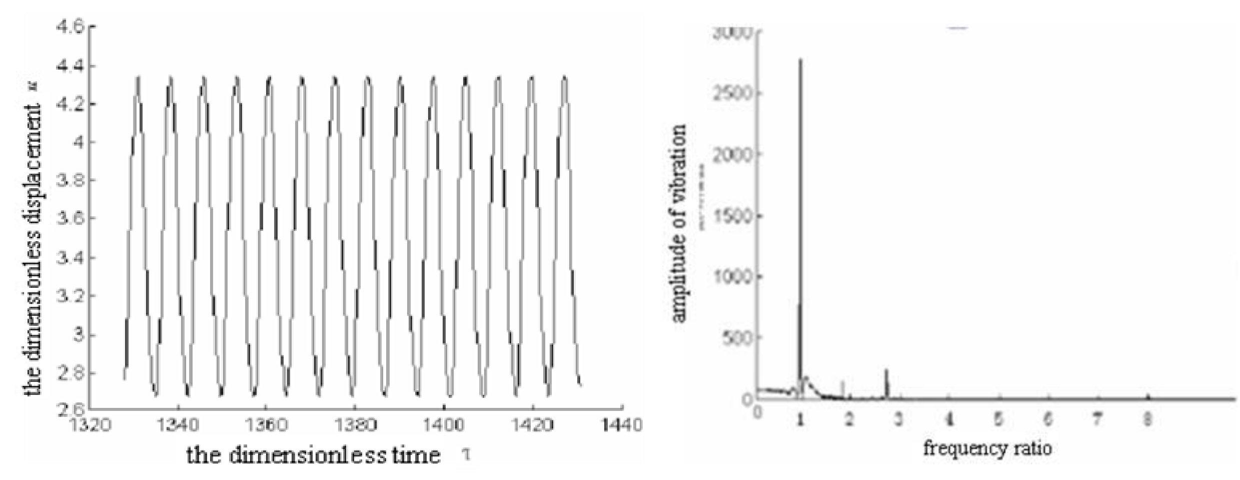


Fig. 4a: Time-domain chart and spectrogram of the system with σ = 0

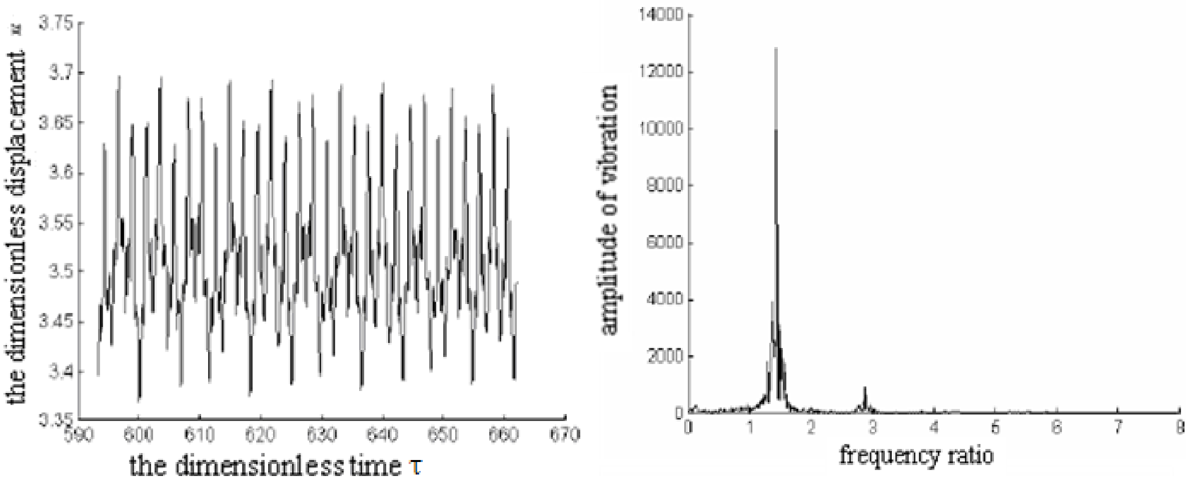


Fig. 4b: Time-domain chart and spectrogram of the system with σ = 0.4

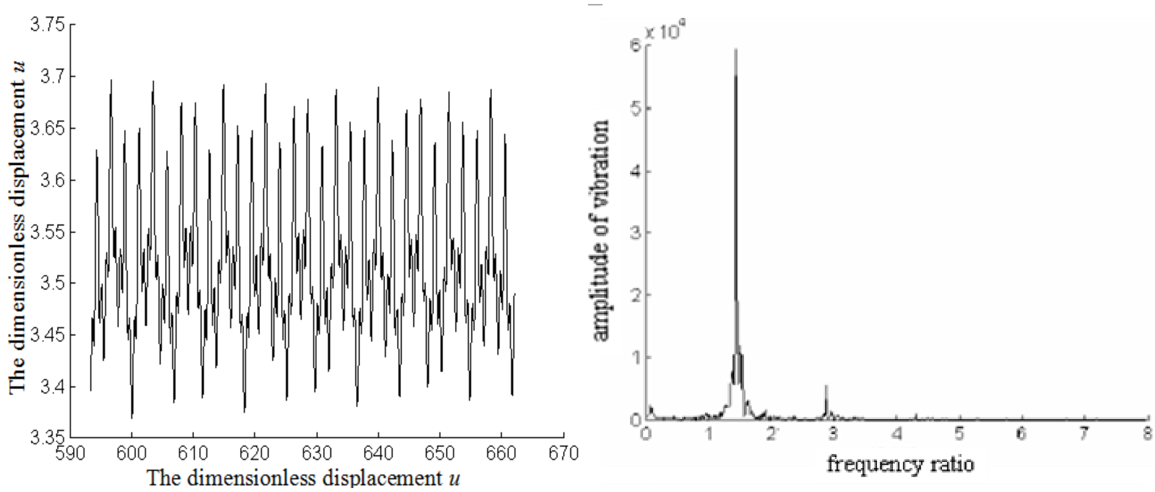


Fig. 4c: Spectrogram and phase plane portrait of the system with σ = 0.6

$$\sigma = \frac{k_1}{k_m} \tag{18}$$

where,

k_1 = The alternating component of the mesh stiffness
k_m = The average component of the mesh stiffness
σ = The fluctuation coefficient of the mesh stiffness

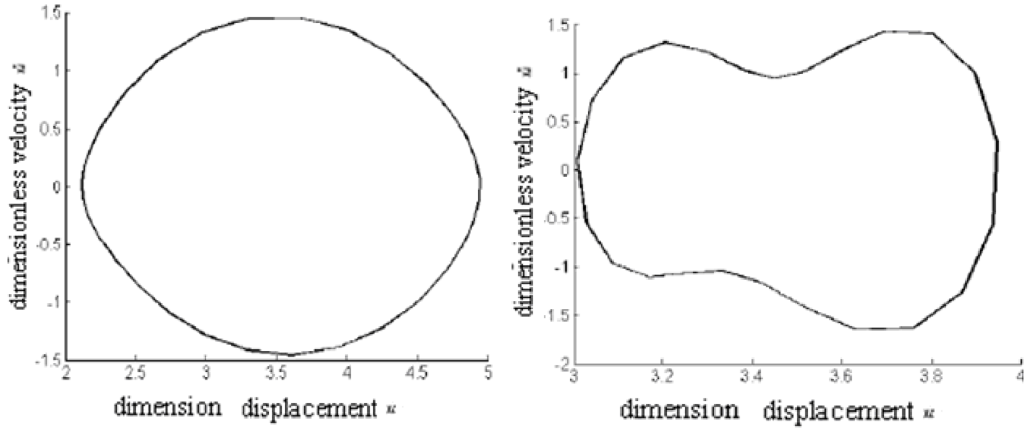
which shows the fluctuation degree of mesh stiffness. It means that the greater σ is, the more intensely the mesh stiffness fluctuates.

Vibration responses are calculated as defining σ = 0, σ = 0.4, σ = 0.6 and the results are shown in Fig. 4a to c, observing the vibration situation with the stiffness amplitude.

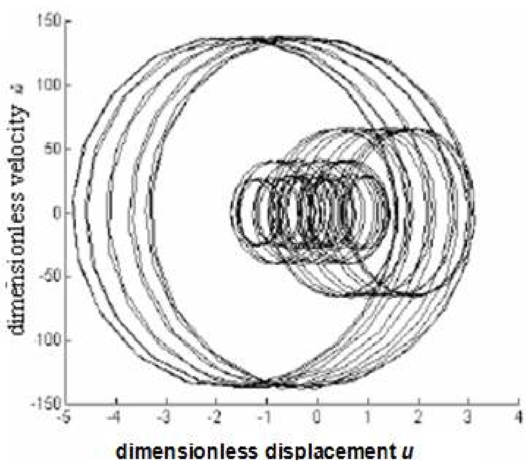
When σ is equal to zero, as shown in Fig. 4a, the mesh stiffness is a constant and the time history of the system is harmonic steady response. When σ = 0.4, as shown in Fig. 4b, the mesh stiffness is quasi-periodic response.

When σ = 0.6, as shown in Fig. 4a, the system produces a chaotic response. Evidently, with σ increasing which is the fluctuation coefficient of the time-varying mesh stiffness, the vibration state experienced harmonic response, quasi-periodic response and chaotic response. In addition, we can also see that the vibration amplitude increases accordingly with the ascent of the fluctuation coefficient σ from the spectrogram Tτ.

- **Gear backlash:** The gear system shows a strong nonlinear vibration characteristics with the gear backlash. The effect of the gear back lash on the vibration of the gear system is various. It is easy to get the influence of gear backlash on vibration cycle from the phase diagram of the response. In order to highlight the clearance of nonlinear effect, assuming the meshing stiffness is a constant. Take the frequency ratio Ω = 0.85 and the response results are shown in Fig. 5 by changing the size of



(a) b = 9μm (b) b = 30μm



(c) b = 120μm

Fig. 5: The response of the system phase diagram with different running clearances

the gear backlash with other parameters unchanged.

As Fig. 5 shown, the gear clearance has slight influence on the aperiodic vibration in a certain range. But the system response quickly converts to chaos response from single frequency response with the increase of gap and the mesh impact is more serious.

- **Meshing error:** From the differential Eq. (13), the external load of the gear is constant and meshing error is alternating incentive. If only change the size of the amplitude of the first step error in the Eq. (8) to study the effect of the error on the system response which is shown in Table 3.

The Fig. 6 is frequency response curve of the gear system under the two different error amplitudes (e_1 = 3.45, e_1 = 6.5), meanwhile, the frequency ratio Ω is equal to 0.85 and other parameters remain unchanged.

Table 3: Error excitation

	Error amplitude e_1 of the first step
1	3.45
2	6.5

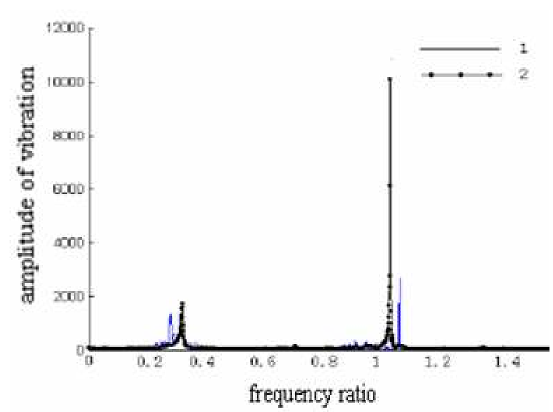
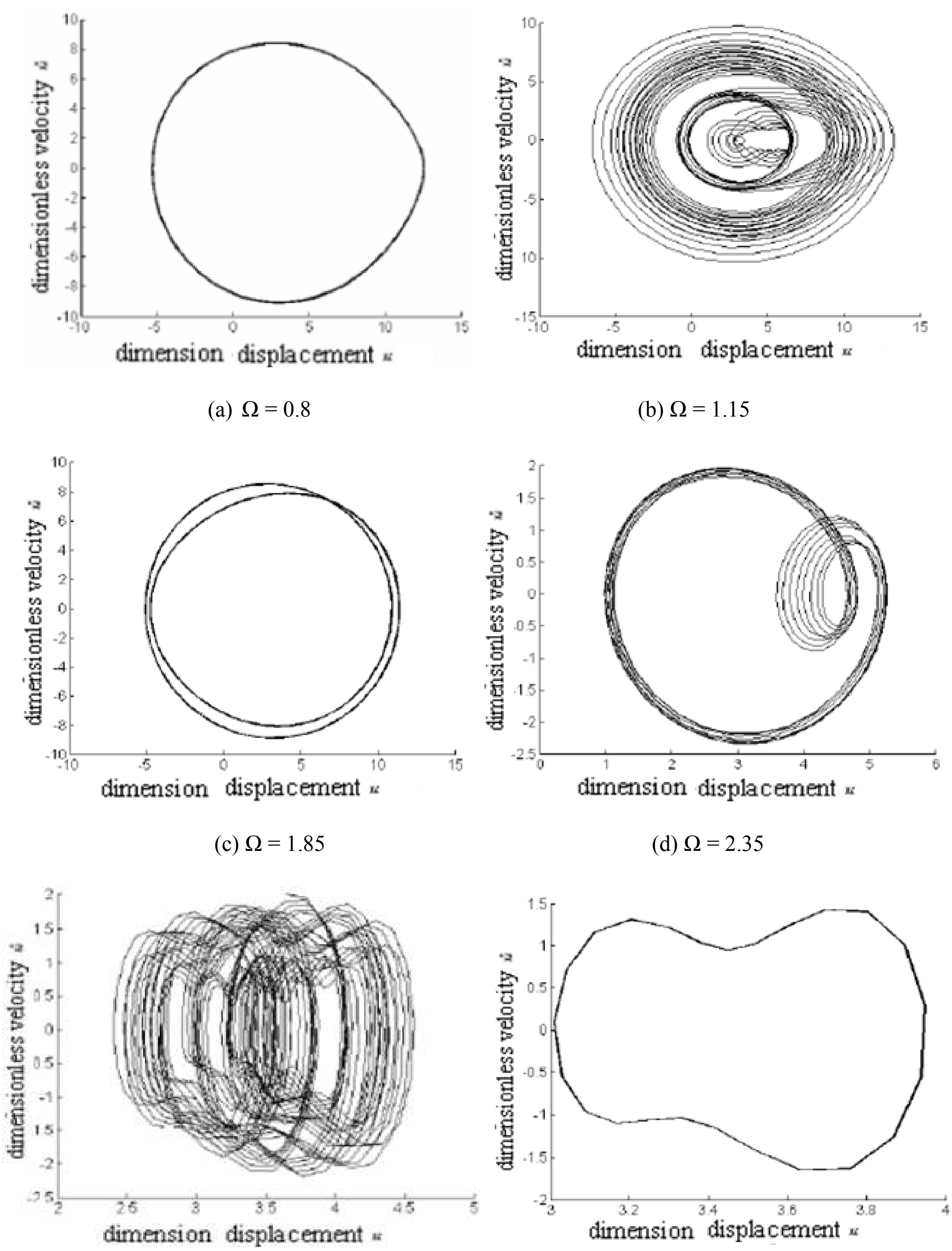


Fig. 6: Frequency response curve under the two different error amplitudes

From the Fig. 6, it is obviously to see that the amplitude of the error excitation has significant impact on the dynamic response amplitude of the system. When the error amplitude increases, the vibration will be greatly enhanced. Thus it can be seen that, in the gear transmission process, the size of the alternating component of the internal incentive also directly affects the non-linear dynamic response in the meshing process. Under the constant average component of internal incentive, the greater alternating error motivation is, the more serious the nonlinear response of the system is and so the vibration is fiercer.

- **Exciting frequency:** Changing the frequency to study the affect of the frequency on the nonlinear dynamical response with the other parameters unchanged. Obviously, it has practical significance to study the vibration response in the near the



(a) Ω = 0.8 (b) Ω = 1.15

(c) Ω = 1.85 (d) Ω = 2.35

(e) Ω = 3.0 (f) Ω = 3.5

Fig. 7: The phase diagram under the different frequency ratio

resonance frequency and high-speed working state, so let the variable Ω change between 0.8 and 3.5 and other parameters remain unchanged. The results are as follows:

As the Fig. 7 shown, when the frequency ratio $\Omega = 0.8$, the Fig. 7a is a periodic solution with the cycle $T = 2\pi/\Omega$, which is known as a single period attractor in the nonlinear vibration theory; when $\Omega = 1.15$, the Fig. 7b is a neither repeated nor closed track filled with a part of the phase space and the system generates a chaotic response; When $\Omega = 1.85$, the system is also harmonic response and its cycle is twice the original. While Ω is equal to 2.35, the system is a curve band with a certain width and it's a quasi cycle response. When $\Omega = 3.0$, the system occurred chaotic response; when $\Omega = 3.5$, the response of the system is the harmonic response and this shows, when the frequency is far from the natural frequency, the response of the system tends to be steady again. It's visible that the exciting frequency has a very significantly effect on the gear vibration system.

CONCLUSION

This chapter used lumper parameter approximation to establish the gear system dynamics model, consider the effection of the time-varying mesh's stiffness, error and gear back lash. Using variable space 4th-order Runge-kutta numerical method solves the gear system's nonlinear kinetics differential equations, to obtain the domain diagram, spectrogram and phase plane of the response of the system by the larger number of numerical calculation analyzed the impact of various parameters on the gear dynamic characteristics, thus to predict the dynamic performance at the design stage, make the system as much as possible to avoid the resonance frequency.

ACKNOWLEDGMENT

This study was supported by the Natural Science Foundation of Shanghai, under the Grants 11ZR1414700.

REFERENCES

Al-Shyyab, A. and A. Kahraman, 2005. Non-linear dynamic analysis of a multi-mesh gear train using multi-term harmonic balance method: period-one motions. J. Sound Vib., 279: 417-451.

Ambarisha, V.K. and R.G. Parker, 2007. Nonlinear dynamics of planetary gears using analytical and finite element models. J. Sound Vib., 302: 577-595.

Dubowsky, S. and F. Freudenstein, 1971. Dynamic analysis of mechanical systems with clearances, part 1: formulation of dynamic model. J. Manuf. Sci. Eng., 93(1): 305-309.

He, S., T. Rook and R. Singh, 2008. Construction of semianalytical solutions to spur gear dynamics given periodic mesh stiffness and sliding friction functions. J. Mech. Design, 130(12): 1-9.

Li, R.F. and W.J. Jun, 1997. Gear System Dynamics. Beijing Science Press, Beijing.

Munro, R.G., 1992. Review of the theory and measurement of gear transmission error. Proceeding of the Inst. of Mechanical Engineers, First International Conference on Gearbox Noise and Vibration, Cambridge.

Raclot, J.P. and P. Velex, 1999. Simulation of the dynamic behavior of single and multi-stage gear systems with shape deviations and mounting errors by using a spectral method. J. Sound Vib., 220 (5): 861-903.

Shaw, S.W., 1985. The dynamics of a harmonically excited system having rigid amplitude constraints. J. Appl. Mech., 52(2): 459-464.

Smith, C.E. and P. Liu, 1992. Coefficients of resultation. ASME Appl. Mech, 59: 663-669.

Tamminana, V.K., A. Kahraman and S. Vijayakar, 2006. A study of the relationship between the dynamic factor and the dynamic transmission error of spur gear pairs. J. Mech. Des., 129(1): 75-84.

Theodossiades, S. and S. Natsiavas, 2000. Nonlinear dynamics of gear-pair systems with periodic stiffness and backlash. J. Sound Vib., 229(2): 287-310.

Veluswami, M.A. and F.R.E. Crossley, 1975. Multiple-impact of a ball between two plates, Parts 2: Mathematical modeling. J. Eng. Ind., 97: 828-835.

Wang, J. and I. Howard, 2005. Finite element analysis of high contact ratio spur gears in mesh. ASME J. Tribol., 127(7): 469-483.

Wang, Y.X. and Y. Liu, 2003. Dynamic load analysis of spur gear pairs with backlash. J. Mech. Strength, 25(4): 373-377.