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Research Article

A Versatile Method for Analyzing the Influence of Track Irregularity on Vehicle-track-bridge Coupled System

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Abstract: A versatile finite element method is applied to analyze the dynamic responses of railway track and bridges under moving railway vehicles. The whole system is divided into two subsystems. The vehicle and railway track are regarded as an integrated subsystem while the railway track and bridge are regarded as the other subsystem. The equations of motion for the two elements are directly derived by means of Hamilton principle. After by assembling the stiffness, damping, mass matrices and the vectors of nodal loads of all elements, the global equations of motion are obtained and solved by Newmark-β method. Numerical examples demonstrate that the method is versatile and correct while dealing with the dynamic responses of vehicle-track-bridge coupled system. These examples also demonstrate that the influence of the track irregularities on the intergraded system is very significant and that the case of several different track irregularities existing at the same time plays more significant on the dynamics responses than the case of one track irregularity.

Keywords: Finite element, Hamilton principle, track irregularities

INTRODUCTION

The dynamic responses of track and bridge structures subjected to moving vehicles have long been an interesting topic in the field of railway engineering. Two kinds of numerical methods, i.e., modal superposition method and finite element method, are widely used to tackle the problem.

For the interaction system between moving vehicles and bridges or beams, the modal superposition has been used in Reference (Frýba, 1999; Hutton and Cheung, 1979; Cheung *et al*., 1999; Yang and Fonder, 1996; Lei, 2002; Lou, 2005a; Zhai, 1998). One separation modal method has been used when the above researchers established the system equations. The method divided the whole system into two subsystems at the interface of the bridge and vehicles while these two subsystems are solved in continuous iteration by wheel-rail force separately. The reponses of the whole system can be obtained but the computational efficiency and convergence of this method are not very good. Then assuming the rigid connection between the wheel and rail when (Lou, 2005c) established the whole system that ignored the normal contact between the wheel and rail and it would cause a lager deviation. Zhai (1998) proposed a new simple explicit integration method when he established a whole system considering the separation between the wheel and rail. This method has been widely used later but there is a little trouble assembling the whole matrixes.

Another numerical method, that is, the finite element method is also very versatile by many researchers (Lin and Trethewey, 1990; Yang *et al*., 1999; Yang and Wu, 2001; Cheng *et al*., 2001; Lou and Zeng, 2005b; Lou, 2007). Cheng *et al*. (2001) used a typical bridge-track-vehicle finite element to investigate the interactions among vehicle and track structure and bridge structure. The two types of equations of motion of finite element form for the entire system are derived by means of the principle of a stationary value of total potential energy of dynamic system in Lou and Zeng (2005b). Lou (2007) also studied the interaction between several vehicles and guide-way systems by the finite element method. The above researchers established the whole system equations by the finite element methods but there is a little trouble assembling the whole matrixes as same as Zhai (1998).

In this study, the dynamic responses of railway track and bridge under a moving railway vehicle are investigated by means of finite element method. The whole system is divided into two subsystems. The vehicle and railway track are regarded as an integrated subsystem while the railway track and bridge are regarded as the other subsystem. The equations of motion for the two elements are directly derived by means of Hamilton principle. After by assembling the stiffness, damping, mass matrices and the vectors of nodal loads of all elements, the global equations of motion are obtained and solved by Newmark (1959) -β method. The whole matrixes can be easily assembled

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by this method and the results can be calculated in less time. This study also studied the effects of several different track irregularities on the dynamic responses of vehicle-track-bridge coupled system. What's more, the paper considered the case of several different types of track irregularity existing at the same time while analyzing the integrated system.

HAMILTON'S PRINCIPLE

It is well-known that Hamilton's principle can be expressed in the form:

$$\int_{t_1}^{t_2} \delta(T-V)dt + \int_{t_1}^{t_2} \delta W_{nc}dt = 0 \quad (1)$$

where, T denotes the kinetic energy for an entire dynamic system, V denotes the potential energy for an entire dynamic system, δW_{nc} denotes the virtual work done by the nonconservative forces for an entire dynamic system and δ is the variation symbol. Equation (1) shows that the sum of the time-variations of the difference in kinetic and potential energies and the work done by the non-conservative forces over any time interval t_i-t_2 equals zero. The application of this principle leads directly to the equations of motion for any given system.

By combining with the finite element method and applying Eq. (1) to an element, the equations of motion for an element can also be established.

Equations of motion for the system of vehicle and bridge considering track structure:

Model of vehicle-track-bridge integrated system: Figure 1 shows a train consisting of a series of identical four-wheel set vehicles moving on a track structure resting on a series of multi-span continuous beams to model railway bridges and the two approach embankments.

The train comprises N_v identical vehicles from left to right and proceeds with speed v and acceleration a at time t along the longitudinal direction. It is assumed that each wheel set of all vehicles always maintains contact with the rails. Each vehicle in the train is modeled as a mass-spring-damper system consisting of a car body, two bogies, four wheel sets and two-stage suspensions. It is assumed that the downward vertical displacements and clockwise direction rotation of vehicle are taken as positive and that they are measured with reference to their respective static equilibrium positions before coming onto the track concerned.

The rail is modeled as a linear elastic Bernoulli-Euler beam supported by discrete viscoelastic supports, the slab is modeled as a linear elastic Bernoulli-Euler beam supported by continuously viscoelastic supports, the bridge are modeled as a series of multi-span continuous Bernoulli-Euler beams, the left and right embankments are modeled as rigid in bridge part.

In the analysis, the whole system is divided into two subsystems. The vehicle and railway track are regarded as an integrated subsystem while the railway track and bridge are regarded as the other subsystem.

Model and equations of track-bridge integrated element: As shown in Fig. 2, the model of track-bridge interaction element consists of rail element, slab element and bridge element. The rail element and the slab element are connected by some discrete viscoelastic supports while the slab element and the bridge element are connected by continuously viscoelastic supports. According to Hamilton's principle, here generate the corresponding kinetic energy T^e_1, potential energy V^e_1 and virtual work δW^e_{nc1}. Then integration for the variation of the kinetic energy, the potential energy and the virtual work over any time interval t_1 to t_2 is:

$$\int_{t_1}^{t_2} \delta T_1^e dt = \int_{t_1}^{t_2} \delta T_r^e dt + \int_{t_1}^{t_2} \delta T_s^e dt + \int_{t_1}^{t_2} \delta T_b^e dt \quad (2)$$

$$\int_{t_1}^{t_2} \delta V_1^e dt = \int_{t_1}^{t_2} \delta U_r^e dt + \int_{t_1}^{t_2} \delta U_s^e dt + \int_{t_1}^{t_2} \delta U_b^e dt + \int_{t_1}^{t_2} \delta U_{rs}^e dt + \int_{t_1}^{t_2} \delta U_{sb}^e dt \quad (3)$$

$$\delta W_{nc1}^e = \delta W_{nc,rs}^e + \delta W_{nc,sb}^e + \delta W_{nc,b}^e \quad (4)$$

where,

T^e_r, T^e_s & T^e_b : The kinetic energy of the i[th] rail beam element, the j[th] slab beam element and the k[th] bridge beam element respectively

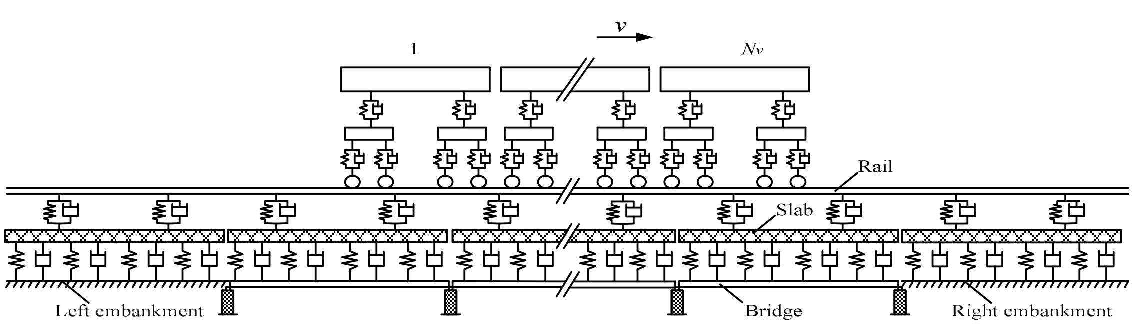

Fig. 1: Model of vehicle-track-bridge integrated system

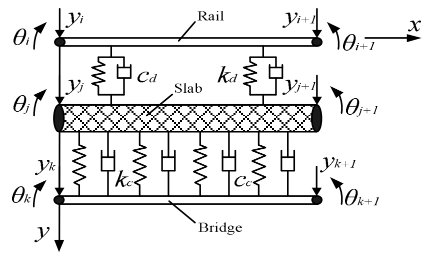

Fig. 2: Model of track-bridge interaction element

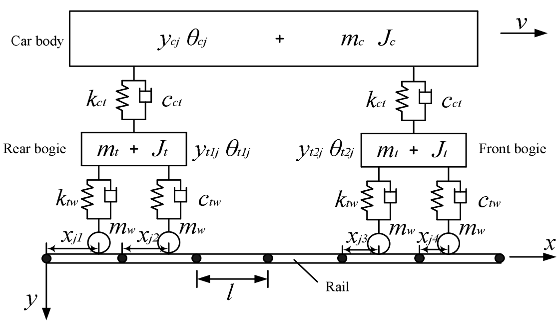

Fig. 3: Model of vehicle-track interaction element

U^e_r, U^e_s & U^e_b : The flexural strain energy of the ith rail beam element, the jth slab beam element and the kth bridge beam element respectively

U^e_{rs} : The spring strain energy of discrete springs between the ith rail beam element and the jth slab beam element

U^e_{sb} : The spring strain energy of continuous springs between the jth slab beam element and the kth bridge beam element

$\delta W^e{}_{nc,rs}$: The internal virtual work performed by no conservative forces between ith rail beam element and jth slab beam element

$\delta W^e{}_{nc,sb}$: The internal virtual work performed by no conservative forces between jth slab beam element and kth bridge beam element

$\delta W^e{}_{ncb}$: The internal virtual work performed by no conservative forces for the kth bridge beam element

Since all variations are arbitrary, one can obtain the vertical equations of motion of a track-bridge interaction element. The equations can be written in partitioned form as:

$$\begin{bmatrix} [M]_{rr} & [0] & [0] \\ [0] & [M]_{ss} & [0] \\ [0] & [0] & [M]_{bb} \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\}^e_{ri} \\ \{\ddot{q}\}^e_{sj} \\ \{\ddot{q}\}^e_{bk} \end{Bmatrix} + \begin{bmatrix} [C]_{rr} & [C]_{rs} & [0] \\ [C]_{sr} & [C]_{ss} & [C]_{sb} \\ [0] & [C]_{bs} & [C]_{bb} \end{bmatrix} \begin{Bmatrix} \{\dot{q}\}^e_{ri} \\ \{\dot{q}\}^e_{sj} \\ \{\dot{q}\}^e_{bk} \end{Bmatrix} + \begin{bmatrix} [K]_{rr} & [K]_{rs} & [0] \\ [K]_{sr} & [K]_{ss} & [K]_{sb} \\ [0] & [K]_{bs} & [K]_{bb} \end{bmatrix} \begin{Bmatrix} \{q\}^e_{ri} \\ \{q\}^e_{sj} \\ \{q\}^e_{bk} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{Bmatrix} \quad (5)$$

where, all matrixes are 4×4 matrixes:

the vector $\{q\}^e_{ri} = [y_{ri} \;\; \theta_{ri} \;\; y_{r(i+1)} \;\; \theta_{r(i+1)}]^T$

the vector $\{q\}^e_{si} = [y_{si} \;\; \theta_{si} \;\; y_{s(i+1)} \;\; \theta_{s(i+1)}]^T$

the vector $\{q\}^e_{bi} = [y_{bi} \;\; \theta_{bi} \;\; y_{b(i+1)} \;\; \theta_{b(i+1)}]^T$

Model of vehicle-track integrated element: As shown in Fig. 3, the model of vehicle-track interaction element consists of vehicle and four rail elements under four wheel-sets. Vehicle also consist a car body, two bogies, four wheel-sets and two-stage suspensions. Each wheel-set of vehicle is assumed to be always in contact with the upper rail. According to Hamilton's principle, here generate the corresponding kinetic energy T^e_2, potential energy V^e_2 and virtual work $\delta W^e{}_{nc2}$. Then integration for the variation of the kinetic energy, the potential energy and the virtual work over any time interval t_1 to t_2 is:

$$\int_{t_1}^{t_2} \delta T^e_2 dt = \int_{t_1}^{t_2} \delta T_c dt + \int_{t_1}^{t_2} \delta T_t dt + \int_{t_1}^{t_2} \delta T_w dt + \int_{t_1}^{t_2} \delta T^e_r dt \quad (6)$$

$$\delta W^e_{nc2} = \delta W_{nc,ct} + \delta W_{nc,tw} \quad (7)$$

$$\int_{t_1}^{t_2} \delta V^e_2 dt = \int_{t_1}^{t_2} \delta V^e_c dt + \int_{t_1}^{t_2} \delta V^e_t dt + \int_{t_1}^{t_2} \delta V^e_w dt + \int_{t_1}^{t_2} \delta U^e_{ct} dt + \int_{t_1}^{t_2} \delta U^e_{tw} dt + \int_{t_1}^{t_2} \delta U^e_r dt \quad (8)$$

where,

T_c : The kinetic energy of the translation of the center of the car body and the rotation about its center

T_t : The kinetic energy of the translation of the center of the two bogies and the rotation about their centers

T_w : The kinetic energy of the four axles due to translation of the center considered separately

T^e_r : The kinetic energy of the four rail beam elements under the four wheels respectively

U_{ct} : The spring strain energy of the vehicle and two bogies

U_{tw} : The spring strain energy of the bogies and wheels

U^e_r : The flexural strain energy of the all rail beam elements under the four wheels

$\delta W_{nc,ct}$: The internal virtual work performed by no conservative force between the vehicle and two bogies

$\delta W_{nc,tw}$: The internal virtual work performed by no conservative force between the bogie and wheels

V_c, V_t & V_w: The potential energy of the gravity of the car body, the gravity of the two bogies and the gravity of the four wheels

Since all variations are arbitrary, one can obtain the vertical equations of motion of a vehicle-track interaction element. The equations can be written in partitioned form as:

$$\begin{bmatrix} [M]_{vv} & [0] \\ [0] & [M]_{rr} \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\}_v^e \\ \{\ddot{q}\}_{rj}^e \end{Bmatrix} + \begin{bmatrix} [C]_{vv} & [C]_{vr} \\ [C]_{rv} & [C]_{rr} \end{bmatrix} \begin{Bmatrix} \{\dot{q}\}_v^e \\ \{\dot{q}\}_{rj}^e \end{Bmatrix} + \begin{bmatrix} [K]_{vv} & [K]_{vr} \\ [K]_{rv} & [K]_{rr} \end{bmatrix} \begin{Bmatrix} \{q\}_v^e \\ \{q\}_{rj}^e \end{Bmatrix} = \begin{Bmatrix} \{F\}_v^e \\ \{F\}_{rj}^e \end{Bmatrix} \quad (9)$$

where, $[M]_{vv}$, $[C]_{vv}$, $[K]_{vv}$ are 6×6 matrixes, $[C]_{vr}$, $[K]_{vr}$ are 6×16 matrixes, $[C]_{rv}$, $[K]_{rv}$ are 6×16 matrixes, $[M]_{rr}$, $[C]_{rr}$, $[K]_{rr}$ are 6×16 matrixes, the vector $\{q\}_v^e = [y_{cj}\ \theta_{cj}\ y_{t1j}\ \theta_{t1j}\ y_{t2j}\ \theta_{t2j}]^T$.

The equations of motion of vehicle-track-bridge integrated system: The conventional assembly process can be employed to form the global equation of motion for the entire vehicle-track-bridge system, which will appear as:

$$[M]\{\ddot{q}\} + [M]\{\dot{q}\} + [M]\{q\} = \{P\} \quad (10)$$

where, the matrices $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices of the entire system respectively, the vector {P} is the global vector of the entire system, the vectors $\{q\}$, $\{\dot{q}\}$ and $\{\ddot{q}\}$ denote the displacement, velocity and acceleration vectors of the entire system. Equation can be solved by step by step integration method such as the Newmark-β method or Wilson-θ method to obtain simultaneously the dynamic responses of vehicle, track or bridge.

NUMERICAL EXAMPLES

An example to verify the versatile method and to analysis the influence of track irregularity on system: A single-span simply supported railway bridge with the two approaches supported on embankments is considered, as shown in Fig. 1. It is assumed that the length of track structure on each approach embankment is equal. The following data are adopted for the vehicle: mass of car body $m_c = 4.175\times10^4$ kg, mass moment of inertia of car body $J_c = 2.08\times10^6$ kg·m^2, mass of one bogie m_t = 3040 kg, mass moment of inertia of one bogie $J_t = 3.93\times10^3$ kg·m^2, mass of one wheel set $m_w = 1.78\times10^3$ kg, spring stiffness of the second suspension system $k_{ct} = 5.3\times10^5$ N/m, damping coefficient of the

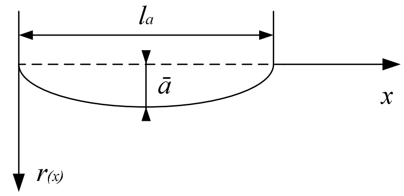

Fig. 4: Irregularity on track surface

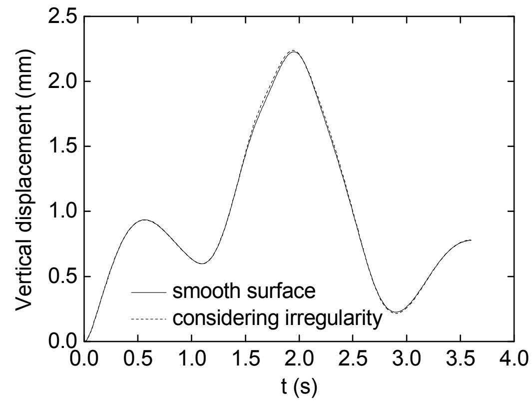

Fig. 5: Vertical displacement of car body

second suspension system $c_{ct} = 9.02\times10^4 N\cdot s/m$, spring stiffness of the primary suspension system $k_{tw} = 1.18\times10^6 N/m$, damping coefficient of the primary suspension system $c_{tw} = 3.92\times10^4 N\cdot s/m$, half of horizontal distance between two bogies $L_c = 8.75m$, half of horizontal distance between two axles $L_t = 1.25m$, vehicle velocity $v = 27.78m/s$ and $a = 0m/s^2$. The following data are adopted for the track structure: the total longitudinal length of track structure $120m$, $E_r = 2.06\times10^{11} pa$, $I_r = 2\times2.037\times10^{-5} m^4$ $\overline{m}_r = 2\times51.5kg/m$, $E_s = 2.1\times10^{10} pa$, $I_s = 10^{-10} m^4$, $\overline{m}_s = 10^{-10} kg/m$, stiffness of discrete springs between rail and slab $k_d = 10^{13} N/m$, coefficient of discrete dampers between rail and slab $c_d = 0N\cdot s/m$, stiffness of continuous springs between slab and bridge $k_c = 2\times6.58\times10^7 N/m$, coefficient of discrete dampers between slab and bridge $c_c = 2\times3.21\times10^4 N\cdot s/m$. The following data are adopted for the simply supported bridge: $L_b = 30m$, $E_b = 2.943\times10^{10} pa$, $I_b = 2.88m^4$, $\overline{m}_b = 1.2\times10^4 kg/m$ and damping ratio of bridge $\zeta = 0.02$. During the analysis, the equations of motion for the integrated system are solved by the Newmark-β method with time step $\Delta t = 0.005s$.

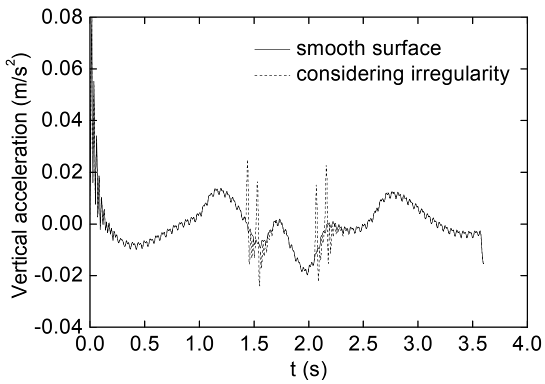

Fig. 6: Vertical acceleration of car body

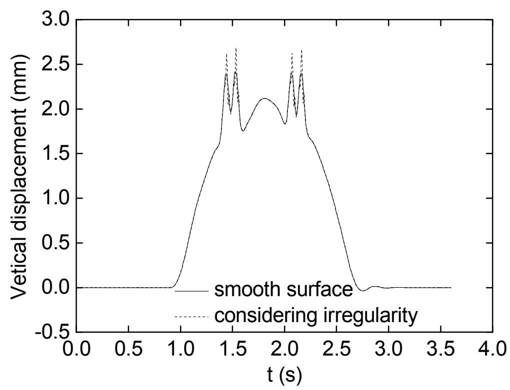

Fig. 7: Vertical displacement of midpoint of the rails

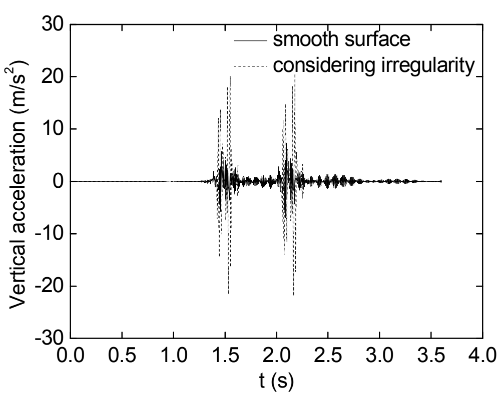

Fig. 8: Vertical acceleration of midpoint of the rails

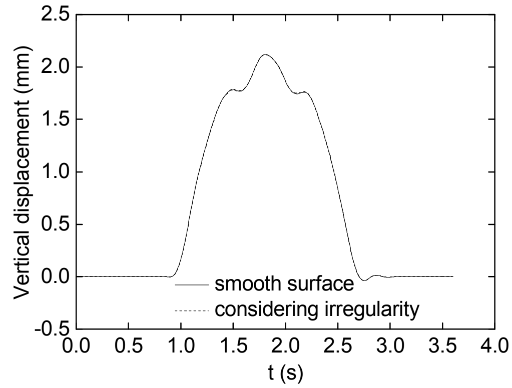

Fig. 9: Vertical displacement of midpoint of the bridge

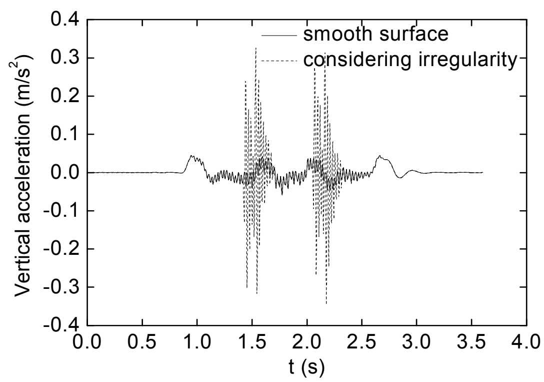

Fig. 10: Vertical acceleration of midpoint of the bridge

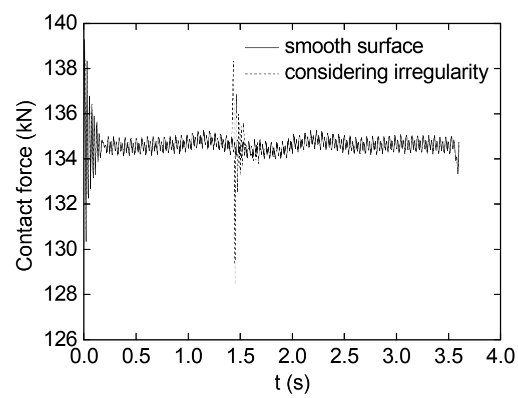

Fig. 11: Contact force between front axle of front bogie and rails

As shown in Fig. 4, one type of irregularity function for the vertical profile of the track is adopted:

$$r(x) = \frac{1}{2}\bar{a}(1 - \cos 2\pi x / l_a) \tag{11}$$

where, x is the along-track distance, $\bar{a}$ is maximum depth of track irregularity and l_a is the length of track irregularity, $\bar{a} = 1.0 \times 10^{-3} m$ and $l_a = 1m$ are used for the local track irregularity. It is assumed that the point of maximum depth of the track irregularity locates at the midpoint of the rails. Considering the track structure irregularity, the time-history response of the center of car body, of the midpoint of the rails, of the midpoint of bridge and two contact forces between wheel-sets and rails have been plotted in Fig. 5 to 12 along with those considering smooth track surface.

As shown in Fig. 5 to 12, the dynamics responses without considering track irregularity are same as those

Table 1: The maximum of dynamics responses by several types of track irregularity

The max. of dynamics responses		First type	Second type	Third type	Fourth type
Car body	Dis. (mm)	2.2280	2.2280	2.6330	2.6330
	Acc. (m/s^2)	0.0196	0.0249	0.0237	0.0237
Midpoint of rail	Dis. (mm)	2.4070	2.6860	3.4690	3.7800
	Acc. (m/s^2)	7.3160	21.8800	8.0260	27.1800
Midpoint of bridge	Dis. (mm)	2.1170	2.1170	2.6770	2.6780
	Acc. (m/s^2)	0.0447	0.3460	0.1058	0.4043
Contact force between front axle of front bogie and rail (kN)		135.3000	138.3000	135.4000	139.6000

Max.: Maximum

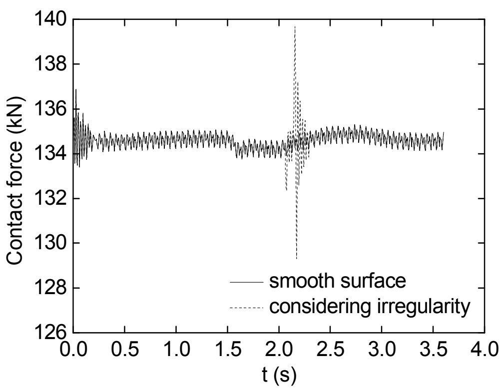

Fig. 12: Contact force between rear axle of rear bogie and rails

in Lou (2005c). Therefore, the versatility and accuracy of this method have been proved.

As shown in Fig. 5 and 9, the dynamics responses for considering local track irregularity are almost same as those for considering smooth surface of track. However, as shown in Fig. 6 to 8 and 10 to 12, the dynamic response, i.e., vertical acceleration of car body, vertical displacement and vertical acceleration of midpoint of the rails, vertical acceleration of midpoint of bridge and two contact forces between wheel-sets and rails, for considering track irregularity have significant variation while the vehicle arrives at the position of local track irregularity. Obviously, maintaining a smooth track surface in railway engineering is very important.

The influence of track irregularity on vehicle-track-bridge coupled system: There are several types of track irregularity to analysis the influence of track irregularity on vehicle-track-bridge coupled system as follows:

The first type: It is assumed that there is smooth surface in rail.

The second type: It is assumed that there is one type of track irregularity as same as this in example 1.

The third type: It is assumed that there is a 20 cm length of subsidence under right of the midpoint of the rails. Now the stiffness of springs and coefficient of dampers between slab and bridge are all zeros.

The forth type: It is assumed that there is above-mentioned of the second and the third type of track irregularity at the same time.

The dynamics responses of vehicle-track-bridge coupled system by the influence of the above-mentioned track irregularity as shown in Table 1.

As shown in Table 1, compared with the first type, the dynamics responses for considering the second type of track irregularity, i.e., vertical acceleration of car body, vertical displacement and acceleration of the midpoint of rail, vertical acceleration of the midpoint of bridge and contact force between front axle of front bogie and rail have significant variation while the other are same as those in the first type; Compared with the first type, the dynamic responses for considering the third type of track irregularity, i.e., vertical displacement and acceleration of car body, vertical displacement and acceleration of the midpoint of rail, vertical displacement and acceleration of the midpoint of bridge have significant variation while the other are same as those in the first type; Compared with the second type and third type, vertical displacement and acceleration of car body and vertical displacement of the midpoint of bridge for considering the fourth type of track irregularity are same as those in the third type while the other dynamic responses all increase relevantly.

CONCLUSION

In this study, the dynamic responses of railway track and bridge under a moving railway vehicle are investigated by means of finite element method. The whole system is divided into two subsystems. The vehicle and railway track are regarded as an integrated subsystem while the railway track and bridge are regarded as the other subsystem. The equations of motion for the two elements are directly derived by means of Hamilton principle. After by assembling the stiffness matrices, the damping matrices, the mass matrices and the vectors of nodal loads of all elements, the global equations of motion for the integrated system are obtained. These equations can be solved by step-by-step integration method, to obtain simultaneously the dynamic responses of vehicle, track and bridge. The results show that the whole matrixes can be easily assembled by this method and the results can be calculated in less time.

From the numerical results obtained in above two examples, two conclusions can be reached:

- The method used in this study can be widely applied in the analysis of vehicle-track-bridge coupled system. What's more, the method has more versatility and correctness on some different objects.
- The effects of track irregularity on the dynamics responses of system are significant. The above two examples demonstrate that vertical acceleration of car body, vertical displacement and acceleration of the midpoint of rail, vertical acceleration of the midpoint of bridge and contact force between front axle of front bogie and rail, considering local track irregularity, have significant variation. The second example demonstrate that vertical displacement and acceleration of car body, vertical displacement and acceleration of the midpoint of rail, vertical displacement and acceleration of the midpoint of bridge, considering local track subsidence, have also significant variation. What's more, the dynamics responses considering above two cases existing at the same time increase relevantly. It shows that the case of several different track irregularities existing at the same time play more significant on the dynamics responses than the case of one track irregularity. So maintaining a good track structure in railway engineering is very important.

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