

## Research Article

# Centralized Fuzzy Data Association Algorithm of Three-sensor Multi-target Tracking System

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**Abstract:** For improving the effect of multi-target tracking in dense target and clutter scenario, a centralized fuzzy optimal assignment algorithm (CMS-FOA) of three-sensor multi-target system is proposed. And on the base of this, a generalized probabilistic data association algorithm (CMS-FOAGPDA) based on CMS-FOA algorithm is presented. The fusion algorithm gets effective 3-tuple of measurement set by using components of several satisfactory solutions of the fuzzy optimal assignment problem and then uses generalized probabilistic data association algorithm to calculate the update states of targets. Simulation results show that, in the aspect of multi-target tracking accuracy, CMS-FOA algorithm is superior to the optimal assignment (CMS-OA) algorithm based on state estimate and CMS-FOAGPDA algorithm is better than CMS-FOA algorithm. But considering the time spent, CMS-FOA algorithm spends a minimum of time and CMS-FOAGPDA algorithm is exactly on the contrary. Therefore, compared with CMS-OA algorithm, the two algorithms presented in the study each has its advantages and should be chosen according to the needs of the actual application when in use.

**Keywords:** Centralized, fuzzy, generalized probabilistic data association, multi-target tracking, the optimal assignment

## INTRODUCTION

With the rapid development of science and technology, monitoring system has been in a progress from single-sensor system to multi-sensor system. Multi-sensor multi-target tracking system is a monitoring system using homogeneous or heterogeneous sensors to describe the environmental condition. Data association problem is a key technique for multi-target tracking. Now the typical multi-sensor multi-target tracking data association algorithms include Joint Probabilistic Data Association (JPDA) algorithm, generalized probabilistic data association (GPDA) algorithm, multiple hypothesis tracking (MHT) algorithm, the optimal assignment algorithm, etc (He *et al.*, 2010; Han *et al.*, 2010).

Each common multi-target tracking algorithm has unique performance characteristics and application environment. Both Joint probabilistic data association algorithm and generalized probability data association algorithm are more applicable to dense target and clutter scenario (Pan *et al.*, 2005, 2009). In general detection scenario, each of them easily causes track's excursion or polymerization due to its whole neighborhood properties of probability calculation. While the optimal assignment algorithm based on certain optimization rule is more applicable to data

association problem under general detection environment. In dense target and clutter environment, one-on-one feasible rule required by the optimal assignment algorithm easily leads to more inaccurate associations, resulting in decreasing accuracy of multi-target tracking (Popp *et al.*, 2001; Zhang *et al.*, 2007). To play the advantages of probabilistic data association and the optimal assignment algorithm and avoid their disadvantages, Zhou and Zhang (2012) proposes a generalized probabilistic data association algorithm based on the optimal assignment algorithm. The results of the studies show that generalized probabilistic data association algorithm based on the optimal assignment algorithm not only can avoid deficiency of error accumulation and loss of information resulted by sequential processing of measurement of multi-sensor in generalized probabilistic data association algorithm, but also can effectively avoid the high wrong association rate on tracking accuracy of the optimal assignment algorithm by using one-to-one feasible rule under dense targets and clutter environment.

Studies have shown that, fuzzy set theory can better deal with data association problem in dense target and clutter scenario (Xu and Chen, 2011). To further effectively improve the anti-jamming capability of the optimal assignment algorithm of data association under

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dense targets and clutter scenario, this study firstly proposes a fuzzy optimal assignment algorithm for multi-sensor multi-target tracking. On this basis, fuses the algorithm above and generalized probabilistic data association algorithm and gets a generalized probabilistic data association algorithm based on the fuzzy optimal assignment algorithm. Simulation results show that, compared with the optimal assignment algorithm based on state estimate, the fuzzy optimal assignment algorithm not only has better multi-target tracking accuracy, but also has less time spent. While compared with the fuzzy optimal assignment algorithm, the generalized probabilistic data association algorithm based on the fuzzy optimal assignment algorithm further improves multi-target tracking and location accuracy in dense target and clutter scenario

### SYSTEM DESCRIPTIONS

Let us suppose that the state equation of discrete-time system is as follows:

$$X(k+1) = \phi(k)X(k) + G(k)V(k) \quad (1)$$

where,  $\phi(k) \in R^{n \times n}$  is the state-transition matrix at time  $k$ ;  $G(k) \in R^{n \times n}$  represents a process noise distribution matrix;  $X(k) \in R^n$  is a state vector of the target at time  $k$ ;  $V(k) \in R^n$  is a sequence of zero-mean, white Gaussian process noise with covariance matrix  $Q(k)$  at time  $k$ , process noises of different time are independent, i.e.:

$$E[V(k)] = 0 \quad (2)$$

$$E[V(k)V'(l)] = Q(k)\delta_{kl} \quad (3)$$

The measurement equation of discrete-time system is:

$$Z(k) = H(k)X(k) + W(k) \quad (4)$$

where,  $Z(k) \in R^m$  is measurement vector at time  $k$ ;  $H(k)$  is the measurement matrix;  $W(k) \in R^m$  is a zero-mean, white Gaussian measurement noise sequence with covariance  $R(k)$  and measurement noises of different time are independent, i.e.:

$$E[W(k)] = 0 \quad (5)$$

$$E[W(k)W'(l)] = R(k)\delta_{kl} \quad (6)$$

The initial state is described as  $X(0)$  which follows a Gauss distribution, with mean  $\hat{X}(0|0)$  and covariance  $P(0|0)$  and:

$$E \left\{ \begin{bmatrix} V(k) \\ W(k) \\ \tilde{X}(0|0) \end{bmatrix} \begin{bmatrix} V'(l) & W'(l) & \tilde{X}(0|0) \end{bmatrix} \right\} = \begin{bmatrix} Q(k)\delta_{kl} & 0 & 0 \\ 0 & R(k)\delta_{kl} & 0 \\ 0 & 0 & P(0|0) \end{bmatrix} \quad (7)$$

where,  $\tilde{X}(0|0) = X(0) - \hat{X}(0|0)$

### THE FUZZY OPTIMAL ASSIGNMENT ALGORITHM

**Fuzzy factor set and membership function:** Let us suppose that the fuzzy factor set is  $U = \{u_1, u_2, \dots, u_k, \dots, u_n\}$ , where  $u_k$  denotes the  $k$ th fuzzy factor. The fuzzy factor set usually includes Euclidean distances between positions, velocities and accelerations of the two targets in  $x$  and  $y$  directions and Euclidean distances between headings and heading change rates of the two targets. Because that these factors have different effects on association judgment, only those factors that play an important role in association judgment can be selected in practical application. This will not only ensure the accurate tracking on the targets with different movements, but also can avoid the algorithm too complex, so as to get the goal of decreasing the computational burden of the information fusion system. Intuitively, the Euclidean distance between positions of targets should be the most important, then the Euclidean distance between headings and direction cosine angles of targets. All these factors constitute the main body of fuzzy association judgment, while the Euclidean distance between accelerations and changing rates of headings or direction cosine angles can be used as the auxiliary judgment, the weight can be taken as a very small value or zero.

Let us suppose that the weight assignment vector of the fuzzy factor set  $U$  is  $A = \{a_1, a_2, \dots, a_n\}$ , where  $a_k$  is the corresponding weight of the  $k$ th factor  $u_k$ . In general,  $\sum_{k=1}^n a_k = 1$ , where the selection of  $a_k$  needs to be decided by the importance or impact of the  $k$ th factor. Usually,  $a_1 \geq a_2 \geq a_3 \dots \geq a_n$  and the weights of the final fuzzy factors are very small or zero. According to the characteristics of fuzzy factors in point-track association system, normal distribution membership function is adopted in this study.

**Determination of the fuzzy factors:** In this study, one selects fuzzy factors between measurement point  $Z_i(l) = [x_i(l), y_i(l), \dot{x}(l), \dot{y}(l)]^T$  and predicted measurement  $\hat{Z}^i(l|l-1)$  as follows:

$$\begin{cases} u_1(l) = [(\hat{x}_i(l) - \hat{x}_i(l|l-1))^2 + (\hat{y}_i(l) - \hat{y}_i(l|l-1))^2]^{1/2} \\ u_2(l) = [|\hat{x}_i(l) + \hat{y}_i(l|l-1)|^{1/2} - |\hat{y}_i(l) + \hat{x}_i(l|l-1)|^{1/2}] \\ u_3(l) = \theta_{it}(l) \end{cases} \quad (8)$$

$i = 1, 2, \dots, m_k; j = 1, 2, \dots, T.$

where  $m_k$  is the number of measurements in confirmation region at present time:

$$\theta_{it}(l) = \left| \tan^{-1}[\hat{y}^i(l) / \hat{x}^i(l)] - \tan^{-1}[\hat{y}^i(l|l-1) / \hat{x}^i(l|l-1)] \right| \quad (9)$$

$$\hat{x}^i(l) = [x_i(l) - \hat{x}_i(l-1)] / T_s \quad (10)$$

$$\hat{x}^i(l|l-1) = \hat{x}^i(l-1|l-1) \quad (11)$$

where  $T_s$  is sampling interval.

**The fuzzy optimal assignment algorithm:** Let us suppose that the membership function describing the similarity between the point and track based on the  $k$ th fact is:

$$\mu_k(u_k) = \exp(-\tau_k(u_k^2 / \sigma_k^2)), k = 1, 2, \dots, n. \quad (12)$$

For the fuzzy weight set  $A = \{a_1, a_2, \dots, a_n\}$ , by using the weighted average method, one can get the integrated similarity degree between measurement point  $Z_i(l)$  and track  $t$  as:

$$f_{it}(l) = \sum_{k=1}^n a_k(l) \mu_k, i = 1, 2, \dots, M_k, t = 1, 2, \dots, T. \quad (13)$$

Then the fuzzy association matrix between measurement point  $Z_i(l)$  and track  $t$  at time  $l$  can be expressed as:

$$F(l) = \begin{bmatrix} f_{11}(l) & f_{12}(l) & \dots & f_{1T}(l) \\ f_{21}(l) & f_{22}(l) & \dots & f_{2T}(l) \\ \dots & \dots & \dots & \dots \\ f_{m_k 1}(l) & f_{m_k 2}(l) & \dots & f_{m_k T}(l) \end{bmatrix} \quad (14)$$

By normalizing each row of matrix (14), one can get the basic probability assignment matrix of measurements on different targets as:

$$M = (m_{it})', i = 0, 1, \dots, m_k; t = 0, 1, \dots, T \quad (15)$$

where  $m_{it}$  is the basic probability assignment function for measurement  $i$  on target  $t$ .

Suppose that the basic probability assignment functions for measurements from different sensors on target  $t$  ( $t = 0, 1, \dots, T$ ) are  $m_{1i_1}$ ,  $m_{2i_2}$  and  $m_{3i_3}$ , respectively, focal elements are  $A_p, B_q$  and  $C_r$ , respectively, then the inconsistent measure of the  $i_1$ th measurement of the first sensor, the  $i_2$ th measurement of the second sensor and the  $i_3$ th measurement of the third sensor can be described as:

$$c_{i_1 i_2 i_3} = \sum_{A_p \cap B_q \cap C_r = \phi} m_{i_1}(A_p) \cdot m_{i_2}(B_q) \cdot m_{i_3}(C_r) \quad (16)$$

where,  $A_r, B_s, C_t \subseteq U$ .  $U$  is the identification frame of target  $t$ . Then  $C = (c_{i_1 i_2 i_3})_{n_1 \times n_2 \times n_3}$  is the association cost matrix of measurement data of three sensors and the data association problem of three-sensor multi-target system can be transformed into a 3-D assignment problem as follows:

$$\min_{\rho_{i_1 i_2 i_3}} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} c_{i_1 i_2 i_3} \cdot \rho_{i_1 i_2 i_3} \quad (17a)$$

subject to:

$$\begin{cases} \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} \rho_{i_1 i_2 i_3} = 1; & i_1 = 1, 2, \dots, n_1 \\ \sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \rho_{i_1 i_2 i_3} = 1; & i_2 = 1, 2, \dots, n_2 \\ \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \rho_{i_1 i_2 i_3} = 1; & i_3 = 1, 2, \dots, n_3 \end{cases} \quad (17b)$$

where,  $\rho_{i_1 i_2 i_3}$  are indicator variables. If a 3-tuple of measurement originates from a real target, it should be 1; otherwise, it is 0.

### GENERALIZED PROBABILISTIC DATA ASSOCIATION ALGORITHM BASED ON FUZZY OPTIMAL ASSIGNMENT

Considering that the fuzzy optimal assignment algorithm is a globally optimal assignment algorithm based on one-to-one feasible rule, in dense target and clutter environment, it will lose a lot of information to implement point-track association by only using components of the optimal solution of (17a-17b) as effective 3-tuple of measurement. And the generalized probabilistic data association algorithm of sequential processing of measurements from multi-sensor is a cascade algorithm. In the sequential process, it will lose some original information and can easily lead to error accumulation, resulting in multi-target tracking precision declining. To solve the problem, this section

proposes a generalized probabilistic data association algorithm based on the fuzzy optimal assignment.

It is easy to see that each component of solution of (17a-17b) happens to correspond to an effective 3-tuple of measurement. All components in the selected good solutions consist of the effective 3-tuple of measurement set. Therefore, one can obtain an equivalent measurement by estimating the position of the corresponding target for each effective 3-tuple of measurement. And by calculating the association probability between the equivalent measurements and target track, one can take advantage of GPDA algorithm to track multi-target. In this way, one gets the generalized probabilistic data association algorithm based on the fuzzy optimal assignment. The following is the detailed description of the fusion algorithm.

For each valid 3-tuple of measurement, one can estimate the position  $(x_t, y_t)$  of the corresponding target  $t$  ( $t = 0, 1, \dots, T$ ) by using the following formula:

$$\begin{cases} \hat{x}_t = \frac{\sum_{s=1}^3 \frac{x_s}{\sigma_{x_s}^2}}{\sum_{s=1}^3 \frac{1}{\sigma_{x_s}^2}} \\ \hat{y}_t = \frac{\sum_{s=1}^3 \frac{y_s}{\sigma_{y_s}^2}}{\sum_{s=1}^3 \frac{1}{\sigma_{y_s}^2}} \end{cases} \quad (18)$$

where,  $\sigma_x^2, \sigma_y^2$  are the location variances in x, y direction, respectively.  $x_s = x_t + u_s$ ,  $y_s = y_t + v_s$  are the location components of target  $t$  measured by sensor  $s$  in two coordinate axes, respectively.

Take the position estimate of corresponding target  $t$  of the  $i$ th 3-tuple of measurement as the comprehensive measurement  $z_i^t$ ,  $i = 1, 2, \dots, m_k$ , then one can calculate the association probability between the  $i$ th 3-tuple of measurement and target track  $t$ . Further, the update state of target can be calculated by using GPDA algorithm. The state and covariance update formula are as follows:

$$\begin{aligned} \hat{X}^t(k|k) &= \hat{X}^t(k|k-1) \\ &+ K^t(k) \sum_{i=0}^{m_k} \beta_{ii}(k) v_i^t(k) \end{aligned} \quad (19)$$

where,  $\hat{X}^t(k|k-1) = F^t(k) \hat{X}^t(k-1|k-1)$  is the state prediction of target  $t$  at time  $k$ ,  $K^t(k)$  is filter gain of target  $t$  at time  $k$ ,  $v_i^t(k) = z_i^t(k) - \hat{Z}^t(k|k-1)$  is measurement of residual:

$$\begin{aligned} P^t(k|k) &= \beta_{0t}(k) P^t(k|k-1) \\ &+ (1 - \beta_{0t}(k)) P_c^t(k|k) + \tilde{P}^t(k) \end{aligned} \quad (20)$$

where,

$$P_c^t(k|k) = [I - K^t(k) H_s(k)] P^t(k|k-1) \quad (21)$$

$$\tilde{P}^t(k) = W^t(k) P_v^t(k) W^{tT}(k) \quad (22)$$

$$\begin{aligned} P_v^t(k) &= \sum_{i=1}^{m_k} \beta_{0i}(k) v_{ii}(k) v_{ii}^T(k) \\ &- \left[ \sum_{i=1}^{m_k} \beta_{ii}(k) v_{ii}(k) \right] \left[ \sum_{i=1}^{m_k} \beta_{ii}(k) v_{ii}(k) \right]^T \end{aligned} \quad (23)$$

## SIMULATION ANALYSES

In order to verify the validity of the proposed algorithms, CMS-OA algorithm, CMS-FOA algorithm and CMS-FOAGPDA algorithm are compared in different simulation conditions. In the simulation, it is assumed that three sensors track eight targets and each target move at a variable speed in a plane. The initial velocity are respectively  $v_x = 40/s$ ,  $v_y = 30/s$ ; angle measurement error and ranging measurement error of the three sensors are respectively the same; detection probability is  $Pd = 0.95$ ; gate probability is  $Pg = 1$ ; the initial value of the weight vector is taken as  $a_1 = 0.7$ ,  $a_2 = 0.2$  and  $a_3 = 0.1$ ; radar sampling time is  $T = 2s$ ; clutter coefficient is  $\lambda = 2$ . The simulation steps are 150 and simulation number is 50. The simulation results are as follows:

- When target-to-target interval is  $\tau = 1000m$ , comparisons of root-mean-square error (RMSE) of CMS-OA algorithm, CMS-FOA algorithm and CMS-FOAGPDA algorithm are as follows:
- When target-to-target interval is  $\tau = 500m$ , comparisons of RMSE of CMS-OA algorithm, CMS-FOA algorithm and CMS-FOAGPDA algorithm are as follows

Table 1 shows the average time spent of different algorithms when the target-to-target interval is 500 m, the ranging measurement error is 200m and the angle error is 0.02rad. Simulation steps are 150 and simulation number is 50.

It can be seen from Fig. 1 that when target-to-target interval is not very small, CMS-FOA algorithm is better than CMS-OA algorithm, but the optimization of magnitude is not big. And one can see from Fig. 1 to 4, with the deterioration of measurement environment, the magnitude of CMS-FOA algorithm being superior to CMS-OA algorithm correspondingly increases. The results show that, compared with CMS-OA algorithm, CMS-FOA algorithm is more suitable for dense target and clutter environment.

It also can be seen from Fig. 1 to 4, in comparison with CMS-FOA algorithm, CMS-FOAGPDA has more accurate target tracking effect and better tracking

Table 1: Comparisons of time spent of different algorithms

Algorithms	CMS-OA	CMS-FOAGPDA	CMS-FOA
Time (s)	4.367	5.1007	3.215

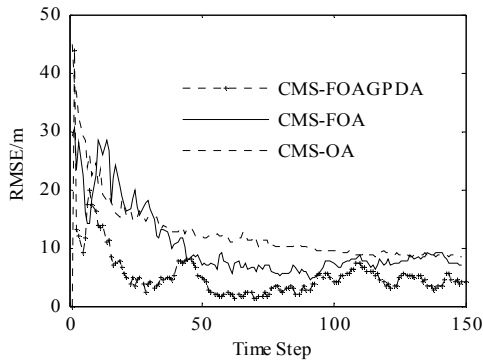


Fig. 1: RMSE of three algorithms under condition of  $e_r = 200m$ ,  $e_\theta = 0.02rad$

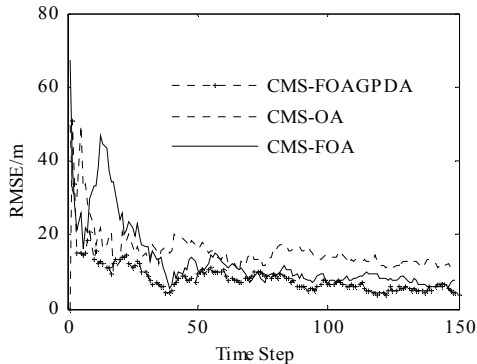


Fig. 2: RMSE of three algorithms under condition of  $e_r = 300m$ ,  $e_\theta = 0.03rad$

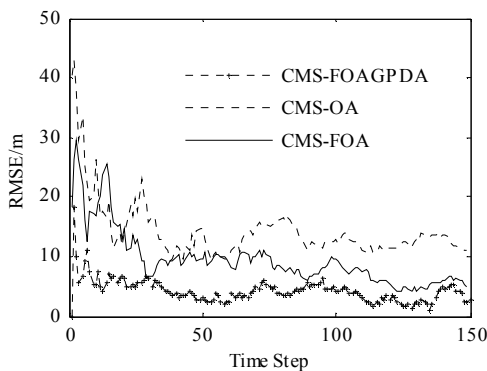


Fig. 3: RMSE of three algorithms under condition of  $e_r = 200m$ ,  $e_\theta = 0.02rad$

stability. It is mainly because that CMS-FOAGPDA not only effectively avoids information loss and error accumulation caused by sequential processing measurements from different sensors in CMS-GPDA algorithm and avoids the deteriorating data association quality by using CMS-FOA algorithm under one-to-one

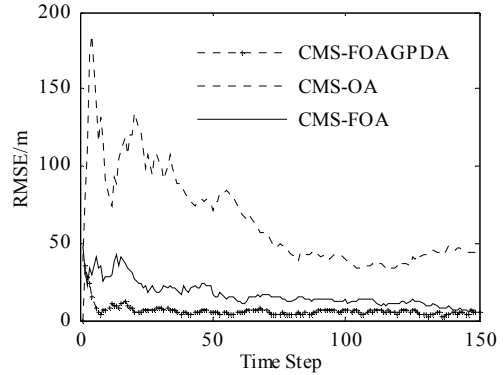


Fig. 4: RMSE of three algorithms under condition of  $e_r = 300m$ ,  $e_\theta = 0.03rad$

feasible rule in dense target and clutter scenario, but also inherits the advantages of anti-jamming performance of GPDA algorithm. Therefore, in various simulation conditions, the fusion algorithm shows better performance of multi-target tracking.

Table 1 show that CMS-FOA algorithm spends less time than the corresponding result of CMS -OA algorithm, while CMS-FOAGPDA spends slightly more time than the corresponding result of CMS -OA algorithm.

## CONCLUSION

In order to better solve multi-sensor multi-target tracking problem, a fuzzy optimal assignment algorithm of data association of three sensor system is proposed. The algorithm first uses fuzzy comprehensive similarity to construct point-track correlation matrix and then uses combination rule of D-S evidence theory to construct measurement data association matrix of three sensors based on fuzzy comprehensive similarity. Simulation results show that the fuzzy optimal assignment algorithm is superior to the optimal assignment algorithm based on state estimate. To further enhance the anti-jamming performance of the fuzzy optimal assignment algorithm, this study puts forward a fusion algorithm which use components of several good solutions of the fuzzy optimal assignment algorithm as effective 3-tuple of measurement and then takes advantage of generalized probabilistic data association algorithm to track multi-target. Compared with the fuzzy optimal assignment algorithm, the generalized probabilistic data association algorithm based on the fuzzy optimal assignment algorithm further improves the accuracy of multi-target tracking in dense target and clutter environment, but the time spent also increases accordingly. How to further optimize the performance of the fusion algorithm is a problem which needs to further study in the future.

**REFERENCES**

- Han, C.Z., H.Y. Zhu and Z.S. Duan, 2010. Multi-source Information Fusion. Tsinghua University Press, Beijing.
- He, Y., G.H. Wang, D.J. Lu and Y.N. Peng, 2010. Multisensor Information Fusion with Applications. Publishing House of Electronics Industry, Beijing.
- Pan, Q., X.N. Ye and H.C. Zhang, 2005. Generalized probability data association algorithm. *Acta Electron. Sinica*, 33(3): 467-472.
- Pan, Q., Y. Liang, F. Yang and Y.M. Cheng, 2009. Modern Target Tracking and Information Fusion. National Defense Industry Press, Beijing.
- Popp, R., K. Pattipati and Y. Bar-Shalom, 2001. M-best S-D assignment algorithm with application to multitarget tracking. *IEEE T. Aero. Elec. Sys.*, 37(1): 22-39.
- Xu, Y.Y. and X.H. Chen, 2011. Fuzzy set theory in the multi-sensor information fusion. *Comput. Appl. Softw.*, 28(11): 2-4.
- Zhang, J.W., Y. He and W. Xiong, 2007. Multisensor multiplied hypothesis algorithm based on data compressing technic. *J. Beijing Univ., Aeronaut. Astronaut.*, 33(12): 1448-1451.
- Zhou, L. and W.H. Zhang, 2012. General probabilistic data association algorithm based on the optimal assignment. *J. Comput. Inform. Syst.*, 8(17): 7241-7248.