# Research Article Deformation Evaluation Due to Poisson's Ratio Variation of Coated Fabric for Airship Envelope

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Abstract: To target to evaluate the deformation due to only Poisson's ratio variation of coated fabric for airship envelop, series biaxial extension tests were carried out for a particular envelop Uretek3216 and the mechanical property parameters were calculated accordingly from the plain composite theory. On the basis of elastic engineering theory, three typical cases of tension ratio 1:1, 1:2 and 1:1~1:2 were proposed for evaluation on Poisson's ratio variation, which is cruciform specimen, inflatable fabric beam and streamline airship. Parametric analysis was carried out for each case through numerical simulation. And the geometry dimension effect was also investigated with scaled model. The significant deformation variation of airship is found from only Poisson's ratio variation is necessary to be considered for accurate deformation predication of large flexible airship. The presented work is valuable to numerical analysis and engineering design for non-rigid airship structures.

Keywords: Airship envelope, coated fabric, deformation evaluation, geometry dimension effect, poisson's ratio, tension ratio

### INTRODUCTION

Non-rigid airship envelopes are also pressured airship, since they maintain their form by pressure stabilization of the lift gas containment, the envelope is similar to the inflatable membrane. They do not have a skeletal frame, since the envelope under pressure acquires a residual membrane tension, becoming stiff enough to resist overall bending effects and carry all other structural features without reducing the membrane tension to zero. Khoury and Gillett (1999) published recent work in Airship Technology, include aerodynamic, flight, envelop material and structural design, application, Miller and Mandel (2000) gave the requirements of airship envelop. Shoji et al. (2007) made tests of tear propagation of high performance of airship envelop material. Kraska (2002) performed structural analysis for CL-160 airship through Abagus software. Over the past, few literatures were about detail numerical analysis and the material tests were mostly used strip tension method to get ultimate strength, strain, creep, etc.

In civil architectural membrane structure field, biaxial extension test and calculation method are fairly advanced, fruitful research results have been achieved. Brian and Marijke (2004) published European design guide for tensile surface structures, as the outcome of European Union Tensinet, in which specify bi-axial extension test method and proposed calculation method. Galliot and Luchsinger (2009, 2011) developed bi-axial test to formulate the nonlinear constitutive of PVCcoated fabric. Bridgens and Gosling (2004) and Gosling and Bridgens (2008) proposed direct stress-strain formula based on bi-axial extension test for coated woven fabrics. Jorg (2011) evaluated the elastic constants variation due to different calculation method for bi-axial extension results. Most work proved that coated weave fabrics behave nonlinear and the elastic constants are dependent on the stress state and vary in much region, include Young's modulus, shear stiffness and Poisson's ratio.

For the coated fabrics of airship envelope, a few literatures were published about bi-axial extension test. Gao *et al.* (2010a) made series tensile strip test of the envelope materials and evaluated the mechanical behavior, elastic constants, Gao (2010b) made fairly thoroughly structural research for airship, include material test, static analysis and dynamic analysis, demonstrate test, Huang *et al.* (2011) proposed the elastic calculation method combined with uni-tensile and bi-axial extension results and analyzed the parameters.

The mechanical behavior of coated fabric for airship has not been investigated comprehensively and the nonlinear constitutive law or elastic constants determination based on linear approximation is still interesting topic, especially on the basis of bi-axial extension test. The variation of Poisson's ratio has not attracted attention and interest in the past. There is not accurate requirement of geometry form in architectural

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membrane structure. There is much margin of power of low altitude airship. The pay load is extreme small for high altitude airship and this requires vigorous constraints such as power, drag, form, etc. Thus Poisson's ratio variation which always is neglected becomes one of concerns except for Young's modulus.

In this study, bi-axial extension test and calculation method are firstly proposed on the basis of our former development, the Poisson's ratio effect on deformation of the membrane with three typical tensile stress state are investigated thoroughly with Poisson's ratio variation through numerical analysis.

# BIAXIAL EXTENSION TESTAND CALCULATION METHOD

**Biaxial extension test method:** Biaxial extension test has been proved to unveil nature of architectural coated fabric over the past recent years. Similarly, the same biaxial extension test would be carried out for airship envelope fabric which is more thinner, lightweight and high strength. The loading protocol can be defined freely and any tension ratio can be realized for two orthogonal directions.

The dimensions of cruciform specimen are shown in Fig. 1. The effective measurement area is  $16 \times 16$  cm<sup>2</sup>. In order to avoid the stress concentration, the corner is rounded with a radius of 15 mm. There are three 150 mm long slits in each tension arm. The samples should be cut along the yarn orientations to avoid lose of yarns. The load can be applied automatically with high precision hydraulic servo system and the strain can be recorded through the electronic displacement extension needles. Normally, biaxial extension test is used to investigate the mechanical behavior of the fabrics in service range rather than limit capacity and the maximum load can be defined from the ultimate strength of specific fabric with respect to rational safety of factor. Figure 2 is a snapshot of the biaxial extension tester focused on the fabric specimen area.

**Elastic constants calculation method:** Airship envelopes are normally made of plain weave fabric which features general orthotropic plane stress behavior. Membrane warp and weft are assumed as *x*-axis and *y*-axis. The stress-strain relationship can be induced according to the orthotropic elastic film theory under the assumption of small strain. The stress-strain relationship can be formulated as follow:

$$\varepsilon_x = \frac{N_x}{E_x t} - \frac{N_y}{E_y t} v_{yx}$$
(1)

$$\varepsilon_{y} = \frac{N_{y}}{E_{y}t} - \frac{N_{x}}{E_{x}t} v_{xy}$$
<sup>(2)</sup>



Fig. 1: Dimension of cruciform specimen (mm)



Fig. 2: Snapshot of membrane biaxial extension tester

$$N_{x} = \frac{E_{x}t}{1 - v_{yx}v_{xy}} (\varepsilon_{x} + v_{yx}\varepsilon_{y})$$
(3)

$$N_{y} = \frac{E_{y}t}{1 - v_{xy}v_{yx}} (\varepsilon_{y} + v_{xy}\varepsilon_{x})$$
(4)

where,

 $N_x$ ,  $N_y$  = The stress in the warp and weft direction (kN/m)

 $\varepsilon_x, \varepsilon_y$  = The strain in the warp and weft direction

 $E_xt$ ,  $E_yt$  = The tension stiffness in the warp and weft direction (kN/m)

 $v_{xy}$ ,  $v_{yx}$  = Poisson's ratio factor

There are four unknown variables, the elastic constant  $E_xt$ ,  $E_yt$ ,  $v_{xy}$  and  $v_{yx}$  in Eq. (1) to (4). A biaxial extension test will yield just two independent equations as Eq. (1) to (4). At least two sets of biaxial extension test with tension ratio unequal to 1 are necessary to be carry out for the nontrivial solution. It can be observed that the elastic constants depend on the stress state. Different stress states will result in different solution, not only one solution. On the contrary, various elastic constants will result in variation of structural response as stress and deformation.

			Poisson's rat	Poisson's ratio				
Parameter	$E_1$ MPa	$E_2 MPa$	V <sub>xy</sub>	V <sub>yx</sub>	Density kg/m <sup>3</sup>	Thickness mm		
Value	1.1e9	8.6e8	0.21-0.35	0.25~0.48	1700	0.38		

Table 1: Properties of the components

The airship envelope fabric Uretek3216 was tested and evaluated thoroughly for the tension ratio 1:1, 1:2, 2:1. The typical elastic constants can be calculated from Eq. (1) to (4) and listed in Table 1. Poisson ratio factor is found to vary in fairly range, from 0.21 to 0.48.

Table 1 Properties of the airship envelope fabric Uretek3216

## POISSON'S RATIO EFFECTS EVALUATIONS ON STRUCTURAL DEFORMATION

In the case of structural analysis of membrane structures, the thin membrane element is employed and the constitutive equations can be defined as follow, on the assumption of geometrical nonlinear, small strain, valid reciprocal rule:

$$\begin{cases} \varepsilon_{1} \\ \varepsilon_{2} \\ \gamma_{3} \end{cases} = \begin{cases} \frac{1}{E_{1}} & -\frac{v_{12}}{E_{1}} & 0 \\ -\frac{v_{21}}{E_{2}} & \frac{1}{E_{2}} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{cases} \begin{bmatrix} \sigma_{1} \\ \sigma_{2} \\ \tau_{3} \end{bmatrix}$$
(5)  
$$\frac{E_{1}}{E_{2}} = \frac{v_{12}}{v_{21}}$$
(6)

where,

 $\varepsilon_1, \varepsilon_2 & \gamma_3$ : Strain,  $\sigma_1, \sigma_2$  and  $\tau_3$  are stress E<sub>1</sub>, E<sub>2</sub>, G<sub>12</sub>, v<sub>12</sub> & v<sub>21</sub>: Elastic constants of coated fabric material

The commercial finite element package Abaqus 6.10 was used for structural analysis of membrane structure. The thin membrane element labeled M3D4 was chosen herein to simulate membrane. Due to variation of Poisson ratio factor dependent on the stress field, generally three typical cases would be evaluated as follow, the tension ratio is 1:1, 1:2 and from 1: 1 to 1:2, respectively.

**Case tension ratio 1:1:** Firstly, in the case of particular tension stress field, tension ration 1:1, Poisson's ratio effects on the structural deformation were analyzed through biaxial extension specimen as shown in Fig. 1. The biaxial extension specimen is a simple basic tensile membrane structure which can develop any tensile stress.

The analytical model is shown as Fig. 3 for the biaxial extension cruciform specimen. The left and bottom side edges are assumed fully fixity and the right and upper side edges are applied uniform tension 1 kN/m. Using Abaqus package, the element M3D4 was



Fig. 3: Cruciform specimen analysis model



Fig. 4: Deformation of cruciform specimen (unit: m) (v = 0.25)

employed for deformation analysis. The material properties are referred to that in Table 1 and moreover, various Poisson's ratio  $v_{12}$  0.25, 0.28, 0.31 and 0.35 are used to evaluate its effects on the deformation.

Figure 4 gives the deformation of cruciform specimen with Poisson's ratio 0.25. The maximum

Table 2: Cruciform deformation of different poisson's ratio (unit: mm)



Fig. 5: Analytic model of the inflatable fabric cantilever beam



Fig. 6: Snapshot of inflatable fabric beam test setup



Fig. 7: Inflatable fabric beam (unit: mm)



Fig. 8: Deformation of the inflatable fabric beam under pressure 5 kPa (unit: m) (v = 0.25)

deformation in x-axis and y-axis is different. The reason can be explained that the fabric has bigger stiffness in x-axis than that y-axis. However, the material properties keep unchanged except for Poisson's ratio, the overall deformation can be observed similarly and the differences between these models can be found further from the Table 2. The maximum deformation decreases with Poisson's ratio increase. The maximum difference can reach about 2.5% with respect to average value of Poisson's ratio case.

**Case tension ratio 1:2:** The conventional rigid airship is cigar, pencil and ellipsoid shaped form, as aquire minimum aerodynamic drag. For most non-rigid airship, the slenderness ratio is usually more than 3, even 4, especially for the case of large scae airship. The central segment of the airship envelope exhibits simular cylinder structural behavior, the overall hoop tension to longitudinal tension is rough 2:1, as feature the deformation of inflatable fabric beam.

The analytical model of inflatable fabric cantilever beam is shown as Fig. 5, the left end is assumed fully fixity and the right end is free. The cantilever length is L and diameter is D. The inflated pressure is P, that is, pressure difference between the inner containment air and atmosphere. According to engineering elastic thin shell theory, the hoop tension  $N_{\theta}$  and longitudinal tension  $N_{\alpha}$  of the inflatale fabric beam can be formulated as follow:

$$N_{\alpha} = PD/4 \tag{7}$$

$$N_{\theta} = PD/2 \tag{8}$$

From Eq. (7) and (8), the ratio of longitudinal tension to hoop tension is 1:2, valid in engineering sense.

Practically, the free end of the inflatable fabric beam is a half spherical cone, which makes tension vary smoothly. Figure 6 shows snapshot of inflatable fabric beam test setup, which consists mainly of an inflatable fabric beam, steel rig, inflation bump, etc. The fabric is airship envelops Uretek3216, warp is longitudinal direction and weft is hoop direction. The dimension is specified as Fig. 7, the length is 2300 mm plus half sphere of radius 213 mm and the pressure is 5 kPa.

Using Abaqus package, the element M3D4 was employed for deformation analysis. The material properties are referred to that in Table 1 and moreover, various Poisson's ratio  $v_{12}$  0.25, 0.28, 0.31 and 0.35 are used to evaluate its effects on the deformation. Figure 8 shows deformation of the inflatable fabric beam under pressure 5 kPa with Poisson ratio 0.25, where  $u_1$  means radius elongation,  $u_2$  means longitudinal elongation. The difference is less 5% between the measured elongation of test set-up and the calculated deformation. Similarly, only the parameter Poisson ratio change,



Fig. 9: D deformation variation of the inflatable beam with different poisson's ratio



Fig. 10: The profile and snapshot of the ZY-1 demonstration airship

other conditions keep unchanged, the inflatable fabric beam exhibits same tendency. With the Poisson ratio varying from 0.25 to 0.35, the longitudinal elongation decrease proportionally linear, as shown in Fig. 9. Moreover, we observed that the displacement in warp direction at the point B decreases by 36% and the radius elongation decreases by 5.8% when Poisson's ratio increases by 40%. The change of Poisson's ratio had large affects on the deformation of the inflatable tube.

**Tension ratio 1:2~1:1:** The general equilibrium of double curvature surface is expressed as follow from the elastic theory:

$$\frac{\sigma_1}{r_1} + \frac{\sigma_2}{r_2} = p \tag{9}$$

where,

 $\sigma_1, \sigma_2$  = The principal stress (kN/m)  $r_1, r_2$  = The principal curvature radius p = The pressure

In the case of typical flexible airship, low drag streamline form is usually employed, the envelope features smooth double curvature from nearly sphere to cylinder, that is, the tension stress field changes from 1:1 to 1:2 overall. The basic stress state can be estimated from Eq. (9). And the detailed stress and deformation evaluation can be performed through numerical analysis with geometrical nonlinear and complicated action taken into account.

ZY-1 airship was targeted to demonstrate the key technologies of stratospheric platform, which was developed by Shanghai Jiao Tong University (SJTU) in 2009 (Gao, 2010b). The airship is a low drag form composed of three-segment polynomial, shown as Fig. 10, the profile and snapshot, respectively. The length is 25 m and the maximum radius is 3.788 m.

By the same computing procedure, the material properties are same as Table 1 except for Poisson's ratio. The airship is assumed free float, the numerical analysis is valid due to full geometry symmetry and only pressure 500 Pa. Figure 11 gives the hoop deformation variation versus various Poisson's ratio. Figure 12 gives the deformation in Y-axis, that is, the elongation of the airship. Poisson's ratio has significant effect on the deformation in Y-axis. The deformation of tail cone is less than nose cone. The significant variation of deformation can be found due to various Poisson's ratio.

Deformation of airship envelope will result in aerodynamic drag performance change except for the fabric strength requirements and flight dynamics change further. The overall performance of non-rigid airship is dependent mostly on the envelope geometry form, is sensitive to deformation. Accurate deformation predication of airship based on rational numerical method with reasonable Poisson's ratio considering will be valuable to engineering design of large airship.

**Geometry dimension effect evaluation:** The Poisson's ratio effects on the structural deformation are obviously found from the former case analysis and test verification. The geometry dimension effect is always concerned issues. Here the scaled model as Fig. 7 is used for geometry dimension effect evaluation.

From the point of engineering theory, the same stress is assumed for the Initial Inflatable Beam (IIB) and the Scaled Inflatable Beam (SIB). The scale factor is taken $\beta$ , the geometry dimension is enlarged  $\beta$ , the pressure should be reduced  $1/\beta$  because of linear relationship from Eq. (7) and (8). Now the scale factor is assumed 10 for the model as Fig. 7, the pressure thus is 500 Pa, other conditions keep unchanged. Figure 13 shows the deformation of the SIB under pressure 500 Pa with Poisson's ratio 0.25. The radius increase 5.698 mm and the length extend 27.380 mm.

Similarly, other numerical analysis was performed for the Poisson's ratio 0.28, 0.31 and 0.35, respectively. The overall deformation of IIB and SIB exhibits same



Fig. 11: The hoop deformation of the airship envelope



Fig. 12: The deformation in Y-axis of the airship envelope



Fig. 13: Deformation of the SIB under pressure 500 Pa (unit: m) (v = 0.25)

Table 3: Deformation of SIB and IIB with different poisson's ratio (unit: mm)

(um	u. mm)				
Poisson's ratio		0.250	0.280	0.310	0.350
Length	IIB	2.636	2.450	2.265	2.018
Extension	SIB	27.380	25.500	23.620	21.110
Radius	IIB	0.573	0.562	0.551	0.536
Elongation	SIB	5.700	5.590	5.480	5.330

phenomena. The U2 displacement at the point B denotes length extension and U1 displacement at the point A, middle point, denotes radius elongation. Table 3 lists the length extension and radius elongation. It can be observed that the length extension and radius elongation decrease with Poisson's ratio increase. The scale factor keeps nearly identical with Poisson's ratio varying. That is, Poisson's ratio effects on the deformation of the inflatable beam have not geometry dimension effects, as mainly depend on the stress state. The law of reasonable value on Poisson's ratio of small-scale model is universal. Therefore the reasonable Poisson's ratio value of the design of large scale inflatable tube could be combined with small scale testing result under specific tensile stress state.

## CONCLUSION

The mechanical behavior of the coated fabric of airship envelope is similar to other plain weave coated fabric of architectural membrane. Except for Young's modulus, the Poisson ratio has fairly variation scope, as is dependent mostly on the tensile stress state. On the contrary, the Poisson ratio variation will result in different structural response, particular deformation in the course of finite element numerical simulation. In the case of tension ratio 1:1, the deformation is insensitive to Poisson ratio variation. For other tensile stress state. the deformation is found to be more sensitive to Poisson ratio variation. The geometry dimension effects do not exist for Poisson ratio variation, that is, the scale factor nearly keeps unchanged with scaled geometry change. Poisson ratio variation is necessary to be considered for accurate deformation predication of large flexible airship.

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