

Research Article

Turbulent Flow over a Flat Plate Using a Three-equation Model

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Abstract: Aim of this study is to evaluate a three-equation turbulence model based on the Reynolds averaged Navier-Stokes equations. Boussinesq hypothesis is invoked for determining the Reynolds stresses. An average turbulent flat plate flow was simulated. Uncertainty was approximated through validation. Results for the mean axial velocity and friction coefficient were within experimental error.

Keywords: Flat plate, friction, reynolds number, turbulence modeling

INTRODUCTION

The problem of turbulence dates back to the days of Claude-Louis Navier and George Gabriel Stokes, as well as others in the early nineteenth century. Searching for its solution, it was a source of great despair for many notably great scientists, including Werner Heisenberg, Horace Lamb and many others. The complete description of turbulence remains one of the unsolved problems in modern physics. A great deal of early work on turbulence can be found, for example, in Hinze (1975).

Recently, Direct Numerical Simulation (DNS) has emerged as an indispensable tool to tackle turbulence directly, albeit at relatively low Reynolds numbers. Several DNS studies on turbulent flow have been performed recently, including Eggels *et al.* (1993), Loulou *et al.* (1997) and Wu and Moin (2008). The latter has carried out DNS on a turbulent pipe flow at Reynolds number of 44,000, which is the largest among the three studies. Mean velocity, Reynolds stresses and turbulent intensities were presented and discussed, along with visualization of flow structure. Good agreement was attained with the Princeton Superpipe data on mean flow statistics and Lawn (1971) data on turbulence intensities. Large Eddy Simulation (LES) is another tool that somewhat bridges between DNS and Reynolds-Averaged Navier-Stokes (RANS) methods. In LES, large turbulent structures in the flow field are resolved, while the effect of Sub-Grid Scales (SGS) is modeled. LES investigation, for example, has been carried out by Rudman and Blackburn (1999) on a turbulent pipe flow at Reynolds number of 38,000. Mean velocity and Reynolds stresses were presented and discussed, along with visualization of flow structure. Results were reported to compare favorably with measurements.

While DNS and LES are fairly accurate for modeling turbulent flows, they remain limited to relatively low-range Reynolds numbers. This drawback explains the wide-spread of turbulence modeling in industrial applications where the use of DNS techniques remains formidable. Turbulence modeling includes eddy viscosity models which utilize the Boussinesq hypothesis for relating the Reynolds stresses to the average flow field, Hinze (1975). In turn, the eddy viscosity is determined by using any of a variety of models, including zero, one and two-equation models, most notably the $k-\epsilon$. While such models vary in complexity, they share several shortcomings, including isotropy of the eddy viscosity and the lack of generality in wall treatment. Such shortcomings lead to poor results in separated flows and other non-equilibrium turbulent boundary layers, Yamamoto *et al.* (2008).

A second-order turbulence model, which also falls under RANS methods, is the Reynolds stress model. While the model relaxes the isotropic assumption, it remains more complicated with many unknown terms. For more on the subject of turbulence modeling, the reader is referred to, for example, Launder and Spalding (1972).

In this study, the accuracy of a three-equation turbulence model is assessed. Using the turbulence model, average turbulent flow over a flat plate was simulated for Reynolds number up to 10^7 . Uncertainty was approximated through model validation. Results for mean axial velocity and friction coefficient are presented and assessed.

THEORY

Starting with the incompressible Navier-Stokes equations in Cartesian index notation and with Reynolds decomposition, averaging and following Boussinesq hypothesis, we have:

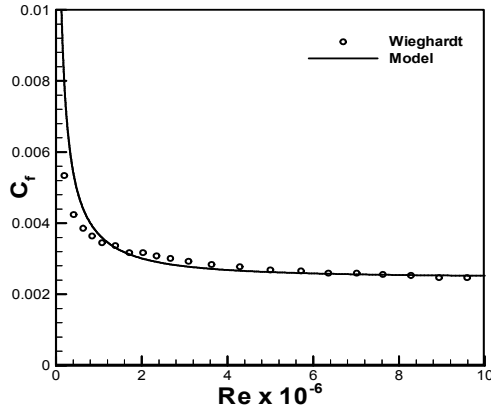


Fig. 1: Distribution of friction coefficient

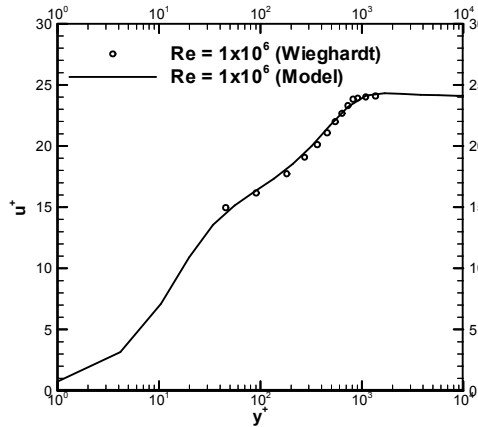


Fig. 2: Mean velocity distribution for $Re = 1.0 \times 10^6$

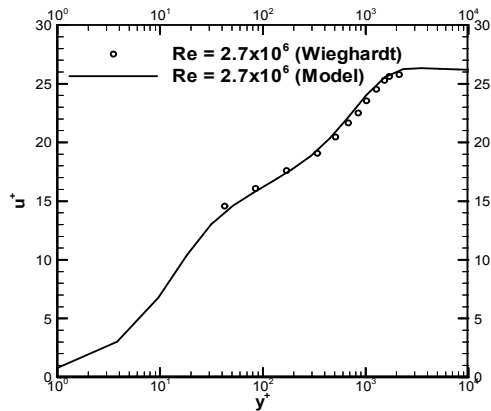


Fig. 3: Mean velocity distribution for $Re = 2.7 \times 10^6$

$$\frac{\partial(\bar{u}_i)}{\partial x_i} = 0 \quad (1)$$

$$\rho \left[\frac{\partial(\bar{u}_i)}{\partial t} + (\bar{u}_j) \frac{\partial(\bar{u}_i)}{\partial x_j} \right] = -\frac{\partial(\bar{p})}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\mu + \mu_t) \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right] \quad (2)$$

For simplicity, the normal stresses (except for the thermodynamic pressure) and body forces are neglected. $\mu_t = Re_t \mu$ is the eddy viscosity, Alammar (2013), where $Re_t = |\bar{u}_i| \rho l_i / \mu$ and l_i is a length scale given by:

$$\rho \left[\frac{\partial(l_i)}{\partial t} + (\bar{u}_j) \frac{\partial(l_i)}{\partial x_j} \right] = \frac{\partial}{\partial x_j} \mu \left(\frac{\partial l_i}{\partial x_j} \right) + C_1^2 \rho \frac{\partial}{\partial x_j} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) + C_2 \frac{\mu}{C_1^2} I_i \quad (3)$$

Hence, we have three equations for the turbulence length scale with their sources being the average strain rate, along with the molecular viscosity. C_1 is a constant length parameter perhaps attributed to the fluid. C_2 is another constant length parameter attributed to wall roughness.

NUMERICAL PROCEDURE

The governing equations were solved with a finite-volume solver using Gauss-Seidel iterative method, in conjunction with second-order schemes. Twenty thousand structured cells were used with y^+ down to 4. Boundary conditions for the length scale were similar to the velocity. No-slip boundary condition was applied at the wall and the inlet turbulence length scale was set to $7.5e^{-5}$ m. C_1 and C_2 were $5.9e^{-6}$ m and $3.5e^{-10}$ m, respectively.

RESULTS AND DISCUSSION

Distribution of friction coefficient over the flat plate is shown in Fig. 1 for Reynolds number up to 10^7 . The model prediction is compared with measurements of (Wieghardt and Tillman, 1951). The agreement is within measurement uncertainty.

The mean velocity distribution for $Re = 10^6$ is depicted in Fig. 2 along with measurements of Wieghardt and Tillman (1951). Again, the agreement is within measurement uncertainty. Y^+ in this simulation is down to 4, which explains the coarse mesh in the laminar sub-layer. The buffer layer is influenced by the strain rate in Eq. (3). The constant source term in Eq. (3) was observed to shift the velocity profile. It's proportional to the surface roughness. The mean velocity distribution for $Re = 2.7 \times 10^6$ is depicted in Fig. 3 along with measurements of Wieghardt and Tillman (1951). Again, the agreement is within measurement uncertainty.

CONCLUSION

In this study, the accuracy of a three-equation turbulence model was assessed. Using the turbulence model, average turbulent flow over a flat plate was

simulated for Reynolds number up to 10^7 . Uncertainty was approximated through model validation. Model results for mean axial velocity and friction coefficient were compared with measurements of Wieghardt and Tillman (1951) and were within measurement uncertainty. While the model was tested on incompressible plane flow, testing of the model is needed on more complex flows.

NOMENCLATURE

C : Constant, m
 I_i : Unit vector
 l_i : Turbulence length scale, m
f : Friction coefficient = $\tau_w/0.5\rho U^2$
U : Area-average velocity, m/s
 u^* : Friction velocity = $\sqrt{\tau_w/\rho}$, m/s
 \bar{u}_i : Mean velocity component, m/s
 u^+ : Normalized mean axial velocity = \bar{u}/u^*
 x_i : Cartesian coordinate, m
 y^+ : Non-dimensional wall distance = $ru^*\rho/\mu$
 μ : Fluid dynamic viscosity, Pa s
 ρ : Fluid density, kg/m³
 τ_w : Wall shear stress, Pa

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